THE SPECTROSCOPY OF LABORATORY AND ASTRONOMICAL PLASMAS

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1.1 Introduction

This course is about plasma spectroscopy. It attempts to inter-relate laboratory and astronomical plasmas and to show how laboratory studies, where the experimenter has some control over the plasma conditions, contribute to an understanding of the mechanisms giving rise to the spectra. The emphasis is on spectral intensities and the important subjects of spectral line classification and spectral line broadening are not discussed. The course starts with some theory which is mostly about atomic collisions and then goes on to some applications. In applying the theory values are required for atomic rate coefficients and some of the experimental methods of measuring these are discussed. The theoretical quantum mechanical methods of calculating these coefficients are outside the scope of the lectures.

It is necessary to start with a few definitions in plasma physics, atomic physics and spectroscopy.

1.2 Plasmas

When matter is heated to temperatures of about ten thousand degrees or greater the particle collisions are sufficiently energetic that electrons are detached and the matter becomes ionised and is called plasma. The electrostatic forces between the charged particles are such that any large scale separation of charge is resisted and plasma neutrality is generally found. Thus it is a good assumption to take the charge densities of electrons and positive ions to be equal (but opposite). The plasma may be characterised by the values of its temperature and electron density. The range of real plasmas in terms of these parameters is shown in figure 1.1. The meaning of temperature in a plasma is not a simple matter and will be discussed in the second half of this lecture. However, for the present purpose a precise definition is not necessary. The diagram shows the enormous range in both temperature and density of plasmas which exists in nature or are produced in the laboratory. Astronomical plasmas range from the plasma of interstellar space to that at the centres of massive stars. Laboratory plasmas have a more restricted range but none the less is quite impressive. An important difference between astronomical and laboratory plasmas is in their sizes and sometimes in their durations. These have important spectroscopic consequences that will be discussed later. The diagram also shows the important way in which the range of laboratory plasmas has been extended by the experiments in the field of thermo-nuclear fusion power research. Many of the problems are similar and this is an important area of interaction with astronomical spectroscopy.
1.3 Atoms

Atoms and ions play a central role in plasma spectroscopy. They are embedded as constituents of the plasma whose properties (such as density and temperature) determine in part the nature of the radiation that they emit. Thus much of the material of this course will be about atomic physics processes. In discussing these processes it is often convenient to make reference to the term schemes of atoms or ions such as that for Ne VII shown in figure 1.2. In such a diagram the various quantum levels available to the atom or ion are represented on an energy scale. The energy difference between pairs of levels is called the excitation potential from the lower to the upper level. The energy between any one of the levels and the ionisation limit is
called its ionisation potential. Energies are measured in electron volts, reciprocal centimetres (cm$^{-1}$) or sometimes in Rydbergs (1 Ry = 13.6 eV). The symbol $\chi$ is used for both excitation potential and ionisation potential, with the distinction between them being made by the symbols inside the associated brackets.

It is not possible to give a complete description of the notation used to designate the levels but this can be found in many standard text books on atomic spectrum. Briefly the notation gives the quantum numbers (principal quantum number $n$, orbital angular momentum quantum number $l$, spin angular momentum quantum number $s$) of each electron (often only one), and the total quantum numbers of the atom or ion. The designation of the latter (based on the LS-coupling approximation) is given in the following form: $m_l$. Where $m$ is the multiplicity, $2S + 1$ (where $S$ is the total spin angular momentum) and $l$ is the total orbital angular momentum quantum number given by the vector sum of the individual electron angular momenta. $^o$ designates odd parity and is left out.
for even parity. Parity is odd or even according as the sum of the individual \( l \)'s of the electrons is odd or even (not the vector sums). \( J \) is the total angular momentum quantum number for the level and is the vector sum of \( L \) and \( S \). Thus \( J \) lies in the range \( L + S > J > |L-S| \). Orbital angular momentum quantum numbers are given letters instead of numbers according to the scheme \( l = s, p, d, f, g, h \) etc for \( l = 0, 1, 2, 3, 4, 5 \), etc. and similarly for \( L \).

There will be much discussion of the population densities of quantum levels for which reference will be made to their statistical weights. This is given the symbol \( \omega \) and has the value \( 2J + 1 \) for a single \( J \)-level or \( \sum (2J + 1) = (2L + 1)(2S + 1) \) for a term (all the \( J \)-levels having the same values of \( n, L \) and \( S \)).

Spectral line radiation is emitted when an upper level makes a radiative transition to a lower level and is absorbed by the inverse process. Nearly all spectral radiation is due to electric dipole transitions for which definite selection rules apply. These are

1. the \( J \) quantum number must change by 0 or \( \pm 1 \) i.e. \( \Delta J = 0, \pm 1 \) except that transitions between 2 levels having \( J = 0 \) are not allowed.
2. there must be a change of parity in a transition i.e. transitions between levels of the same parity are not allowed.

Selection for systems in strict LS coupling

- spin change transitions are not allowed
- \( \Delta S = 0 \) is a selection rule.
- \( \Delta L = 0 \) or \( \pm 1 \) with \( S = S \) forbidden.

The LS-coupling breaks down the intermediate-coupling allows some spin change transitions which are known as intercombination transitions. Sometimes electric dipole or magnetic dipole transitions are possible but these typically have transition probabilities that are \( 10^{-5} \) times electric dipole probabilities.

Spectroscopy

Spectroscopy illustrates some of the concepts and magnitudes associated with spectroscopy. It is to be familiar with both wavelength and energy units for photons. The wavelengths (\( \approx 10^{-8} \) cm = 1/10 nm) and electron volts are used in these units it is convenient to remember that a 100 \( \mu \) photon has an energy of about one whole range of photon wavelengths from the visible to the x-ray include spectral lines of all elements and their ions. The available techniques for the study of spectra are to divide the region into spectral ranges by which reference is made to the visible, the quartz ultraviolet, the scale at the bottom of the diagram gives the temperature in the mean energies of the photon scale. In a very crude way it can indicate the magnitude of plasma temperature required to excite the spectral wavelengths.

The concept of temperature is bound up with the idea of thermodynamic equilibrium in the plasmas to be discussed are very far from thermodynamic equilibrium and the concepts are important in what follows.
2.1 Maxwellian distribution of particle velocities

Consider a system of particles contained in some way such that there is no loss or gain of particles or energy. The particles exchange energy with each other by making collisions in which the total kinetic energy and momentum are conserved - elastic collisions. Eventually the velocities will take up an equilibrium distribution where the movement of particles out of one region of velocity space is balanced by an equal movement into the same space. The details of the argument leading to the specific derivation of the velocity distribution are to be found in various texts on statistical mechanics such as Ter Haar’s "Elements of Statistical Mechanics" (1955). The outline of the argument is as follows.

Particles are identified by their position and velocity co-ordinates \((x,y,z)\) and \((u,v,w)\) respectively. A velocity distribution function \(f(u,v,w)\) \(du\ dv\ dw\) is introduced and is the number of particles per unit volume having velocities in the interval \(u\) to \(u + du\), \(v\) to \(v + dv\) and \(w\) to \(w + dw\). The equilibrium distribution function has the property that \(df/dt = 0\). The method is to identify two regions of velocity space \((u_1,v_1,w_1)\) and \((u_2,v_2,w_2)\) and consider collisions that transfer particles between them assuming conservation of momentum and kinetic energy and any reasonable law of force between them. In this way it may be shown that

\[
\frac{df}{dt} = - \int \left( f_1 f_2 - f_1 f_2' \right) a du_2 dv_2 dw_2 dw
\]
where the primes relate to the instant after the collision, $a$ is determined by the law of force between the colliding particles and $\omega$ is the angle of scatter. Thus the equilibrium condition is met if $f_{12} = f_{1}^{'}f_{2}^{'}$, no matter what the value of $a$ and $\omega$. In other words the details of the individual collisions do not influence the nature of the equilibrium distribution function. On the other hand the assumptions of the conservation of kinetic energy and momentum are important. The requirement that $f_{12} = f_{1}^{'}f_{2}^{'}$ is met by the Maxwellian velocity distribution thus:— (see figure 2.1)

$$f(u, v, w) = n \left(\frac{m}{2\pi kT}\right)^{3/2} \exp \left(-\frac{m(u^2 + v^2 + w^2)}{2kT}\right)$$

where $n$ is the total number of particles per unit volume and $\frac{3kT}{m} = \frac{c^2}{m}$ = mean value of $u^2 + v^2 + w^2$. If $m$ is the mass of an individual particle (all of which are assumed to be the same) then $T$ is defined as their kinetic temperature ($k$ is Boltzmann's constant). If there is more than one kind of particle then in equilibrium they will take up the same temperature and their mean velocities will be related inversely as the square roots of their masses. In the case of a hydrogen plasma composed of free electrons and protons the ratio of the mean (R.M.S) velocities is

$$\sqrt{\frac{m_p}{m_e}} = \frac{\sqrt{3}}{2\sqrt{2}} \cdot \sigma$$

Fig. 2.1

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i.e. in an equilibrium mixture of electrons and protons the electrons on average have velocities 43 times greater than the protons (see figure 2.2). This will be seen to have a number of important consequences in what follows.

Maxwellian distributions for electrons and protons at the same temperature.

Protons

Electrons

Speed.

Fig. 2.2

For the concept of temperature to be applicable requires that there be Maxwellian distributions of particle velocities. The extent to which this is realised in practice is one of the important matters discussed in later lectures.

2.2 Saha-Boltzmann population distribution

If account is taken of the atomic structure of the particles in the previous example so that some of the energy of the system goes to exciting and ionising the atoms then a quantum statistical argument may be used to derive the equilibrium population distribution among the excited levels. This is expressed in the Saha-Boltzmann equation.

\[
\frac{n_e n(z+1,g)}{n(z,p)} = \frac{\omega(z+1,g)}{\omega(z,p)} \frac{(2\pi m kT)^{3/2}}{e^3} \frac{e}{h^3} \exp \left( \frac{-\chi(z,p)}{kT} \right)
\]

where \( n(z+1,g) \) is the population number density in the ground level \( g \) of ions having charge \( z+1 \).

\( n(z,p) \) is the same thing for level \( p \) of ions \( z \).

\( \omega(z+1,g) \) and \( \omega(z,p) \) are the corresponding statistical weights.

\( \chi(z,p) \) is the ionisation potential of level \( p \) of ion \( z \).
By expressing the distribution in the above manner it is possible to avoid some difficulties that arise in the more usual formulation (the Saha equation) which is in terms of partition functions. The partition function is the statistical weight summed over all levels of the ion and depends on particle density and temperature.

When attention is confined to one species of ion then the Boltzmann equation is useful

\[ \frac{n(z,p)}{n(z,q)} = \frac{w(z,p)}{w(z,q)} \exp \left( -\frac{\chi(z,p-q)}{kT} \right) \]

\( \chi(z,p-q) \) is the difference in excitation potential between levels \( p \) and \( q \) of ion of charge \( z \).

2.3 Black-body radiation and the Planck function

So far in discussing equilibrium distributions account has been taken of elastic and inelastic collision processes in redistributing the available energy between the available energy states. Since charged particles and atoms are involved there is the additional possibility that energy appears in the form of radiation. To meet the basic equilibrium requirement means that radiation energy must also be contained within the system - say by having hypothetical fully reflecting walls. Under these conditions the radiation builds up to a distribution given by the Planck function and is called black-body radiation

\[ B(\nu) = \frac{2\nu^3}{c^2} \frac{1}{\exp(\nu/kT) - 1} \]

\( \nu \) is the energy of a photon of frequency \( \nu \).

2.4 Complete thermodynamic equilibrium and local thermal equilibrium

A system that meets all these statistical relations is in the complete thermodynamic equilibrium. It is a hypothetical situation although it comes close to being met in practice in the laboratory where black-body furnaces are used for the calibration of absolute radiation detectors.

In the case of plasmas discussed in these lectures the radiation escapes relatively easily but the particles are at least partially retained by such forces as magnetic or gravitational fields. Under these conditions the Planck function does not describe the radiation but it sometimes happens that the particle collision rate is sufficient to ensure that the Saha-Boltzmann and Maxwellian distributions are good descriptions. Such a plasma is said to be in Local Thermal Equilibrium - LTE. The conditions necessary for LTE to apply are discussed later in these lectures.

2.5 The principle of detailed balance

It follows from these considerations of a system in complete thermodynamic equilibrium that the rate at which particles move from one state to another is exactly balanced by the rate in the inverse direction. It may be shown that transitions from the initial to the final state through a third state must be treated separately and that only the direct processes must balance. This principle results in a number of useful relationships between the coefficients used to describe individual processes (such as collision rate coefficients) in forward and inverse directions in systems that are very far from thermodynamic equilibrium. It is frequently easier to calculate the rate of an inverse process and use the principle of detailed balance to
evaluate the required direct process.

3.1 Low density plasmas

Most of the plasmas discussed in these lectures depart significantly from any of the equilibrium configurations discussed above and since, of these LTE is the most common, they are often called non-LTE plasmas. However, it is usually better to refer to them as low density plasmas. There are two main physical reasons for departures from equilibrium viz:

(a) At densities lower than that required for LTE it generally happens that when an electron moves from a level of higher energy to one of lower it does so by giving energy to radiation. In contrast movement from lower to higher levels requires a collision process involving particles only. Such processes are not inverse in the detailed balance sense and cannot therefore give rise to LTE population distributions. The best known example is the ionisation balance in the solar corona where for hydrogen-like ions (to take the simplest case) ionisation by electron collision is balanced by radiative recombination. In general the balance is between a multitude of collisional and radiative processes to be discussed later.

