# Time-Distance Helioseismology Hands-on Exercise

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The goal of this exercise it to develop familiarity with the basic concepts of practical time-distance helioseismology (Duvall et al. 1993): the basic features of the time-distance diagram and their physical origins, the connection between the time-distance diagram and the power spectrum, and the role of phase-speed filtering.

This exercise uses the *td_exercise* IDL widget written by J. Zhao. In order to run this code start IDL and then type “td_exercise”. There is a separate handout describing the use of this package. Many of the IDL programs in this package can be called from the command line.

We will focus on an example *SOHO*/MDI data set (see Scherrer et al. 1995, for a description of *SOHO*/MDI). The data set shows the line-of-sight Doppler velocity as measured in the Ni I 6768 Å line. This example data set is a tracked data cube. The pixels are about 1.46 Mm across and the cadence is one Dopplergram per minute.

This exercise contains five main parts. Each section contains a little bit of background information and then a list of questions. Section 1 covers some of the basic properties of the data and is to some extent a review of the material covered by Doug Braun. Section 2 covers the spatial average of the cross-covariance, i.e. the time-distance diagram. Section 3 covers phase-speed filtering. We examine travel-time maps in section 4 and section 5 provides some additional challenges.

****READ THIS PARAGRAPH: In none of these questions are we looking for “exact” numerical answers; we are interested in getting rough, eyeball, estimates of things for the sake of basic understanding. This exercise will hopefully provide starting points for your own investigations. Don’t feel any pressure to get through all of these questions; work on things to your own satisfaction.**

## 1. The Data and the Power Spectrum

Recall that the pixels in the example data set are 1.46 Mm/pixel at disk center and the cadence is one minute.

**Questions**:
1) Use the “Open File” button. Accept the default file name.

- What extent of the Sun does this data set cover ?
- How does the compare to the radius of the Sun ? Are we looking at tiny patch of the Sun ? Almost the full disk ?
- For how long, very roughly, can you track a region of this size across the visible disk ? (assume it is near the equator). This puts a limit of how long a continuous time series can be for any particular co-rotating patch of the Sun.

2) Use the “Dopplergram” button to look at movies of Dopplergrams.

- Can you see any individual wave sources ?

3) Use the “compute power spectrum” button. The units for the horizontal wavenumber axis are $dk_\mathrm{h} = 2\pi/L$ where $L$ is the length of a side of the original data cube. The units for the vertical axis is cyclic frequency in units of $df = 1/T$ where $T$ is the total time duration of the data cube.

- What are the Nyquist wavenumber and frequency for this data set ?
- Check that the f-mode ridge appears in the correct place in the power spectrum. Note that you can use “oplot” to overplot a curve on the power spectrum. Remember that for f modes $\omega^2 = gk_\mathrm{h}$, where $g = 274$ m/s is the surface gravitational acceleration. Here $\omega$ is angular frequency, which is related to the cyclic frequency $f$ by $\omega = 2\pi f$.

## 2. The Time-Distance Diagram

The time-distance diagram is the spatial average of the temporal cross-covariance. The point-to-point cross-covariance is defined as

$$C(\boldsymbol{r}_1, \boldsymbol{r}_2, t) = \sum_{t'} \phi(\boldsymbol{r}_1, t')\phi(\boldsymbol{r}_2, t' + t) \tag{1}$$

where $\phi$ is the Doppler data cube. Kosovichev & Duvall (1997) showed that the positive time lag part of the cross-covariance can be written approximately as

$$C(\boldsymbol{r}_1, \boldsymbol{r}_2, t) \approx A \exp\left\{-\frac{\delta\omega^2}{4}(t - t_\mathrm{g})\right\} \cos\left\{\omega_0(t - t_\mathrm{p})\right\} \tag{2}$$

where $t_\mathrm{p}$ and $t_\mathrm{g}$ are the phase and group travel times from $\boldsymbol{r}_1$ to $\boldsymbol{r}_2$.

**Questions**:
1) Use the “compute time-distance (TD) diagram button”. Examine the resulting plot.

- What physical processes do the different ridges correspond to ?