(b) The second reason for departure from LTE (which may be additional to the first) is that the plasma conditions (particularly temperature and density) are changing on a time scale shorter than the relaxation times for the atomic processes establishing the population distributions.

In discussing non-LTE population distributions it is convenient to separate to some extent the establishment of the distribution among the ions (designated by their charge) from that among the bound levels of individual ions. When considering the stage of ionisation it is generally adequate to take account only of the population in the ground level and sometimes a metastable level. This point will be returned to later when the distribution among the excited levels is considered but is assumed in the meantime.

3.2 Collisional ionisation and excitation

When an electron of sufficient kinetic energy strikes an atom or ion one of the things that can happen is that a bound electron is knocked out and becomes free-ionisation. A measure of this process is the ionisation cross-section \( \sigma \) that the target ion offers the electron in units of \( \text{cm}^2 \) (sometimes \( \sigma \omega^2 - \sigma \omega \); radius of first Bohr orbit in Hydrogen = 0.529.10^{-8} \text{ cm}). Clearly such a cross-section has a threshold below which the electron energy is not sufficient to cause ionisation \( = \) ionisation potential). Above threshold ionisation cross-sections rise to a maximum at an energy about two or three times threshold and then, for electric dipole transitions, fall off to an asymptotic value proportional to \( \log E/E \). Each of the bound electrons of the atom or ion contributes to the total cross-section.

For plasmas it is generally more convenient to use ionisation rate coefficients \( S \) rather than cross-sections \( Q \). These are related by \( S = Q \omega \) where the average is taken over the velocity \( (v) \) distribution of the electrons assumed to be Maxwellian. Rate coefficients are generally expressed in \( \text{cm}^3 \text{ s}^{-1} \).

Theoretical calculations and experimental measurements of these cross-sections have been reviewed by Bely and Van Regemorter (1970), by Dolder (1969) and by Kunze (1972). A more recent review by Seaton (1975) also touches on this work. On the basis of a comprehensive survey of all available experimental measurements of cross-sections
Lotz (1967) has devised an expression to represent electron-impact ionisation cross-sections. The expression has three free parameters which have been tabulated to give cross-section values that the author claims to represent the measured values to +40-30%. The method has been extended to atoms and ions for which there are no measurements by interpolation and extrapolation of the parameters and by using theory as a guide. An extensive tabulation of values is given in a report IPFL/62 by the same author.

Figures 3.1 and 3.2 illustrate comparative values according to three authors whose work will be discussed later. The values ascribed to Jordan are based on an

![Graph illustrating the ionisation rate coefficients for O I to O VIII proposed and used by various authors.](image)

**Fig. 3.1**

Comparison of the ionisation rate coefficients for O I to O VIII proposed and used by various authors

2. ———— Summers MNRAS in press.
expression for near threshold ionisation due to Seaton (1964) based on some theoretical and some experimental measurements for positive ions and using classical theory as a guide for extrapolation. Summers' values are based exclusively on theoretical calculations using the exchange classical impact parameter approximation (Burgess 1964). Lotz values are based on his 3 parameter empirical fit formula that is very useful because so little ambiguity arises in its use. Jordan and Summers but not Lotz include autoionisation contributions to the total coefficients.

Fig. 3.2

Comparison of ionisation rate coefficients for some ions of Sulphur according to various authors.

2. ——— Summers MNRAS in press
The comparison (figures 3.1 and 3.2) shows the good agreement between Lotz and Jordan at low temperatures. The expression used by Jordan is expected to be wrong at high temperature. Summers' values are about a factor $\frac{3}{2}$ of those by Lotz but Kunze (1972) points out that more recent experimental values for ions are also about a factor $\frac{3}{2}$ by those by Lotz.

It is often adequate and certainly simpler to use an empirical formula developed by Wilson and White (unpublished) thus:

$$S = \zeta \frac{0.90 \times 10^{-5}}{\chi^{2/3}} \frac{(kT_e/\chi)^{1/2}}{(4.88 + kT_e/\chi)} \exp \left\{ -\frac{\chi}{kT_e} \right\} \text{cm}^3 \text{s}^{-1}$$

where $\chi$ is the ionisation potential in eV
$kT_e$ is the electron temperature in K and
$\zeta$ is the total number of outer electrons having the same principal quantum number.

When compared with values calculated by Lotz this simple formula gives results that agree within a factor 3.

It is convenient to consider also at this stage the related process of collisional excitation where the impinging electron causes a bound-bound transition from a level of lower energy to one of higher. In these lectures the term de-excitation will be used for the inverse process.

An useful review of electron impact excitation of positive ions has been completed recently by Seaton (1975). It updates and extends earlier related reviews by Gabriel and Jordan (1972), Bely and van Regemorter (1970), Kunze (1972).

A remarkable feature of excitation cross-sections for positive ions is their finite value at threshold. Typical of these are the two sets of curves shown in figure 3.3. Notice that for atomic hydrogen these calculations predict zero cross-section at threshold and that there is a greater difference between the atom case ($Z = 1$) and hydrogen-like helium ($Z = 2$) than between hydrogen-like helium and hydrogen-like ion of infinite charge.

Again for plasmas it is more convenient to use rate coefficients than cross-sections. The following simple expression (Seaton 1964) allows values to be calculated for dipole transitions and for $kT_e < \chi$

$$\chi(p,q) = \frac{1.17 \times 10^{-3}}{T_e} \frac{g_f(p,q)}{\chi(p,q)} \exp \left\{ -\frac{1.16 \times 10^4 \chi(p,q)}{T_e} \right\} \text{cm}^3 \text{s}^{-1}$$

$g$ is the effective mean Gaunt factor which Seaton suggests should be put equal to 0.2 to get results that agree with more sophisticated calculations to within a factor 3. Seaton's value applies to threshold excitation i.e. $kT_e < \chi$. For excitation at higher values of $(kT_e/\chi)$ reference should be made to a paper by Van Regemorter (1962) $f(p,q)$ is the absorption oscillator strength for the transition $\chi(p,q)$ is the excitation potential in eV and $T_e$ is the electron temperature in K.

Specific calculations and some measurements of greater accuracy have been done for a number of transitions and for these reference should be made to the review papers mentioned above.
Cross-sections for transitions in hydrogenic ions calculated in the Coulomb-Born approximation (Born Z = 1). $Z^\alpha Q$ is plotted against ($W/AE$), $W$ being the incident electron energy and $AE$ the transition energy difference.
(a) Is $\rightarrow$ 2s calculations by Tully (1960);
(b) Is $\rightarrow$ 2s calculations by Burgess (1961).

3.3 Radiative recombination and spontaneous radiative decay

Radiative recombination takes place when a free electron in the field of a positive ion makes a radiative transition to a bound level thus forming an ion or atom of charge one less than initially.

$$N(z) + e \rightarrow N(z-1) + hv$$
The photon carries away the excess energy of the interaction i.e. the kinetic energy of the electron before recombination plus the ionisation potential of the final bound level. Thus the recombination spectrum is characterised by discontinuities corresponding to the energies of the ionisation potentials of the relevant bound levels. These lie to the long wavelength end of the recombination continuum spectrum.

Radiative recombination unlike collisional recombination is a two body process so that the rate at which it takes place is proportional to the product of the electron and ion densities. The remaining factor in the product is the radiative recombination coefficient and is usually given the symbol $\alpha$. Its value depends on the level to which recombination takes place, i.e. on the atomic structure of the ion and the temperature of the free electrons. Since it is essentially a two body process it can be evaluated with better precision than collisional ionisation for example. The difficulty of the calculation is the determination of the atomic structure. The problem of calculating radiative recombination coefficients for the formation of hydrogen-like ions has been treated by Menzel and Pekeris (1935) and corrected by Burgess (1958). On the basis of their treatment Seaton (1959) has derived a three term expression for the Gaunt factor giving for the radiative recombination coefficient into all levels of a hydrogen-like ion of nuclear charge $Z$:

$$\alpha = 5.20 \times 10^{-14} \ Z^{1/2} \ (0.43 + \frac{1}{2} \ln \lambda + 0.47 \ \lambda^{-1/3}) \ cm^3 \ s^{-1}$$

where $\lambda = 1.58 \times 10^5 \ Z^2 / T$.

This formula has been used to calculate the values plotted in figure 3.4 which compares them with collisional ionisation coefficients from the ground levels of hydrogen-like ions.

Recombination coefficients into the individual levels of $H$-like ions are also given in the paper by Seaton.

For radiative recombination to form ions other than $H$-like ions (or hydrogen) the calculation of rate coefficients requires account to be taken of the more complex structure of the ions involved. However by assuming a simple Coulomb field and using non-integral effective quantum numbers results of good accuracy are obtained. This method is discussed by Burgess and Seaton (1960).

The inverse of radiative recombination is photo-ionisation and the rate at which it takes place is proportional to the radiation flux and in order that it balances radiative recombination to give a Saha-Boltzmann population distribution (principle of detailed balance) the flux density and spectrum must be that of a black-body at the temperature of the electrons. For the plasmas discussed here this is never the case.

The spectral lines whose study is the subject of this lecture series arise from the spontaneous radiative decay from an upper to a lower bound level. The coefficient used to describe the rate of this process is the spontaneous transition probability. It is given the symbol $A(p,q)$ and measured in $s^{-1}$. Its calculation and measurement for many atoms and ions is an important branch of atomic physics. This work is continually kept under review by a group at the Bureau of Standards in Washington. The latest addition to their bibliography is by Fuhr and Wiese (1973) and their main compilations are by Wiese, Smith and Glennon (1966) and Wiese, Smith and Miles (1969). From a study of systematic trends among known values Wiese and Weiss (1968) have suggested methods of extrapolation from existing data to estimate values that are not otherwise available. A useful generally applicable method of estimating transition probabilities for electron dipole transitions is due to Bates and Damgaard (1949).
Fig. 3.4

The basis of the method is similar to that of Burgess and Seaton for radiative recombination. The latter was developed from the former.

Oscillator strengths, f-values and radiative lifetimes are directly related to spontaneous transition probabilities. Also related through the principle of detailed balance are the two coefficients for induced transitions: photo-excitation $B(q,p)$ and stimulated emission $B(p,q)$ defined here in terms of radiation density. These are all inter-related as follows

$$\omega(p) A(p,q) = \omega(p) \frac{8\pi h v(p,q)^3}{c^3} B(p,q)$$

$$= \omega(q) \frac{8\pi h x(p,q)^3}{c^3} B(q,p)$$

$$= \frac{8\pi^2 e^2 v(p,q)^2}{mc^3} \omega(q) f(q,p)$$

where $\omega(p)$ is the statistical weight of level $p$.
$v(p,q)$ is the frequency of the radiation.
$f(q,p)$ is the absorption f-value or oscillator strength.

$$\omega(q) f(q,p) = \omega(p) f(p,q)$$

where $f(q,p)$ is the emission f-value or oscillator strength.

The radiative lifetime $\tau$ is the reciprocal of the sum of all the radiation coefficients affecting the level in question. Usually

$$\tau(p) = \frac{1}{\sum_{q} A(p,q)}$$

2.4 Collisional-radiative recombination and ionisation

In calculating the radiative recombination coefficient it was tacitly assumed that the total coefficient for an ion was the sum of the coefficients into each of the quantum levels separately. On the other hand only ground level ionisation was considered. The basis of these assumptions will now be considered and this will lead to a discussion of the collisional-radiative model first described in the following series of papers. Bates, Kingston and McWhirter (1962), Bates and Kingston (1963) and McWhirter and Hearn (1963).

These authors considered the atomic interactions between the particles of a system composed of hydrogen-like ions of charge $(Z-1)$, electrons and fully stripped nuclei of charge $Z$. The processes taken account of, and which contribute to the interaction, are as follows:

(a) collisional excitation (and de-excitation) from (or into) any bound level $q$ to (or from) an upper level $p$

$$N(Z-1,q) + e \rightarrow N(Z-1,p) + e$$

Rate coefficients $X(q,p), X(p,q)$
(b) collisional ionisation from, and three body recombination into, any level \( p \)

\[ N(Z-1, p) + e \rightleftharpoons N(Z) + 2e \quad S(p), B(p) \]

(c) spontaneous radiative decay from any level \( p \) into a lower level \( q \)

\[ N(Z-1, p) + \gamma N(Z-1, q) \quad \text{hv} \quad A(p, q) \]

(d) radiative recombination into level \( p \)

\[ N(Z) + e + \gamma N(Z-1, p) + \text{hv} \quad \alpha(p) \]

These processes are illustrated in figure 3.5.

\[ \text{Fig. 3.5} \]

Processes included in collisional-radiative calculation.
It is assumed that the density of radiation is small (including that arising from processes (c) and (d) so that photo-ionisation and photo-excitation may be neglected. Note however that the second paper by Bates, Kingston and McWhirter mentioned above takes account of the plasma being optically thick. In order to limit the problem to a manageable size the possible presence of He-like and other ions has been ignored.

The time rate of change of the population density \( n(p) \) of level \( p \) may be expressed thus:

\[
\frac{dn(p)}{dt} = - n(p) \left\{ n_e \left[ S(p) + \sum_{q \neq p} X(p,q) \right] + \sum_{q < p} A(p,q) \right\} \\
+ n_e \sum_{q \neq p} n(q) X(q,p) + n_e n(q) A(q,p) \\
+ n_e n(Z) \alpha(p) + n_e^2 n(Z) \beta(p)
\]

The level \( p \) or \( q \) is identified by its principal quantum number where the relative populations of the sub-levels, identified by their inner quantum numbers, are assumed to be statistical. In principle \( p \) and \( q \) take all values between 1 and infinity leading to an infinite set of equations and containing sums to infinity. Nevertheless it has been possible to obtain a solution because of the following two properties of the physical system.

(a) The reciprocal lifetime of a level \( p \) is the sum of radiative and collisional components. For all levels but the ground level the lifetime is therefore shorter than or equal to the radiative lifetime. For hydrogen the latter are about \( 10^7 \) secs and for hydrogen-like ions are shorter (\( \tau \) proportional to \( Z^{-4} \)). The lifetime of the ground level which cannot decay radiatively, is determined by collisions and depends particularly, therefore, on the electron density. In the solar atmosphere ground level lifetimes are between \( 10^2 \) and \( 10^4 \) secs and even in most laboratory plasmas it is a good approximation to say that excited levels lifetimes are very much shorter than for the ground level. This leads to the quasi-steady-state approximation where \( \frac{dn(p)}{dt} \) is equated to zero except for \( p = 1 \). It may be shown that this leads to valid solutions for times greater than about \( 10^{-7}/Z^4 \) secs.

(b) Inspection of the numerical values of the various coefficients shows the following trends: the radiative coefficients (\( A \) and \( \alpha \)) decrease with increasing \( p \) whereas the collisional coefficients (\( X, S \) and \( \beta \)) increase. Thus, to any desired accuracy there is always some level \( p \) above which it is a good approximation to neglect radiative processes. Since each collision process is accompanied by its inverse the principle of detailed balance may be applied and leads to the populations of these levels being adequately given by the Saha-Boltzmann equation. This connects the population densities of these levels to the continuum of free electrons and population density of stripped ions \( n(Z) \).

With these two approximations the infinite set of differential equations reduces to a finite set of simultaneous equations and one differential equation. Thus a solution is possible by numerical methods and the results may be expressed in the following simplified form:

\[
n(p) = n_0(p) + n_1(p) n(1)
\]
\[ \frac{dn(l)}{dt} = -n_e n(l) S_{CR} + n_e n(Z) \alpha_{CR} \]

The coefficients \( n_0(p) \) and \( n_1(p) \) have been tabulated and allow the calculation of the population density of level \( p \) under circumstances where the plasma is undergoing dynamic ionisation or recombination. \( S_{CR} \) and \( \alpha_{CR} \) are the collisional-radiative ionisation and recombination coefficients respectively. At very low densities they reduce to the collisional ionisation coefficient for the ground level and the radiation recombination coefficient summed over all levels.

The results of these calculations can be presented in a more general way by adopting reduced temperatures \( \Theta \) and densities \( n_e \). These are related to their actual equivalents by the following \( Z \) scaling laws

\[ \Theta = \frac{T}{Z} \]

\[ n_e = \frac{n_e}{Z} \]

The population density of an excited level \( p \) may also be presented in reduced form thus

\[ n(p) = \frac{X n(p)}{Z} \]

Where \( X = n_e/n(Z) \) and for plasma neutrality is less than or equal to \( Z \).

The table in figure 3.6 gives values of the excited state lifetimes and shows that only in very extreme conditions of high density and temperature does the lifetime of the ground level approach 10% of that of the excited levels. However the table does illustrate the limits of validity of the quasi-steady-state approximation.

<table>
<thead>
<tr>
<th>Relaxation Time Constants ( Z' \tau(p) ) in seconds</th>
<th>( \Theta \times 10^{-2} )</th>
<th>( \Theta )</th>
<th>( p = 1 )</th>
<th>( p = 2 )</th>
<th>( p = 3 )</th>
<th>( p = 15 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>( 10^8 )</td>
<td>3.1 \times 10^{-11}</td>
<td>2.1 \times 10^{-8}</td>
<td>1.0 \times 10^{-8}</td>
<td>1.0 \times 10^{-8}</td>
<td>1.0 \times 10^{-8}</td>
</tr>
<tr>
<td>4</td>
<td>( 10^{10} )</td>
<td>3.1 \times 10^{-9}</td>
<td>1.0 \times 10^{-8}</td>
<td>1.0 \times 10^{-8}</td>
<td>1.0 \times 10^{-8}</td>
<td>1.0 \times 10^{-8}</td>
</tr>
<tr>
<td>16</td>
<td>( 10^8 )</td>
<td>1.2 \times 10^{-12}</td>
<td>1.0 \times 10^{-12}</td>
<td>1.0 \times 10^{-12}</td>
<td>1.0 \times 10^{-12}</td>
<td>1.0 \times 10^{-12}</td>
</tr>
<tr>
<td>16</td>
<td>( 10^{10} )</td>
<td>1.2 \times 10^{-9}</td>
<td>1.0 \times 10^{-9}</td>
<td>1.0 \times 10^{-9}</td>
<td>1.0 \times 10^{-9}</td>
<td>1.0 \times 10^{-9}</td>
</tr>
<tr>
<td>64</td>
<td>( 10^8 )</td>
<td>5.4 \times 10^{-14}</td>
<td>1.0 \times 10^{-14}</td>
<td>1.0 \times 10^{-14}</td>
<td>1.0 \times 10^{-14}</td>
<td>1.0 \times 10^{-14}</td>
</tr>
<tr>
<td>64</td>
<td>( 10^{10} )</td>
<td>5.4 \times 10^{-11}</td>
<td>1.0 \times 10^{-11}</td>
<td>1.0 \times 10^{-11}</td>
<td>1.0 \times 10^{-11}</td>
<td>1.0 \times 10^{-11}</td>
</tr>
<tr>
<td>256</td>
<td>( 10^8 )</td>
<td>1.3 \times 10^{-12}</td>
<td>1.0 \times 10^{-12}</td>
<td>1.0 \times 10^{-12}</td>
<td>1.0 \times 10^{-12}</td>
<td>1.0 \times 10^{-12}</td>
</tr>
<tr>
<td>256</td>
<td>( 10^{10} )</td>
<td>1.3 \times 10^{-9}</td>
<td>1.0 \times 10^{-9}</td>
<td>1.0 \times 10^{-9}</td>
<td>1.0 \times 10^{-9}</td>
<td>1.0 \times 10^{-9}</td>
</tr>
<tr>
<td>256</td>
<td>( 10^{12} )</td>
<td>1.3 \times 10^{-6}</td>
<td>1.0 \times 10^{-6}</td>
<td>1.0 \times 10^{-6}</td>
<td>1.0 \times 10^{-6}</td>
<td>1.0 \times 10^{-6}</td>
</tr>
</tbody>
</table>

The indices give the power of ten by which the entries in the time constant columns must be multiplied.

Fig. 3.6

Table illustrating the range of the validity of the quasi-steady-state approximation that requires \( \tau(p) < \tau(1) \). This table shows that the approximation is least well met at high temperature and high density.
The validity of second approximation is shown in figure 3.7 which illustrates the way in which radiative processes give way to collisions as \( p \) increases. The Collision Limit marked in this figure is defined as the lowest level from which an electron has a greater probability of making a transition upwards as compared with downwards (excluding \( p = 1 \)). Above the collision limit collisions dominate. As the electron density increases collisions become more dominant at lower and lower levels as shown in figure 3.8 where at a density of \( n_e = 10^{16} \) cm\(^{-3} \) collisions exceed neutrals.

The relative magnitudes of the five processes populating the first thirteen principal quantum levels for a plasma with \( \Theta = 64000^\circ \text{K} \) and \( \eta(\zeta) = 10^8 \) cm\(^{-3} \).

![Graph showing relative magnitudes of processes](image)

Fig. 3.7

The relative magnitudes of the four processes which populate the ground level at \( \Theta = 32000^\circ \text{K} \).

![Graph showing relative magnitudes of processes](image)

Fig. 3.8

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radiative processes in populating the ground level. This value is relative independent of temperature. The approach of the excited level population densities to their Saha-Boltzmann values is shown in figure 3.9.

Figure 3.9 gives values for the collisional radiative recombination coefficient and shows how at higher electron density the density independent radiative coefficient increases as the collisional processes take over. At high density the coefficient is proportional to density in the same way as a three-body recombination coefficient.

The dependence of collisional-radiative ionisation coefficients on density is much less dramatic and resembles a step function. The step corresponds to the onset of stepwise excitation as the collision limit comes down to \( p = 2 \). Values are plotted in figures 3.11.

3.5 Dielectronic recombination

If an electron collides with an ion at an energy just below the threshold for excitation it is possible for this excitation to take place but because the impinging electron does not then have the energy to escape from the Coulomb field it becomes captured in an outer orbital. This is the first step of dielectronic recombination. It has already been noted that excitation cross-sections for positive ions at threshold are finite so their extrapolation below threshold means that there is a relatively high probability of capture. The inverse process to capture is autoionisation where the ion makes a radiationless transition to its ground level and an electron is ejected. Competing with autoionisation is the so-called stabilisation process that makes dielectronic recombination effective. Stabilisation takes place when the excited electron (the lower of the two) makes a spontaneous radiative decay thus
Reducing the total internal energy of the recombined ion to below its ionisation potential*. Finally the captured electron cascades down to a lower level.

The dielectronic recombination process has been worked out in detail by Burgess (1965 and 1964). The quantitative calculation of the rate of dielectronic recombination is presented as follows. Dielectronic capture and auto-ionisation may be

*In some circumstances radiative decay does not take the ion below its ionisation potential and gives rise to the possibility of secondary autoionisation (Blaha, 1972).
represented thus:

\[ \text{N}(z,i) + e \leftrightarrow \text{N}(z-1, n\ell) \]

The symbols \( C_d \) and \( A_a \) are used to represent the coefficients for these two processes such that their rates are respectively \( n_e n(z,i) C_d \) and \( n(a-1,j,n\ell) A_a \). Since these are inverse processes their rate coefficients are related by detailed balance thus

\[ \frac{A_a}{C_d} = S(T) = \frac{2(2\pi mkT)^{3/2}}{h^3} \frac{\omega(z,i)}{\omega(z-1,j,n\ell)} e^{E_0/kT} \]

where \( S(T) \) is calculated using the Saha-Boltzmann equation.

---

**Fig. 3.11**

The ratio of the collisional-radiative ionisation coefficient to its value at the limit of low density at various values of the reduced temperature \( T_e/Z^2 \)
The stabilisation process may be represented thus:

\[ N(z-1, j, n\ell) + N(z-1, i, n\ell) + h\nu (i, j) \]

and its rate coefficient is given by the symbol \( A_r \).

Thus the recombination rate is the rate of dielectronic capture multiplied by the ratio of stabilisation on the total of auto-ionisation plus stabilisation thus

\[ \eta_e n(z, i) \alpha_d = \sum \eta_e n(z, i) \frac{A_r}{A_a + A_r} \]

The summation sign indicates the need to sum over all the possible alternative routes for the recombination process to take. Thus there may be a number of different levels \( j \) available for excitation by the impinging electron. Also there are many alternative levels \( n\ell \) into which it may be captured. Introducing the Saha-Boltzmann ratio \( S(T) \) and cancelling the particle densities gives

\[ \alpha_d = \sum \frac{1}{S(T)} \frac{A_a A_r}{A_a + A_r} \]

\[ \alpha = \frac{h^3}{(2\pi m k T)^{3/2}} \left( \sum \eta_e n(z, i) \frac{A_a A_r}{A_a + A_r} \right) \]

The problem now resolves itself into the determination of the terms where \( A_a > A_r \) when \( \frac{A_a}{A_a + A_r} \approx 1 \) and those where \( A_a < A_r \) when \( \frac{A_a}{A_a + A_r} \approx 0 \) and the process may be neglected. Thus paradoxically it is the levels that auto-ionise most strongly that contribute most to dielectronic recombination.

An approximate calculation of the dielectronic recombination coefficient gives the following expression

\[ \alpha_d = \frac{8.2 \times 10^{-4}}{T^3/2} \left( \frac{z(z+1)}{z^2 + 13.4} \right)^2 \exp \left( -\frac{E_o}{kT_e} \right) \]

\[ \sum f(i, j) E_o^{1/2} \]

\[ j \left( 1 + 0.105x + 0.015x^2 \right) \text{cm}^3 \text{s}^{-1} \]

where \( x = 0.0735 (E_o/z+1) \)

\( E_o \) is the excitation potential \( i-j \) in eV

\( f \) is the absorption oscillator strength for \( i\rightarrow j \)

\( z \) is the charge on the recombining ion.

Detailed calculations have been carried out by H P Summers (1974) for all ions of elements from hydrogen to argon. Further data is available in an Appleton Laboratory Report Number IM 367.
When dielectronic recombination and collisional-radiative recombination are coupled together as was done by Summers, in the work mentioned above, quite complicated effects result as shown in figure 3.12, coefficients are not additive but the full set of equations describing the processes affecting the populations of each individual level must be treated in detail. The effect of density on the collisional-dielectronic recombination coefficients may be seen from the curves in figure 3.12.