2) Use the “Display and Fit TD Curve” button to plot a slice through the time-distance diagram.

- what is the time-scale of the fine structure in the different ridges ? why ?
- what, roughly, is the width in time of the ridges? why ?
- At a distance of 30 Mm, not 30 pixels, what, by eye, is the first bounce group travel time ?
- Use the “Display and Fit TD Curve” button to fit the TD curve to obtain a first bounce group travel time. Is the answer reasonable ?
- What is the horizontal group speed of the waves responsible for this part of the cross-covariance ?

3) Let's check that the number we just got for the horizontal group speed is reasonable. Go back to the power spectrum. If you are somewhat familiar with IDL: to answer the next few questions it may be easier if you call some of the IDL programs from the command line, in particular the commands to read the data file and compute the power spectrum (see the instructions handout). In this way you will be able to make custom plots of the power spectrum. If you prefer, you can continue with the widget and use "oplot" to overplot things.

- Which modes have this group speed ? recall that for any particular ridge, the horizontal group speed is $v_g = \frac{\partial\omega}{\partial_{k_{\rm h}}}$.
- Is there appreciable power in any of these modes ?
- What are the lower turning points of the rays corresponding to these waves ? Use figure 1 and the local dispersion relation $\omega^2 \approx c^2k^2 = c^2(k_z^2 + k_h^2)$ for acoustic waves in the case where the acoustic cutoff frequency is negligible compared to $\omega$.
- For a polytrope, $z_t \approx \Delta/\pi$ (e.g. Bogdan 1997) where $z_t$ is the depth of the lower turning point and $\Delta$ is the first skip distance. Compute $\Delta$ for the cases we are considering. Does this make sense ?

## 3. Phase-Speed Filtering

Phase-speed filtering was originally introduced by Duvall et al. (1997) in order to improve time-distance measurements at small distances. As we showed in the previous section, the lower turning point $z_t$ can be estimated from $\omega/k_{\rm h} = c(z_t)$. Thus, in order to isolate the signal due to a particular ray path, we can filter the data to isolate a particular range in horizontal phase speed. In this exercise we will use filters of the form

$$\mathcal{F}(k,\omega) = \exp\left[-(\omega/k - v)^2/\delta v^2\right] \tag{3}$$

where $v$ is the target phase speed and $\delta v$ the width in phase-speed. An additional filter is used to remove the surface-gravity wave.

**Questions**:
1) First try the example of $v = 21$ km/s, $dv = 2$ km/s. Use "Input Parameters and Do Filtering" under "Test Phase-Speed Filtering" to filter the data. Then use "Compute and Display Filtered Power"

- Check that the filter in fact isolated the part of the power spectrum with the target phase speed. Make an estimate of the group speed along this ridge, you will need it for later.

2) Use "Compute and Display Filtered TD Diagram" to compute the cross-covariance of the filtered data

- Estimate the group speed and phase speed from the TD Diagram. Do these make sense ?