![Graph showing the temperature and density dependence of the recombination coefficients of some ions of oxygen.](Summers, H.P., Mon. Not. R. A. S. 169 pp 667-680 (1974))

The numbers on the curves are electron densities in cm⁻³

- Recombination to form Be-like ions
- Recombination to form He-like ions
- Recombination to form H-like ions

**Fig. 3.12**

Dielectronic recombination does not take place for H-like ions. The curves show only a slight influence of density due to collisional-radiative recombination where the \( n_e = 10^{16} \text{ cm}^{-3} \) curve lies slightly above the others at the low temperature end. The same effect is seen for He-like ions but now dielectronic recombination has enhanced the total between \( 10^5 \) and \( 10^7 \) K. However, an increase of density reduces the dielectronic contribution as can be seen in the figure. The physical reason for this is that the captured electron in the outer orbital (nλ) suffers collisional ionisation before it can cascade down. The Be-like curves show the same effects more dramatically.
3.6 Auto-ionisation

When an atom or ion suffers a collision it is possible for an inner electron to be raised to an excited level which may lie above the ionisation potential of the unperturbed atom or ion. An atom or ion excited in this way will interact with the adjacent continuum and autoionisation may take place whereby the electron becomes free leaving the ion in its ground level. As in the case of dielectronic recombination the excited ion may decay radiatively when no change in stage of ionisation takes place. Thus the rate at which autoionisation takes place may be expressed thus

$$ S_a = \sum_p \frac{A_a(p)}{A_a(p) + A_r(p)} X(g,p) $$

where $p$ is a level from which autoionisation can take place. $A_a(p)$ and $A_r(p)$ are the probabilities of autoionisation and radiative decay from level $p$, and $X(g,p)$ is the rate of collisional excitation of level $p$ from the ground level of the atom or ion. For the lower levels

$$ A_a(p) \gg A_r(p) $$

and the ratio of the probabilities becomes unity. The autoionisation rates should be added to the direct ionisation rates to give the total rates.

Autoionisation has been studied by Goldberg, Dupree and Allen (1965) and Bely (1968). Summers (1974) has used the same methods to obtain numerical values for some of the most important cases. These are illustrated in figure 3.13 which shows total ionisation rates over the direct rate only. In some circumstances it may be seen that autoionisation increases the direct rate by almost ten times.

**Autoionization corrections. Tabulation of $S_{ed}$ (including autoionization)/$S_{ed}$ (omitting autoionization). ($z-1$ is the ion charge)**

<table>
<thead>
<tr>
<th>$Te/e^2$</th>
<th>$2s^22p$ ionization</th>
<th>$2p^63s$ ionization</th>
</tr>
</thead>
<tbody>
<tr>
<td>$2.0^4$</td>
<td>1.01 3.49 2.33 1.92</td>
<td>1.00 1.00 1.00 1.00</td>
</tr>
<tr>
<td>$5.0^4$</td>
<td>1.12 1.57 1.40 1.00</td>
<td>1.00 1.11 2.58 1.80</td>
</tr>
<tr>
<td>$1.0^4$</td>
<td>1.31 1.36 1.25 1.00</td>
<td>1.26 2.57 6.89 4.74</td>
</tr>
<tr>
<td>$2.0^4$</td>
<td>1.19 1.32 1.18 1.12</td>
<td>1.02 2.80 5.81 9.25</td>
</tr>
<tr>
<td>$5.0^4$</td>
<td>1.12 1.16 1.06 1.06</td>
<td>1.32 4.95 6.80 7.67</td>
</tr>
<tr>
<td>$1.0^4$</td>
<td>1.09 1.10 1.05 1.03</td>
<td>1.09 4.81 5.29 5.33</td>
</tr>
<tr>
<td>$2.0^4$</td>
<td>1.06 1.07 1.04 1.03</td>
<td>2.06 3.97 4.15 4.07</td>
</tr>
<tr>
<td>$5.0^4$</td>
<td>1.05 1.06 1.03 1.02</td>
<td>1.97 3.15 3.15 3.02</td>
</tr>
</tbody>
</table>

Fig. 3.13

4.1 The steady-state ionisation balance

In order to calculate the state of ionisation of a plasma that does not meet the requirements of LTE it is necessary to take account of individual ionisation and recombination mechanisms. In this chapter consideration is given to steady-state plasmas which have been sufficiently constant with time that their state of ionisation has settled to a steady value. Such conditions are seldom found in the laboratory for low density plasmas but the theory is widely assumed for astronomical plasmas such as the solar corona. Its specific application to the solar corona is discussed later.

In treating this problem all possible ionising and recombination processes have to be included but in practice only the processes discussed in the previous chapter are included. The dominant processes are collisional ionisation and radiative recombination but di-electronic recombination and auto-ionisation are also very important. At larger density collisional radiative processes take over.

In its formal treatment the problems required the solution of a set of equations as follows.

\[
\frac{dn(z)}{dt} = n_e \left\{ - n(z) S(z) + n(z-1) S(z-1) - n(z) a(z) + n(z+1) a(z+1) \right\}
\]

where \( z = 0 \) to \( Z \) where \( Z \) is the nuclear charge.

In the steady-state \( \frac{dn(z)}{dt} = 0 \) for all values of \( z \) and the equations reduce to

\[
\frac{n(z)}{n(z+1)} = \frac{a(z+1)}{S(z)}
\]

\( S(z) \) and \( a(z+1) \) are the ionisation and recombination coefficients for ions of initial charge \( z \) and \( z+1 \). They include all the appropriate components such as collisional- and auto-ionisation and radiative and di-electronic recombination.

It is generally assumed that the abundances of the element in question remains constant for all temperatures and densities, i.e.

\[
\frac{n(El)}{n(H)} = \sum \frac{n(z)}{z n(H)}
\]

where \( n(H) \) is the hydrogen atom-and ion number density. (In both astronomical and laboratory fusion plasmas the main constituent is hydrogen with small additions of heavier elements.)

Calculations have been done for the most abundant elements by a number of authors. The most widely used is that by Jordan (1969) and includes all the significant processes mentioned above. A more recent calculation by Summers treats the recombination process in a more sophisticated manner but uses ionisation rate coefficients that are approximately \( \frac{1}{2} \) of the values more widely accepted on theoretical grounds (see previous discussion of collisional ionisation). It should be noted however that the experimental evidence such as it is favours the lower values. For ions of low charge the discrepancy between Jordan's and Summers' results is small but it increases with charge and for iron there are large differences. At this time such differences
should be treated as the approximate range of uncertainty in the calculations. The results of these two sets of calculations is illustrated in figure 4.1 for neon.

The fractional abundances of the ions of neon \( \frac{n(z)}{n(\text{El})} \) as functions of temperature.


Fig. 4.1

These illustrate the relatively large differences between the results for Li-like ions (Ne VIII) but good agreement for H-like (Ne X). Figure 4.1 also illustrates a number of other features common to both calculations. Thus the discontinuity at Ne VIII of the otherwise regular progression of the curves is due to the extra difficulty in ionising a Ne-like ion which starts a new shell. The shoulder on the Li-like ion population to the high temperature side of the peak arises because of the relatively high population of Ne-like ions at these temperatures and available therefore for recombination to form Li-like ions. This shoulder has important consequences for the sun's spectrum.

When these calculations are applied specifically to the solar atmosphere account has to be taken of a number of additional effects. The main dependence of the ionisation and recombination coefficients is on temperature. However, there is a smaller dependence in density but Summers' calculations are for constant values of density (Fig. 4.1 is for \( n_e = 10^8 \text{ cm}^{-3} \)). Jordan's on the other hand includes the density variations in the solar atmosphere. For atoms and singly charged ions the photospheric radiation has a significant effect in causing photoionisation and photoexcitation. Summers' calculations do not include this but Jordan's do.
A remarkable feature of the solar atmosphere is the steep temperature gradient in the transition zone between the top of the chromosphere at a temperature less than $10^{4}$K and the corona at a temperature of more than $10^{5}$K. The presence of this gradient means that the atmosphere is stratified into layers each having a characteristic ion species. The resulting steep concentration gradients mean that diffusion is important including thermal diffusion.

At this stage the discussion leaves the solar atmosphere and returns to a consideration of the theoretical problem of ionisation balance in a homogeneous plasma of higher density. The theory of collisional-radiative recombination and ionisation shows how as the density increases the state of ionisation and population density of excited levels approaches the description given by the Saha-Boltzmann equation. It has already been shown that this happens when the density rises to a value where collisions completely dominate over radiative decay processes. The plasma conditions required for this to happen have been calculated and give rise to criteria for LTE to be a valid description of the plasma. This problem has been discussed by Griem (1963), Wilson (1962) and McWhirter (1968). The latter defines the criterion as the density at which the rate of transitions between all pairs of levels caused by collisions is ten times the corresponding radiative rates i.e

$$n_e X(p,q) > 10 A(p,q)$$

Using the expression given earlier for the excitation rate coefficient and the standard relation between f-values and transition probabilities this reduces to

$$n_e > 1.6 \times 10^{12} T_e^{1/2} X(p,q)^3 \text{ cm}^{-3}$$

$T_e$: electron temperature in K,
$X(p,q)$: excitation potential in eV.

For a given ion to be in LTE this inequality must be met by all pairs of levels of the ion. Since it is least easily met by the pair having the greatest excitation potential it is sufficient if the criterion is satisfied by this largest value of $X(p,q)$ for the ion.

When this criterion is compared with the results of more detailed collisional-radiative calculation for 10% departure from LTE very good agreement is found. The criteria developed by Griem and Wilson (references above) give densities that agree within a factor 2 with the above despite differences in their derivation.

McWhirter (loc. cit.) also discusses the rate of approach of the population densities of the ions of a plasma to LTE following a change in the plasma conditions. In these lectures this question is more conveniently considered later.

5.1 Equilibrium times for electrons and positive ions

Before considering the problem of the time dependence of ionisation and recombination it is convenient to consider the times for equipartition of kinetic energy between the constituent particles of a plasma. In this case it is the rate of elastic collisions that matter. The problem is discussed by Spitzer (1956). He expresses the time required for the distribution of kinetic energies of identical particles to approach a Maxwellian distribution as the self-collision time $t_c$ thus
\[ t_c = \frac{m^2}{5.71 \pi n e^4 Z^4 \ln A} \left( 3kT \right)^{3/2} s \]

where \( m \) is the mass of the particles
\( T \) is their temperature
\( n \) is their number density
\( e \) the electronic charge
\( Z \) the ionic charge,

and \( \ln A \) is defined by Spitzer and depends on the Debye shielding distance \((\ln A \approx 20)\).

Values based on this expression are plotted in figure 5.1 for electrons and protons. This shows that electrons come into equipartition with themselves more quickly than do protons in the ratio of the square root of their mass ratio \((\approx 50 \text{ times faster})\). This is another consequence of the much higher velocities of electrons compared with ions.

Spitzer also gives an expression for the equipartition time for test particles in a field of particles of another kind, viz.

\[ t_{eq} = \frac{3mm_l k^{3/2}}{8(2\pi)^{1/2} n_1 Z_2^2 Z_1^2 e^4 \ln A} \left( \frac{T}{m} + \frac{T_1}{m_1} \right)^{3/2} s. \]

The symbols have the same meaning as before with the subscript \( 1 \) referring to the field particles. This expression was used to calculate the values of the equipartition time between electrons and protons plotted in figure 5.1.

### 6.1 Time-dependent ionisation and recombination

Most laboratory plasmas are produced by pulsed electrical discharges and are therefore of short time duration. In these circumstances it turns out to be necessary in predicting the stage of ionisation of the plasma to solve the full set of differential equations describing the rates of ionisation and recombination

\[ \frac{dn(z)}{dt} = n_e \left( n(z) S(z) + n(z-1) S(z-1) - n(z) a(z) + n(z+1) a(z+1) \right) \]

\( z \) is the charge of the ion and runs from \( z = 0 \) to \( z = Z \) (the nuclear charge). \( S(z) \) and \( a(z) \) are the appropriate ionisation and recombination coefficients that have already been discussed. For laboratory plasmas of the type discussed here it turns out that the rates of the atomic processes are not fast enough for the plasma to approach a steady-state balance within its lifetime. Recent satellite observations of the sun show that there are time variations of intensity in periods short compared with ionisation or recombination times leading to the need to use the full time rate equations here too. This possibility is only rarely considered in the astronomical literature but is fundamental to most treatments of pulsed laboratory plasmas.

Satisfactory analytical solutions of the set of differential equations are not available but the problem can be solved by numerical methods using computers. Since in general density \( n_e \) and temperature \( T \) are varying with time the ionisation and recombination coefficients also vary with time. Thus a numerical solution is the
only satisfactory approach.

To illustrate the nature of the solution and indicate the magnitude of the atomic times consider the solution for a hypothetical plasma of constant temperature and density into which is introduced, at time zero, a small sample of some element (say nitrogen). Figure 6.1 shows how the ionisation takes place as successive electrons are stripped off. It is clear that recombination plays a negligible role until the system approaches the steady-state. From this type of calculation it is possible to predict the time required by such a plasma to reach the steady-state of ionisation balance. An approximate estimate of this may be made as follows. It may be seen from

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The time dependent ionisation of neon at $T = 5 \times 10^5 K$ and $n_e = 10^{14}$ cm$^{-3}$.

Figure 6.1

Figure 3.4 that the values of the coefficients $\alpha$ and $S$ at which corresponding values are equal lie along a band of values in the neighbourhood of $S = \alpha = 10^{-12}$ cm$^3$ s$^{-1}$. At these temperatures neighbouring ions have equal populations and are the dominant ions in the plasma. Thus the time for the last ion to be produced is given by

$$t_{\text{ion}} = \frac{1}{n_e 10^{-12}} \text{ s}.$$  

And this is about equal to the time required for the plasma to reach its steady-state of ionisation.

Time resolved observations of spectral intensities from pulsed laboratory plasmas can provide reassuring evidence of the essential applicability of the type of analysis of ionisation given above. Thus figure 6.2 shows traces of the intensities of lines showing the development of ionisation and then recombination during the discharge cycle. The full understanding of these observations requires also some consideration of the mechanism of excitation which will be discussed more fully below. Processes of plasma loss or in-flow may also modify the data. These are discussed later. Usually the spectrum of a pulsed plasma is dominated by ionisation only (such as a theta pinch) or recombination only (such as a laser-produced plasma).