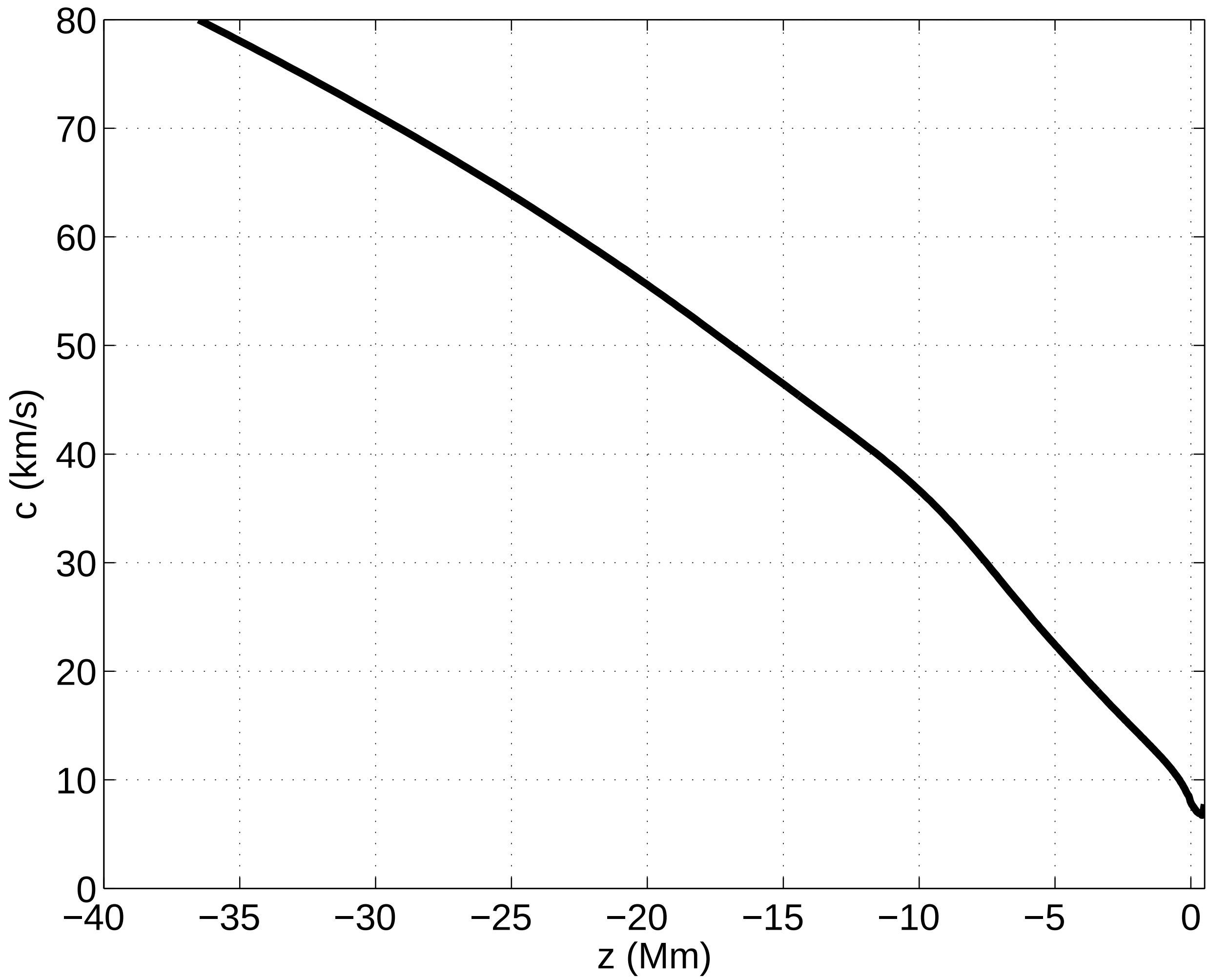


Fig. 1.— Sound speed as a function of depth, from Christensen-Dalsgaard et al. (1996)

## 4. Travel-Time Maps

1) Use the "Compute TD Map" map button. This computes maps of travel times. Examine the results using "Display TD Map".

- What is the length scale of these features ? what are these features ? (notice the change in pixel size, it has been increased by a factor two in both spatial dimensions)
- What is the relationship between the east-west, north-south, and io travel times. Sketch the basic flow geometry that you would infer from one of these features in the travel time maps.

2) Load one of the data cubes from Doug's exercise and get a travel time map (use "Open File" and then "Compute TD Map").

- What do you see ? How would you interpret these maps ? For some time-distance studies of sunspots see Duvall et al. (1996); Kosovichev et al. (2000); Jensen et al. (2001); Zhao et al. (2001); Hughes et al. (2005).

## 5. Some Challenges

Here are some additional challenges.

1) Write the temporal Fourier transform of the cross-covariance. Can you relate this to the power spectrum ? The power spectrum is

$$P(\boldsymbol{k},\omega) = |\phi(\boldsymbol{k},\omega)|^2 \tag{4}$$

2) Can you simplify this when the power spectrum is azimuthally symmetric, $P(\boldsymbol{k},\omega) = P(k,\omega)$ ? (hint: $\int \mathrm{d}\theta e^{ix\cos\theta} = 2\pi J_0(x)$ where $J_0$ is the zero-order Bessel function.)

3) Verify your calculation numerically.

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