7.1 Excitation and spectral line intensities

The practical measurement of a spectral line intensity either in the laboratory or from an astronomical object requires the setting up of a spectrometer with possibly a light collector (telescope) and a detector. The signal measured by this system depends not only on the sensitivity of this system but also the volume of the source viewed by, and the acceptance cone of, the system. It also depends on the extent to
Observed waveforms of the output of a photomultiplier when set to detect the line indicated (each the average of 5 shots).

which the radiation is attenuated by the intervening space including re-absorption by the source itself. This last point of radiation trapping is discussed later in a brief treatment of optically thick plasmas but the other more practical questions are outside the scope of these lectures.

The remaining question of the atomic excitation mechanisms is discussed here and, taking account of these aspects only, the intensity is a spectral line is given by

\[ I = \frac{1}{4\pi} n(p) A(p, q) \, \text{erg cm}^{-3} \text{s}^{-1} \text{sterad}^{-1} \]

Thus the atomic physics problem reduces to the calculation of the population density of the upper excited level \( p \). For plasmas in LTE this is relatively simple and requires the application of the Saha-Boltzmann equation. Such plasmas are liable also to be optically thick and it is in taking account of this that the major problem of interpretation of spectral intensities lie for LTE plasmas. The case of low density optically thin plasmas is treated in this chapter.

Before discussing specific kinds of ions some mention should be made of continuum radiation. This arises from free-bound and free-free transitions of electrons in the fields of ions and gives rise to recombination radiation and bremsstrahlung respectively. For low density plasmas it is relatively weak compared with spectral lines. It will not be discussed further in this chapter but is mentioned later in
connection with other aspects.

7.2 Hydrogen and Hydrogen-like ions

The excitation of hydrogen and hydrogen-like ions has already been discussed under the heading of collisional-radiative recombination and ionisation. However, it is useful to isolate some of the principal features of that somewhat complex treatment in order to build up a physical picture of the excitation mechanism. In the treatment of collisional-radiative processes the concept of the quasi-steady-state was introduced to obtain a solution for times longer than the radiative plus collisional relaxation times of the excited levels. Some modification of this approach may be necessary for ions having metastable levels but this will be left out of account at the moment and returned to later.

The population density of an excited level \( p \) is given by the expression quoted earlier

\[
n(p) = n(l) n_{l}(p) + n_{o}(p).
\]

The coefficients \( n_{l}(p) \) and \( n_{o}(p) \) are tabulated in the papers quoted previously for a number of values of \( p \) and for ranges of electron density and temperature.

For low levels \( p \) and small values of density the first term is very much greater than the other. Physically this corresponds to the situation where \( p \) is excited from the ground level and decays again radiatively with negligible interference by other processes. Thus

\[
\frac{dn(p)}{dt} = n_{e} n(l) X(l,p) - n(p) \sum_{r<p} A(p,r)
\]

Applying the quasi-steady-state approximation gives

\[
n(p) = n_{e} n(l) \frac{X(l,p)}{\sum_{r<p} A(p,r)}
\]

and for the intensity

\[
I = \frac{1}{4\pi} n_{e} n(l) X(l,p) \frac{A(p,r)}{\sum_{r<p} A(p,r)} hv.
\]

If \((p,q)\) is the dominant transition the ratio of the \( A \)'s is very near to unity. Thus the intensity is determined by the excitation rate coefficient from the ground level.

For higher levels \( p \) (but below the collision limit) the contribution from \( n_{o}(p) \) increases until above the collision limit it dominates. However these higher transitions are much weaker and it is seldom that the spectral lines are sufficiently intense to be studied for the low density plasmas with which we are concerned.

Hutcheon and McWhirter (1973) have extended the collisional-radiative theory to plasmas of lower density where the metastable 2s level modifies the trend of the earlier results. They identify three regions of electron density between which there
is a change in the intensity ratio of Ly α/Ly β. Figure 7.1 identifies the regions and figure 7.2 gives the calculated values of the ratio. It may be noted that the mid-density region gives the maximum intensity ratios. The physical reason for the change at low density is due to the possibility of 2 photon decay of the 2s level. This provides an alternative escape path to the emission of Ly α by the 2p level to which electrons in the 2s level are transferred by collisions with both positive ions and electrons. The low densities to which these modifications to collisional-radiative theory apply are found only in astronomical plasmas.

7.3 Lithium-like ions

Of all ions in a low density plasma lithium-like are the simplest because they have no metastable levels. Nitrogen V has been studied extensively and its term scheme is shown in figure 7.3. All the spectral lines with which we are concerned arise from transitions by the single electron lying outside the 1s² inner shell.

As before the intensities of the spectral lines are given under the quasi-steady-state approximation by their rates of collisional excitation from the ground (2s) level. Accurate quantum mechanical excitation rate coefficients for many of these
Variation of the theoretical steady state Lyman α/Lyman β ratio temperature

Curve A is calculated from the original CR model but with improved cross-sections. Curve B is calculated from the two-photon CR model with the same cross-sections as Curve A. Curve C is calculated from the Saha equation.
levels have been published so the prediction of intensities is possible with good reliability. Some excitation rates for N V are shown in figure 7.4. From this it may be seen that the temperature dependence of the 2s 2p excitation is quite different to excitations to higher levels. This leads to the important possibility of using a measurement of the intensity ratio to determine the temperature of the plasma. The
The curves are theoretical excitation coefficients
based on cross-sections calculated by Burke et al. (Proc. Phys. Soc. B7, 209, [1966]). The points are our measured values.

Fig. 7.4

The ratio of the intensity 2s 2p to any other line of the ion may be used say the 2s 3p transition. The intensities are

\[ I(2s\ 2p) = \frac{1}{4\pi} n_e n(2s) \times (2s\ 2p) \text{photons cm}^{-3} \text{s}^{-1} \text{sterad}^{-1} \]

and

\[ I(2s\ 3p) = \frac{1}{4\pi} n_e n(2s) \times (2s\ 3p) \text{photons cm}^{-3} \text{s}^{-1} \text{sterad}^{-1}. \]
Thus the ratio is

\[
\frac{I(2s\ 2p)}{I(2s\ 3p)} = \frac{X(2s\ 2p)}{X(2s\ 3p)} \quad \text{(Intensities in photons).}
\]

The dependence of this ratio on the plasma temperature is illustrated in figure 7.5 for a number of different Li-like ions. The steep dependence makes this a particularly sensitive method of determining temperature.

![Graph showing the ratio of ion line intensities as a function of temperature.](image)

**Lithium-like ion line intensity ratios for electron temperature measurement.**

**Fig. 7.5**

For plasmas that are in steady-state ionisation balance such as is generally assumed for the solar atmosphere above the chromosphere the particular population distribution of the Li-like ions means that special care has to be taken in interpreting measurements. It was noted when discussing the ionisation balance that there was a sholder on the density/temperature distribution of these ions to the high temperature side of the maximum. This gives rise to a weighting of the intensities to high temperatures when these are averaged over the full depth of the plasma.

In the solar plasma electron densities are sufficiently low that there is no need to take account of any of the additional mechanisms that complicate the simple excitation of these ions. This is not always the case for laboratory plasmas where the high collisional excitation rate of the 2p level and its relatively low radiative decay rate (coupled possibly with radiation trapping) can lead to the 2p level having
a population comparable to the ground level. Thus excitation to the higher levels includes a contribution from the 2p level while the intensity of the 2s 2p line is reduced. However sufficient atomic data is available to take account of these complications and successful interpretation is possible (figure 7.5).

![Graph showing the effect of electron density on the 2s-2p ratio of N/V.](image)

Fig. 7.6

Spectra arising from the decay of doubly excited Li-like ions produced in the di-electronic recombination of He-like ions are considered with the spectral lines of the latter ions.

7.4 Beryllium-like ions

Beryllium-like ions have two electrons in the second shell making four in total. The excitation of the spectral lines is complicated over the relatively simpler lithium-like ions by the presence of the low-lying metastable 2s2p3P level. This is
illustrated in figure 1.2 which shows term scheme of Ne VII. Because the metastable level is relatively low-lying (unlike the 2s level of hydrogen-like ions) it can be populated to an extent comparable with the ground level. (This can be seen by calculating the Boltzmann factor between the metastable and ground levels and is confirmed by the detailed consideration of the populating processes that follow.) The relatively large population density of the metastable level means that excitation from this level also influences the populations of all the other levels. Figure 7.7 illustrates the process.

\[ I(b,q) = n_e \left( n(g)X(g,p) + n(m)X(m,p) \right) \frac{A(p,q)}{\sum_r A(p,r)} \]

Fig. 7.7

Excitation processes in the presence of a metastable level

Thus the first step in calculating the spectral intensities from such an ion (and the same considerations apply to other ions such as boron-like) following the determination of its stage of ionisation, is to determine the relative populations of the metastable and ground levels. The processes that need to be considered for this purpose may be discussed with reference to the partial term diagram of a beryllium-like ion illustrated in figure 7.8.

Radiative transition probabilities for the $2s^2 \, ^1S_o - 2s2p \, ^3P_1$ lines have been calculated by Garstang and Shamey (1967) on the intermediate coupling approximation. Values for the magnetic quadrupole transition $2s^2 \, ^1S_o - 2s2p \, ^3P_2$ have also been calculated by Garstang (1967). The values are compared with the allowed $2s^2 \, ^1S - 2s2p \, ^1P$ probabilities below.
Fig. 7.8
Partial term scheme of a beryllium-like ion

<table>
<thead>
<tr>
<th></th>
<th>(2s^2 , ^1S - 2s2p , ^1P)</th>
<th>(2s^2 , ^1S_o - 2s2p , ^3P_1)</th>
<th>(2s^2 , ^1S_o - 2s2p , ^3P_2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Be I</td>
<td>(5.5 \times 10^8 , \text{s}^{-1})</td>
<td>(0.71 , \text{s}^{-1})</td>
<td>(1.6 \times 10^{-4} , \text{s}^{-1})</td>
</tr>
<tr>
<td>Si XI</td>
<td>(6.4 \times 10^9 , \text{s}^{-1})</td>
<td>(6.5 \times 10^5 , \text{s}^{-1})</td>
<td>(0.21 , \text{s}^{-1})</td>
</tr>
</tbody>
</table>

These three sets of values serve to define the boundaries between four density region. At high density where the rates of collisional transitions exceed the fastest radiative rates the ion meets the LTE criterion and the Saha-Boltzmann equation gives the population densities. A more interesting situation arises at somewhat lower densities when all the lower levels but the metastable and ground levels decay radiatively and their collisional decay may be neglected. In this density regime the metastable level has a relatively small probability of radiative decay. Its population density is determined by the balance between its collisional excitation from the ground level and its collisional de-excitation into singlet levels \(2s2p \, ^1D\) and \(2s2p \, ^1P\). Thus although collisions dominate in these processes the
population ratio departs from its Boltzmann value.

The next lower density regime starts when the radiative decay of the metastable level via the intercombination line $2s^2 \, ^1S_0 - 2s2p \, ^3P_1$ is equal to or greater than its rate of collisional de-excitation. Important in this part of the calculation is the collisional mixing between the sub-levels of $2s2p \, ^3P_{0,1,2}$. There are three important mechanisms by which this takes place.

(a) Electron collisions causing transitions directly
(b) Proton " " 
(c) Electron collisions exciting $2p^2 \, ^3P$ from which radiative decay back to $2s2p \, ^3P$ redistributes the populations among the $J$ levels.

Finally the next lower density regime starts when collisional de-excitation of the $J = 2$ sub-level gives way to its radiative decay by the magnetic quadrupole transition.

Results of a preliminary calculation for OV are presented in figure 7.9. In this the upper LTE density regime is excluded (it starts about $n_e = 5 \times 10^{17}$ cm$^{-3}$). The values for the various excitation rate coefficients upon which this graph is based were calculated by Eissner and are unpublished. More recent calculations by Malinovsky (1975) show that Eissner under-estimated the $2s2p \, ^3P - 2p^2 \, ^3P$ collision rate thus giving rise to an under-estimate of the metastable radiative decay rate. Thus the curve labeled $n(m)/n(g)$ should be lowered by about $\frac{1}{2}$ in the range $10^7 - 10^{11}$ cm$^{-3}$. At the same time this correction extends the region where $\lambda 760/\lambda 629$ intensity ratio is sensitive to electron density to lower values of density ($\sim 10^{10}$ cm$^{-3}$). Such changes in the values of basic data lead to uncertainty in the interpretation of astronomical spectra.

Fig. 7.9
Intensity ratios of oxygen V against electron density

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The spectra of beryllium-like ions have been studied in the laboratory but it has only been possible to do this in the density regime of about $10^{14} - 10^{15}$ electrons cm$^{-3}$ whereas astronomical spectra are produced in lower density regimes. Nevertheless useful measurements can be made but at this stage the accuracy is not adequate to help in choosing between alternative theoretical rate coefficients. The laboratory experiments are discussed later.

Since the metastable $2s2p^3P$ level can develop population densities comparable with the ground level the basis of the quasi-steady-state approximation has to be examined again. In a recent calculation (unpublished) I estimated the population ratio $n(m)/n(g)$ as a function of time for an ionising plasma consisting initially of boron-like ions and being ionised through beryllium-like to lithium-like. It turned out that the population ratio quickly reaches a value within 25% of the steady-state value calculated by the simpler method. Thus the error introduced by assuming the quasi-steady-state is less than the uncertainty in the values of the rate coefficients.

In the boundary regions between the density regimes discussed above some of the line ratios of these beryllium-like ions are sensitive to the value of the electron density. At the same time intensity ratios including one of the transitions from an upper level are sensitive to the electron temperature. These ions are therefore particularly useful diagnostic probes of plasma conditions and the determination of reliable rate coefficients are particularly important.

7.5 Helium-like ions

The excitation processes considered in this section fall into two categories. In the first the collisional excitation of helium-like ions in much the same way as for other ions is considered while in the second the excitation of the di-electronic satellites is discussed. In both cases the initial target ions are helium-like although the satellite lines are strictly from lithium-like ions since a recombination process is involved. All the lines lie close in wavelength to the main $1s^2 1S - 1s2p^1P$ resonance line and this is an additional reason for considering them together under the helium-like heading.

The features which distinguish helium-like ions from others are as follows:

(a) Compared with ions having more bound electrons, helium-like ions have exceptionally large ionisation potentials corresponding to the relatively greater difficulty of removing the first electron from the innermost shell.

(b) Compared with the ionisation potential the excitation potential of the lowest excited levels are relatively large (excitation potentials are about 0.8 of the corresponding ionisation potentials). In this respect they resemble hydrogen-like ions.

(c) There are two metastable levels of which one ($1s\,2s\,3S$) is the lowest level of the triplet series.

Figure 7.10 illustrates the term scheme of O VII (not to scale) and the collisional and radiative processes taken into account in discussing its excitation. Because of the relatively small differences between the excitation potentials of the excited levels the line ratios of this ion are not used as indicators of electron temperature (however, see the later discussion of satellite lines) and attention is concentrated on calculating intensities of lines arising from levels having principle quantum number 2. For these in a similar way to beryllium-like ions the small radiative transition probabilities associated with the metastable levels make the line ratios
sensitive to the electron density in specific density ranges. The relevant transition probabilities are as follows:

(a) $1s^2 1S - 1s2p^1P$ is an allowed electric dipole transition scaling as $Z^4$.
(b) $1s^2 1S - 1s2p^3P_1$ is allowed through a breakdown of L-S coupling at a rate that scales as $Z^5$.
(c) $1s^2 1S - 1s2p^3P_2$ is a magnetic quadrupole transition and scales as $Z^8$.
(d) $1s^2 1S - 1s2s 1S$ decays by 2 photon emission scaling as $Z^6$.
(e) $1s^2 1S - 1s2s 3S$ can take place by a relativistic magnetic dipole transition and scales like $Z^{10}$.

Because of the large scaling powers with $Z$ the last four become relatively more important than the main resonance line at high $Z$. The following table gives some sample value for their radiative transition probabilities (Gabriel and Jordan, 1972).

<table>
<thead>
<tr>
<th>Ion</th>
<th>$1^1S - 2^1P$</th>
<th>$1^1S - 2^3P_1$</th>
<th>$1^1S - 2^3P_2$</th>
<th>$1^1S - 2^1S$</th>
<th>$1^1S - 2^3S$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Scaling:</td>
<td>$Z^4$</td>
<td>$Z^9$</td>
<td>$Z^8$</td>
<td>$Z^6$</td>
<td>$Z^{10}$</td>
</tr>
<tr>
<td>CV</td>
<td>$8.87 \times 10^{11}$</td>
<td>$2.84 \times 10^7$</td>
<td>$2.58 \times 10^4$</td>
<td>$3.31 \times 10^5$</td>
<td>$4.86 \times 10^3$</td>
</tr>
<tr>
<td>O VII</td>
<td>$3.30 \times 10^{12}$</td>
<td>$5.53 \times 10^8$</td>
<td>$3.26 \times 10^3$</td>
<td>$2.31 \times 10^6$</td>
<td>$1.04 \times 10^3$</td>
</tr>
<tr>
<td>Ne IX</td>
<td>$8.87 \times 10^{12}$</td>
<td>$5.43 \times 10^9$</td>
<td>$2.23 \times 10^6$</td>
<td>$1.00 \times 10^7$</td>
<td>$1.09 \times 10^4$</td>
</tr>
<tr>
<td>A XVII</td>
<td>$3.18 \times 10^8$</td>
<td></td>
<td></td>
<td></td>
<td>$4.71 \times 10^6$</td>
</tr>
<tr>
<td>Fe XXV</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$2.00 \times 10^8$</td>
</tr>
</tbody>
</table>
Where the electron collision rate exceeds the fastest of these radiative rates the ion meets the requirements for LTE and the Saha-Boltzmann equation may be used. For somewhat lower densities the singlet levels (except \(2^1S\)) decay radiatively and the metastable levels suffer collisional transfer to the levels that can radiate or are ionised. Thus in this regime the metastable level population densities are determined by collisions which however are not in detailed balance because of the part played by levels that have a high probability of radiative decay.

In the next lower density regime the intercombination transition \((1^1S - 2^3P)\) takes over as the most probably decay route for the triplet metastable level \(2^3S\). The \(2^3P\) level is rapidly populated from the metastable level by collisions which also ensure through this process \((2^3S - 2^3P)\) that the sub-levels \((J = 0, 1, 2)\) are statistically populated. At lower densities the \(1^1S - 2^3P\) magnetic quadrupole transition adds its contribution to this decay mechanism - the levels being so close that this cannot be distinguished as a separate spectral line. The on-set of the magnetic quadrupole does not modify the intensity of \(2^1S - 2^3P\) since all the triplets decay by this route anyway.

The next lower density regime is entered when the singlet metastable level \(2^1S\) has a greater probability of decay by two-photon emission than by collisions to the \(2^1S\) level. When this happens the resonance line becomes relatively weaker and the 2 photon continuum appears. There has been one report of the latter having been detected in the laboratory but none from an astronomical source.

Finally at even lower densities the relativistic magnetic dipole decay of the \(2^3S\) level takes over from \(1^1S - 2^3P\) as the most probably decay route for the triplets. This gives rise to the forbidden lines of ions such as O VII first identified correctly in the solar spectrum.

Together these three lines and the two photon continuum give rise to a varying pattern of spectral intensities from which it is possible to deduce the plasma electron density over a wide range. This is illustrated in figure 7.11 for case of O VII. The calculations for this figure required values for many collision rate coefficients. For these theoretical calculations based on the Coulomb-Born-Oppenheimer approximation have been used (Bely, 1968 and Burgess et al, 1970). Such calculations suffer from relatively large uncertainties and further theoretical work on these ions is required.

These calculations do not include contributions to the rate of populating the levels having principal quantum number 2 by cascade from upper levels nor that due to recombination particularly into the triplet levels. Bely (in the paper quoted in the previous paragraph) has estimated the effect of the former to be an increase of up to 40% in the population density of the triplet levels. It appears that no account has been taken of the latter. This is the population coefficient corresponding to the \(n_0(p)\) term in the hydrogen-like case where it is generally negligible in the plasmas we are considering. However, it may make a significant contribution to the triplets in the helium-like ions.

The second part of this section on helium-like ions concerns the theory of the satellite lines found to the long wavelength side of the \(1^1S - 2^1P\) resonance line. These arise primarily from the di-electronic recombination process but also from inner-shell excitation of the corresponding lithium-like ions. The theory of these lines has been developed by Gabrieli (1972) and by Bhalla, Gabrieli and Presnyakov (1975). They show that the relative intensities of these lines with respect to the \(1^1S - 2^1P\) resonance line depend on the electron temperature of the plasma and that the extent to which the ion is in a state of transient ionisation may also be deduced. Thus when taken together with the intercombination line \((1^1S - 2^3P)\) and the forbidden
Fig. 7.11

Oxygen VII intensities v. electron density.

The line (\(^{1}\text{S} - 2^{3}\text{S}\)) these together provide a very powerful diagnostic probe for the plasma from which they are emitted.

Inspection of the expression given earlier for the di-electronic recombination coefficient leads to the intensity of one of the satellite lines arising from this process having an intensity given by

\[
I_{s} = \frac{1}{4\pi n_{e} n_{\text{He}}} \frac{\hbar^{3}}{2(2\pi m kT)^{3/2}} \frac{\omega(L_{i}, j, n_{l})}{\omega(H_{e}, g)} \frac{A_{r} A_{a}}{A_{r} + A_{a}} \exp \left(-\frac{E_{s}}{kT}\right) \text{ph cm}^{-3} \text{s}^{-1} \text{ster}^{-1}
\]

where \(E_{s}\) is the energy difference between the upper level of the satellite line and the helium-like ground level designated by the symbols (quantum numers) \((L_{i}, j, n_{l})\) and \((H_{e}, g)\) respectively. When the ratio of the intensity of the satellite line is taken to the resonance line in the electron and ion densities cancel and there remains only a term dependent on the electron temperature and the particular satellite line.

For high \(Z\) ions, such as iron, inner shell excitation of lithium-like ions also contributes to the excitation of the satellite lines. This contribution is proportional to the population density of the lithium-like ions as well as electron temperature and is therefore a measure of the extent to which the ion departs from steady-state

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ionisation balance. Since different satellite components depend differently on the
two basic excitation mechanisms it is possible to distinguish between them by mea-
suring selected components.

8.1 Radiation trapping and the equation of radiative transfer

In the formal treatment of the theory of the intensities of spectral lines and con-
tinuum the equation of radiative transfer is sometimes taken as the starting point.
It represents a mathematically simpler statement of the intensity of a spectral
feature in the most general case but since it does not help to elucidate the most
important physics of the problem considered here this approach has been avoided.
However when it comes to a discussion the re-absorption of radiation in an optically
thick plasma it can be helpful.

So far there has been very little discussion of the effects of opacity and this chapter
will deal with it only very briefly. Radiation which is emitted within the volume
of a plasma has a finite probability of being reabsorbed or scattered as it escapes
from the plasma and passes through the intervening medium on its way to the spectro-
meter and detection system. Although absorption by the intervening medium can be very
important, as for example the absorption of the ultraviolet component of star light
by interstellar hydrogen at wavelengths less than λ915Å, it is a matter outside the
general range of this paper and will not therefore be discussed further.

The reabsorption of radiation within the plasma can also be very important. Of the
possible processes the reabsorption of resonance radiation is the most important here.
By this process a photon, which is produced by the radiative decay of an excited level
to the ground level of the ion, has a high probability of finding a similar ion in
its ground level and can be absorbed by it with a relatively high probability - hence
the name of resonance radiation. Reabsorption of this nature has a number of effects
on the emitted spectral line radiation viz.

(a) It can become an important mechanism of populating the upper level of the
resonance transition. Thus the statistical calculation of the population
densities of the excited levels may have to be modified to take account of
it.

(b) Since the extent of the reabsorption depends on the depth of plasma through
which the radiation has to pass it is possible that for plasmas that are far
from spherical the angular distribution of radiation is modified by the plasma
shape. Trapped radiation has the greatest probability of escape through the
shortest dimension.

(c) Where the upper level has an alternative means of de-excitation (either by
collision or by the emission of an alternative photon) the resonance line
intensity is reduced by the effect of opacity. Note however that the intensity
of a line from a level that has no other means of decaying is not reduced by
the effect of opacity (but see (b) above).

(d) Where there is an alternative radiative transition for the upper level of an
optically thick line (for example the Hα - Ly β combination in hydrogen) the
effect of trapping the resonance line is to enhance the intensity of the
alternative transition.

(e) Finally it should be mentioned that opacity modified the spectral shape of
a line. For most of the lines from the plasmas considered there thermal
Doppler effect is the dominant broadening mechanism. The full calculation
of the diffusion of a Doppler broadened line through a plasma is complex but
it is possible to see qualitatively that there is a greater probability of absorption of a photon near the line centre and less in the wings where the probability of finding another ion with the same component of velocity is much less. Thus radiation trapping has the effect of broadening the lines. It may also be noted in this connection that usually a plasma is surrounded by a layer of cooler material. Optically thick lines emitted by such a plasma will have the narrower cores of their profiles depleted in intensity by reabsorption in this boundary layer. Such profiles are said to be self reversed and it may be shown that the effect is even possible although the temperature of the outer layers is not less than the interior.

8.2 The equation of radiative transfer

It is necessary to start with a number of new definitions that will be related to some of the earlier concepts as the argument develops.

$J(\nu)$ is the coefficient of emissivity which is defined as the power radiated by unit mass of the plasma per unit frequency ($\nu$) interval per unit solid angle.

$\chi(\nu)$ is the absorption coefficient such that if intensity $I(\nu)$ falls on a layer of thickness $ds$, mass density $\rho$, the change of intensity in the layer is

$$dI(\nu) = - I(\nu) \chi(\nu) \rho ds + J(\nu) \rho ds$$

$\tau(\nu)$ is the optical depth at frequency $\nu$ and is given by

$$\tau(\nu) = \chi(\nu) \rho ds$$

Thus the first equation may be re-written in terms of the optical depth

$$\frac{dI(\nu)}{d\tau(\nu)} = - I(\nu) + \frac{J(\nu)}{\chi(\nu)}$$

This is the equation of radiative transfer and $J(\nu)/\chi(\nu)$ is called the source function.

In its simplest application consider a uniform plasma where the particle collision rate is sufficiently great as to maintain the population distribution in LTE. Thus the emission and absorption coefficients are related such that the ratio $J(\nu)/\chi(\nu)$ is the Planck function. Because the plasma is uniform the integration of the equation of radiative transfer is simple and gives

$$I(\nu) = \frac{J(\nu)}{\chi(\nu)} \left(1 - \exp \left[-\chi(\nu) \rho D\right]\right)$$

where $D$ is the physical depth of the plasma. This rather trivial result shows that for large optical depths ($\tau(\nu) \equiv \chi(\nu)\rho D >> 1$) the plasma radiates like a black body whereas for $\tau(\nu) \ll 1$ it reduces to

$$I(\nu) = J(\nu) \rho D$$

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which is the result used hitherto for optically thin plasmas. By substituting the expression for the absorption coefficient for hydrogen continuum radiation given by Cillière (1932) viz.

\[ \chi(v) = \frac{\frac{n_e n_p}{4\pi \rho}}{k T_e} \cdot \frac{2^6 \pi^3}{3/2} \left( \frac{m_e}{k T_e} \right)^{1/3} \left[ \frac{e^6}{h c m^2} \right] \left( \frac{1 - \exp \left[ -\frac{h v}{k T_e} \right]}{v^3} \right) \]

it is possible to see how the continuum spectrum of a plasma moves from being black body at long wavelengths to optically thin bremsstrahlung at short wavelengths. A more general treatment would include recombination radiation.

8.3 Population densities in an optically thick plasma

In this section of the modofication to the expression obtained earlier for the population density of an excited level due to one of the lines arising from the decay of that level having an opacity greater than unity. Stimulated emission is neglected in this treatment. The previous expression for the rate of change of the population density of an excited level \( p \) is modified for absorption at the frequency of the transition \( (p, q) \) where \( q \) is the lower level thus

\[
\frac{dn(p)}{dt} = -n(p) A(p, q) + n(q) B(q, p) \frac{4\pi}{c} \int_{0}^{\infty} I(v) f(v) \, dv + \text{collisional and other radiative terms}
\]

where \( I(v) \) is the intensity of radiation averaged over direction and \( f(v) \) is the form factor of the absorption profile of the transition \( (p, q) \)

\[
\int_{0}^{\infty} f(v) \, dv = 1
\]

The first two terms of the right hand side of this equation may be expressed thus

\[
\frac{dn(p)}{dt} = -n(p) A(p, q) \, g(\tau_o) + ...
\]

where

\[
g(\tau_o) = 1 - \frac{n(q)}{n(p)} \, \frac{\omega(p)}{\omega(q)} \, \frac{c^2}{2\hbar v_o^3} \int_{0}^{\infty} I(v) f(v) \, dv
\]

In deriving this the relationship between the Einstein coefficients has been used. \( g(\tau_o) \) is known as the escape factor.

Use is now made of the expression derived for a uniform plasma

\[
I(v) = \frac{J(v)}{\chi(v)} \{ 1 - \exp \left[ -\chi(v) \, p \, \tilde{b} \right] \}
\]

where this time the source function may not be equated to the Planck function since the plasma is not in LTE. \( \tilde{b} \) is of the order of the smallest linear dimension of the plasma (for a sphere it is the radius).

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It is now assumed that the source function is independent of frequency and that the line is Doppler broadened such that

\[ \chi(\nu) = \frac{n(q)}{\sqrt{\pi} \Delta \nu_D} \frac{h\nu}{\rho c} B(q,p) \exp\left\{-\frac{(\nu-\nu_0)^2}{2\Delta \nu_D^2}\right\} \]

where

\[ \Delta \nu_D = \sqrt{\frac{2RT}{M}} \frac{\nu_0}{c} \]

\( \nu_0 \) is the central frequency of the line where the average opacity

\[ \tau_o = \frac{n(q)}{\sqrt{\pi} \Delta \nu_D} \frac{h\nu_0}{c} B(q,p) \]

Substitution gives finally

\[ g(\tau_o) = 1 - \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} \exp\left(-x^2\right) \left[ 1 - \exp\left(-\tau_o \exp\left(-x^2\right)\right) \right] dx \]

Holstein has derived the following approximate value for

\[ \frac{1}{g(\tau_o)} = \tau_o \left(1 + \ln \tau_o \right)^\frac{1}{2} \]

For \( \tau_o \) greater than 2 it gives values within a few percent of values obtained by numerical integration of the full integral. Thus by using the escape factor it is possible to take approximate account of the effect of opacity on the populations of excited levels. The escape factor is a function of the average optical depth \( \tau_o \) at the frequency of the line centre given by the expression above.

A full treatment of the problem requires the simultaneous solution of the equations of radiative transfer of all the optically thick lines together with the equations describing the collisional processes affecting the population densities.

9.1 Radiation Cooling

In the development of the subject so far the requirements of plasma spectroscopy as a diagnostic technique for the measurement of electron temperature, density, abundance etc. have been foremost. In this section consideration is given to radiation cooling as an energy loss mechanism. As will be seen much of the same basic physics enters this problem.

Radiation cooling is important in laboratory plasmas particularly in the field of fusion research where the influx of impurities and subsequent power loss is a problem currently receiving a lot of attention. In astronomy radiation cooling where the spectrum resembles a black body is clearly fundamental to the energy balance of many objects. In the present context consideration is given to the radiated power from more tenuous
black body is clearly fundamental to the energy balance of many objects. In the present context consideration is given to the radiated power from more tenuous atmospheres which emit spectral lines. As an example the solar corona and transition region down to the top of the chromosphere is taken. Figure 9.1 summarised current ideas about the overall energy balance in this part of the solar atmosphere. The primary heating mechanism for the corona is some form of mechanical waves - probably m.h.d or acoustic. The energy thus deposited in the upper layers is conducted back down again except for a small part which provides the energy for the
solar wind. As the conducted energy passes down towards the surface it establishes the steep temperature gradient and provides on its way the energy for the radiation loss from this increasingly more dense region of the atmosphere. The mechanical waves in passing through this region cause additional broadening of the spectral lines so that the spectroscopist has a chance of seeing the effects of the energy twice - once on its way up as mechanical energy and then as the power in spectral lines after it has been conducted back down towards the surface.

In the case of the solar atmosphere the radiated power loss has been calculated with the approximation of the steady-state ionisation balance. In the fusion plasma case the steady-state approximation may be adequate for a rough initial estimate of radiation loss but detailed time dependent calculations will be needed at some stage in designing these experiments.

9.2 Radiated power loss in the steady-state approximation

Three sets of calculations have been done for astronomical plasmas and these will serve to illustrate the method of the calculation which may then be adapted for any comparable low density plasma. Two of the calculations have been done for general cosmological plasmas and use cosmic abundances of the elements (Pottasch, 1965 and Cox and Tucker, 1969). The third was done with the sun in mind and will be discussed in more detail (McWhirter, Thonemann and Wilson, 1975). The calculation is an attempt to add together the intensities of all the spectral lines and continuum emitted by a steady-state low density plasma containing elements in the abundances found in the sun. It is further assumed that the plasma is optically thin in all this radiation although the criterion for it being optically thin is not as severe as it often is. The radiation may be absorbed and re-emitted many times as it diffuses out of the plasma provided the probability that an excited level so produced suffers de-excitation by collision is negligibly small, i.e. it does not matter how long it takes the radiation to diffuse out of the plasma so long as it all escapes.

The power radiated by a spectral line from unit volume is given by

\[ P(p, q) = n_e n(g) X(g, p) \frac{A(p, q)}{\sum_r A(p, r)} \frac{h \nu (p, q)}{v} \text{ erg s}^{-1} \text{cm}^{-3} \]

From an analysis of the solar spectrum the five or six strongest lines for each ion present were identified and these were added together in the calculation to give the power for each ion thus

\[ P(z, El) = \sum_p n_e n(g) X(g, p) \frac{A(p, q)}{\sum_r A(p, r)} \frac{h \nu (p, q)}{v} \]

\[ = n_e \frac{n(H)}{n(El)} \frac{n(El)}{n(H)} \sum_p X(g, p) \frac{h \nu (p, q)}{v} \]

For ions with important metastable levels care was taken to include excitation from metastable levels in the calculation

\[ n(H) \] is the number density of hydrogen atoms and ions
\[ n(El) \] " " " " " " the ions of the element in question.

In evaluating the sum excitation rate coefficients were calculated using the \( g \) approximation where the value of \( g \) was taken from the best available source especially for lines that were strong in the spectrum and therefore accounting for an important fraction of the radiation of that ion.
The next stage was the sum over all the ions of the element. The ratio \( \frac{n(Z)}{n(EL)} \) was assumed constant over temperature and was taken from the steady-state ionisation balance calculations of Summers (1974) or of Jordan (1969). It may be noted that the factors following \( n_e \frac{n(H)}{n(H)} \) is a function of electron temperature and of the ions of the element only. Thus for each element there is a power loss curve such as those illustrated in figure 9.2, 9.3 and 9.4 for hydrogen, carbon and neon respectively. The total power curves are designated the radiated power functions for the elements in question. They apply to all steady plasmas of low enough density for the excitation model to apply.

The radiated power functions have a number of common features that mainly reflect the shell structure of atoms. This can be helpful in interpolating between elements. The hydrogen curve is the simplest and is made up of two components. The greater at lower temperature is due to the excitation of atomic hydrogen while the other is due to bremsstrahlung and recombination continua resulting from the interaction of electrons and positive ions of hydrogen. At temperatures lower than \( 10^4 \) K radiation from negative ions of hydrogen becomes important but this has not been included in these calculations. The carbon curve illustrates the importance of excitation of the electrons in the second shell from particularly boron-like, beryllium-like and lithium-like ions. The lines which contribute most are the transitions between levels having principal quantum number 2. This pattern is repeated for neon but with everything shifted to higher temperatures. Elements with electrons in three shells show a third peak at lower temperature but this is less dramatic than the difference between shells one and two.

To calculate the total radiated power loss function for the sun it is now necessary to multiply the individual radiated power functions by the appropriate elements abundances \( \frac{dn(EL)}{dn(H)} \) and add them all together giving finally

\[
P(Sun) = n_e \frac{n(H)}{n(H)} \sum_{EL} \frac{n(EL)}{n(H)} \sum_z \frac{n(z)}{n(EL)} \sum_p X(g,p) \ h\nu \ (g,p) \ erg\ s^{-1}\ cm^{-8}
\]

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Fig. 9.3

Fig. 9.4
The final result is shown in figure 9.5 and also for comparison the results of the other two calculations. The differences between them are consistent with the different abundances used and different rate coefficients.

![Graph showing the total radiated power loss function for the solar corona.](image)

**Fig. 9.5**

### 9.3 Radiation cooling in a time dependent situation

The useful question to answer for a laboratory plasma (such as is used in fusion research) is how much energy is required to bring a neutral atom introduced into the plasma to its steady state ionisation balance? Energy is required for three reasons viz. (a) radiation loss (b) ionisation and (c) to heat the electrons released in the ionisation process to the ambient temperature. For simplicity the temperature and density of the plasma will be assumed to be constant. Consider the power required per unit volume when the atom has reached a stage of ionisation of charge $z$.

$$P = \frac{n_e}{\rho} \sum_{\nu} X(g,p) \, \nu \, (g,p) + n_e \, S(z) \left[ \chi(z) + 3/2 \, kT \right]$$

It will spend a time of order $\frac{1}{n_e S(z)}$ in this stage of ionisation so the energy required to get it to the next stage is

$$\frac{1}{n_e S(z)} \left\{ \frac{\sum_{\nu} X(g,p) \, \nu \, (g,p) + n_e \, S(z) \left[ \chi(z) + 3/2 \, 2kT \right]}{\rho} \right\}$$

Thus the energy required to take an ion from its neutral state to its maximum charge
for the plasma $z_{\text{MAX}}$ (determined possibly in the steady-state ionisation balance) is

$$z_{\text{MAX}} \left\{ \sum_{z=0}^{\Sigma} \frac{X(g,p) \nu(g,p)}{S(z)} \right\} + x(z) + \frac{3}{2} kT \text{ erg}$$

Difficulties arise with numerical calculations due to the problem of defining $z_{\text{MAX}}$ in a realistic manner. However calculations for hydrogen-like ions have been done by McWhirter and Hearn (1963). (Note that their calculation does not include the $3/2kT$ term.) These calculations show that for $T/z^2 = 3 \times 10^9 K$ the energy required for each ionisation is about ten times the ionisation potential of the ion. At higher temperatures less energy is required. See figure 9.6.

![Figure 9.6](image)

**Fig. 9.6**

The energy required to ionise a hydrogen-like ion

10.1 Laboratory measurements of excitation and ionisation rate coefficients

Four experimental methods of measuring excitation rate coefficients or cross-sections have been proposed but only two of them have yielded values of interest in the present context. The other two methods are mentioned only for completeness. In the first of these use is made of the fact that collisions causing excitation broaden the levels and the corresponding rate may be measured from the broadening of the spectral lines arising from that level. The broadening may be understood in
terms of the uncertainty principle where the collisions shorten the lifetime of the level with a corresponding spread in its energy. In the second of these methods (Burgess and Skinner, 1974) a laser is tuned to the same wavelength as some transition connecting the level in question and causes the population ratio between the levels to be modified by photo-excitation. In circumstances where the populations are determined (in the absence of the laser beam) by collision processes the intensity change of a spectral line originating from the perturbed level gives a measure of the collision rate. Both these methods are likely to be limited in their applicability to relatively small numbers of transitions and since neither has been developed to the extent of the other two they will not be considered further.

The two methods which have been used in practice to measure excitation rate coefficients or cross-sections are closely related to similar methods to measure ionisation rate coefficients or cross-sections. The first makes use of the techniques of plasma spectroscopy and the second of crossed-beams of electrons and ions. The first of these will now be considered in some more detail.

10.2 Measurement of excitation rate coefficients by the technique of plasma spectroscopy

The application of this technique depends on having a laboratory plasma of sufficiently low density that the intensities of the spectral lines are adequately described by the low density excitation model discussed in earlier sections (7). It was shown there that the intensity of a spectral line in the low density limit is given by an expression as follows.

\[ I = \frac{1}{4\pi} n_e n(g) X(g,p) \frac{A(p,q)}{L A(p,r)} \frac{hv L}{r} \text{erg s}^{-1} \text{cm}^{-2} \text{sterad}^{-1} \]

The factor \( L \) has been multiplied into the previous expression in order to account for the geometry of the spectrometer-plasma combination. It will be defined in detail below. When a metastable level contributes significantly to the excitation of the line being measured the situation becomes more complicated and this is discussed below but at the moment attention is confined to the simpler situation such as lithium-like ions. In such cases the experimental problem breaks down into the following components.

(a) Determination of the intensity of the spectral line \((p,q)\) - usually in the vacuum ultra-violet.
(b) Determination of the plasma density including the concentration of the element from which the ion is formed.
(c) Determination of the geometrical factor \( L \).
(d) Determination of the fractional abundance of the relevant ion with respect to all ions of the element.
(e) Determination of the plasma electron temperature of which the excitation rate coefficients are functions and whose determination is therefore necessary for a meaningful measurement.

These individual components of the experiment will now be considered in turn.

(a) The measurement of spectral intensity in the vacuum ultraviolet region of the spectrum requires an intensity calibration of a suitable spectrometer. The most suitable method is the braching-ratios method since the same plasma source can be used as for the main experiment. The method requires that the intensities of two spectral lines be observed at the same time. They must arise from the same upper level and have well determined transition probabilities.
Finally one of the lines must lie in a region of the spectrum where a good intensity calibration is established. Thus the method really amounts to a transfer of intensity calibration from one spectral region to another. The region of known calibration has been the visible where a standard tungsten filament lamp provides the intensity calibration. Equally it could be the soft X-ray region ($\lambda < 100\AA$) where an intensity calibration against a proportional counter is relatively reliable.

Figure 10.1 shows a diagram of the apparatus used for one of the first measurements made using this technique. Calibration of the grazing incidence spectrometer was done against the Ebert instrument which was calibrated against a standard tungsten filament lamp. The lines used in this particular experiment

![Diagram](image)

**Fig. 10.1**

The apparatus

for the branching-ratios calibration are shown in figure 10.2. The $1216\AA$ line of He II was detected with the help of the normal incidence spectrometer since the grazing incidence instrument was sensitive to the stronger signal due to the fourth order of He II $\lambda 304\AA$. The time resolution available with the photo-electric detectors enabled He II $\lambda 1216\AA$ to be distinguished from H I $\lambda 1216\AA$ present as impurity. The calibration was transferred to the grazing incidence instrument by setting both on H I $\lambda 1216\AA$ from a hydrogen plasma. For this particular study the calibration was required to cover the range from $\lambda 162\AA$ to $\lambda 266\AA$ as well as $\lambda 1240\AA$ for a study of the NV spectrum. The authors estimate the accuracy of the calibration as ± 30%.

Before leaving the question of the intensity calibration it is necessary to consider the question of the geometrical factor that enters the expression for the spectral intensities. Although they are arranged to view as closely
Table 1

<table>
<thead>
<tr>
<th>Wavelength (Å)</th>
<th>Ion</th>
<th>Transition</th>
<th>( A ) (s(^{-1}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>6562.8</td>
<td>H</td>
<td>2-3</td>
<td>4.39 \times 10(^5)</td>
</tr>
<tr>
<td>1023.7</td>
<td>H</td>
<td>1-3</td>
<td>5.54 \times 10(^5)</td>
</tr>
<tr>
<td>4685.7</td>
<td>He II</td>
<td>3-4</td>
<td>1.44 \times 10(^6)</td>
</tr>
<tr>
<td>1215.1</td>
<td>He II</td>
<td>2-4</td>
<td>1.35 \times 10(^6)</td>
</tr>
<tr>
<td>243.0</td>
<td>He II</td>
<td>1-4</td>
<td>2.04 \times 10(^6)</td>
</tr>
<tr>
<td>3203.1</td>
<td>He II</td>
<td>3-5</td>
<td>3.52 \times 10(^6)</td>
</tr>
<tr>
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<td>He II</td>
<td>1-5</td>
<td>6.60 \times 10(^6)</td>
</tr>
<tr>
<td>256.3</td>
<td>He II</td>
<td>1-3</td>
<td>8.91 \times 10(^6)</td>
</tr>
<tr>
<td>1640.4</td>
<td>He II</td>
<td>2-3</td>
<td>7.50 \times 10(^6)</td>
</tr>
<tr>
<td>4603.7</td>
<td>N v</td>
<td>3S-3P</td>
<td>4.13 \times 10(^6)</td>
</tr>
<tr>
<td>4620.0</td>
<td>N v</td>
<td>2S-3P</td>
<td>1.20 \times 10(^6)</td>
</tr>
<tr>
<td>5081.5</td>
<td>C v</td>
<td>3S-3P</td>
<td>3.18 \times 10(^6)</td>
</tr>
<tr>
<td>312.4</td>
<td>C iv</td>
<td>2S-3P</td>
<td>4.56 \times 10(^6)</td>
</tr>
<tr>
<td>3811.3</td>
<td>O vi</td>
<td>3S-3P</td>
<td>5.10 \times 10(^6)</td>
</tr>
<tr>
<td>3834.2</td>
<td>O vi</td>
<td>2S-3P</td>
<td>2.59 \times 10(^6)</td>
</tr>
</tbody>
</table>

Fig. 10.2

as possible the same region of plasma the two instruments have different values for the geometrical factor. For the visible instrument this factor can be most readily determined. Since the geometry does not change between calibration and measurement it is not necessary to measure it for the vacuum UV instruments and it may be regarded as an additional component of the calibration factor of these instruments.

(b) The measured intensity of a line is proportional to the product of the electron density and the density of the particular ions in their ground state. The latter is proportional to the number density of the particular element which in this type of study is present as a small and well determined fraction of the main gas used to fill the discharge chamber. The latter is usually hydrogen but may be helium. It must also be borne in mind that discharges of the type being discussed are subject to magnetic field compression which may cause the plasma density to be different from the density of the initial filling gas.

Two methods have been used to determine the plasma density and both have their advantages. In the first the total intensity of light scattered from a laser beam by the free electrons (Thomson scattering) is measured. This is combined with the measurement of electron temperature and is discussed in detail in a later section. The other method requires the measurement of the intensity of continuum radiation from the plasma in the visible region of the spectrum. Continuum arises from the interaction of free electrons with positive ions either through free-free transitions giving rise to bremsstrahlung or through free-bound
transitions to give recombination continuum. The theory of these two-body processes is well established so that a measure of the intensity may be expressed,

\[
I = \frac{1}{4\pi} n_e n^+ \gamma(T_e) \Delta \lambda L
\]

\(\gamma(T_e)\) is the coefficient whose value can be calculated with good accuracy. By choosing to make the measurement in the visible region of the spectrum the dependence of \(\gamma\) on \(T_e\) is made small and the accuracy of the measurement of the latter does not influence that of the density. \(\Delta \lambda\) is the bandwidth of the measurement. \(L\) is the geometrical factor that has already been mentioned. It is the presence of this factor that gives this method its great advantage over the other since this is the same geometrical factor that is required for the measurements of the spectral line intensities. Indeed the method also removes the need for an absolute calibration of the visible instrument. A relative calibration between the wavelengths used for the branching ratios calibration and that of the continuum measurement is all that is required.

In making a measurement of the continuum intensity it is important to avoid even weak spectral lines. The method of selecting a suitable region of line-free continuum is done in two stages (a) from the photograph of the spectrum a selection is made of a region apparently free of lines, (b) two similar spectrometers fitted with photomultipliers are set to observe this region from the same plasma volume (using a half-silvered mirror). One of these is set with a wide exit slit and the other with a narrow slit which is then scanned across the region of the wide slit with the photomultiplier signals being subtracted so that they almost cancel. Such a system is a very sensitive detector of weak lines and can be used to make the final selection of a line free region.

There may be a need to correct the value of \(\gamma\) which is based on the assumption that the plasma is pure hydrogen (or maybe helium) for the presence of other ions. Generally for the small quantities that are present it is sufficient to add fractions scaled as \(z^2\) where \(z\) is the ionic charge.

The major difficulty with the method is due to the possibility of the plasma being inhomogenous. The weighting of contributions from different regions in continuum is different than in the spectral lines. In over-coming this there is some advantage in combining with measurements of electron density by laser scattering.

(c) If the electron density is determined solely by laser scattering there is need to measure the geometrical factor \(L\) independently. Provided the plasma radiation fills the same aperture and the length of slit as the standard lamp the factor \(L\) reduces to the average depth of plasma viewed by the spectrometer and may be taken as the diameter of the plasma column. This is best measured by scanning across a diameter of the plasma column in one of the lines to be observed.

(d) In order to determine the fractional abundance of the ion on which measurements are being made compared with the total of all ions of the element it is necessary to resort to a calculation of ionisation rates. As already mentioned such calculations for pulsed laboratory plasmas require the solution of the time dependent equation of ionisation and (less important) recombination. For this values are required for the temperature and electron density of the plasma as a function of time. Laser scattering is used to
obtain this information as discussed below. The coefficients used in the ionisation equation are based on theory. However, the experiment is self-checking in this respect since the ionisation coefficients determine the details of the time sequence of the various spectral lines of the element studied. By multiplying the calculated ion populations by the appropriate excitation coefficients, the time sequence of the intensities may be predicted and compared with the observed sequence. Note that the excitation coefficients do not strongly influence the time sequence and that their absolute values have no relevance to the time history. Good agreement between prediction and observation is found after suitable adjustment and give a reliable estimate of the fractional ionisation. Figure 10.3 gives some indication of the extent of the agreement for the NV experiment already discussed.

Fig. 10.3

Predicted and observed intensities of nitrogen V lines
A laser scattering technique is used to determine the electron temperature and density (if need be) during the period of the plasma lifetime. Figure 8.1 also shows the arrangement of the laser apparatus for the NV experiment used to illustrate the method so far. Care has to be taken to minimise the amount of laser light scattered off the walls of the discharge vessel and is the reason for the laser dump tube. The laser light that is scattered off the free electrons is detected after dispersion in an Ebert spectrometer by a series of photomultipliers fitted with a system of fibre optics. In order to optimise the signal to noise ratio the optical system also includes (a) a polariser to discriminate against the unpolarised plasma light in favour of the polarised scattered laser light, (b) a piece of optically active quartz to rotate the plane of polarisation to coincide with the plane of greatest sensitivity of the spectrometer, and (c) a filter to exclude second order light at about $\lambda$3500$\AA$ from the plasma. It was important also to ensure that the photomultiplier circuits had equal response times to the very fast laser pulses. Finally steps had to be taken to minimise the effect of the magnetic field of the plasma on the sensitivities of the photomultipliers.

The optical arrangement of the fibre optics feeding the multipliers is illustrated in figure 10.4. Also in this figure is plotted the signal ratios from

![Diagram of fibre optics arrangement.]

which the temperature is deduced. Different channel combinations are sensitive in different temperature ranges and this also serves as a check on departures from Maxwellian distributions (which were never observed in this experiment).

From the total scattered signal the electron density is deduced. This requires the intensity calibration of the complete arrangement and is most conveniently
done by scattering the laser light off cold gas (Rayleigh scattering at a carefully measured density).

10.3 Measurements of ionisation coefficients by the technique of plasma spectroscopy

The method discussed above in section 10.2 for the determination of the fractional abundance of the ion being studied compared with all others of that element may be used to determine the ionisation coefficients for the ions of the element. To illustrate the method some recent measurements on the ions of neon using a theta-pinchi discharge will be used.

As before use is made of the time sequence of appearance of the spectral lines of the ions being studied. By comparing the observed sequence with the predicted sequence values can be deduced for the correction factors for the ionisation rate coefficients which are the most influential factors in determining the time sequence. These appear in the differential equation determining the ionisation of this element thus: (as presented earlier),

\[
\frac{dn(z)}{dt} = n_e \left\{ -n(z) S(z) + n(z-1) S(z-1) - n(z) n(z) a(z) + n(z+1) a(z+1) \right\}
\]

For a plasma that is varying with time in temperature and density it is necessary to take account of this in finding a solution and it must be borne in mind that the ionisation coefficients are often steep function of temperature. It is worth noting however that in the hypothetical case of constant temperature and density the time between instants defined by the positions of the centres of gravity of two consecutive curves describing the variation of ion population with time is about equal to the reciprocal of \( n_e S(z) \). This result is used later in correcting the theoretical values adopted in predicting the time sequence to fit the observation.

The measured temperature and density of the theta-pinchi plasma as determined by laser scattering are shown in figure 10.5. Also shown are the predicted ion populations using rate coefficients close to the values proposed by Letz (1967).

In calculating the spectral intensities the usual low density assumption is made that the intensity is given by

\[
I = \frac{1}{4\pi} n_e n(g) X(g,p) \sum_{p \gamma} A(p,\gamma) L
\]

Since there is no need to make absolute measurements of intensity or even relative intensities between lines account need only be taken of variations in the factors appearing in this formula. Their absolute values are not relevant. Variations in \( n_e \) and \( n(g) \) have already been established. But it is necessary to measure how the geometrical factor \( L \) varies by measuring the plasma diameter in spectral lines of the ions being observed. The variation of the excitation coefficient with temperature must also be included. The functional form of its variation with temperature is generally taken to be \( T^{\frac{3}{2}} \exp(-\chi/\Delta T) \) where \( \chi \) is the excitation potential of the upper level. Care must also be taken to include contributions from excitation from metastable levels.

The comparison between the observed and uncorrected predicted time sequences are shown in figure 10.6. It may be seen that the rate coefficients used for the predictions require to be decreased in order to bring them into agreement with experiment. Because of the result derived above concerning the times between the centres of gravity of
neighbouring curves the first approximation to the correction may be obtained by decreasing the corresponding ionisation coefficients by the ratio of the times between predicted and observed peaks of intensity. With the modified coefficients the time sequence can be predicted once more and, if need be, iterated to agreement. This has not yet been done for the present data which is still being worked on.
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