I. Introduction

The sun is a unique astronomical object in that its proximity allows its electromagnetic spectrum to be observed from the γ ray to the radio regions, corresponding to 18 decades in energy. Additionally, for significant portions of this range, it is possible to combine high spectral, temporal, and spatial resolutions. For example, the ultraviolet spectrometer (Woodgate et al., 1980) on the Solar Maximum Mission Satellite has a resolving power of typically 10⁴ and is capable of obtaining a significant signal in an emission line in 64 msec from a spatial area of 3'' × 3'' on the solar disk. Hence, the spectroscopic information available for the sun is more detailed than that for any other astronomical plasma.

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Fig. 1. A typical model of the electron density and temperature structure in the solar atmosphere. The photospheric region results are from Vernazza et al. (1976) and transition region and corona from Dupree (1972).
In order to interpret these observational results, it is necessary to have high-quality atomic physics data, the type of data depending on the region of the solar atmosphere being studied. For example, the visible and UV continuum radiation arise mainly from deep layers of the solar atmosphere, known as the photosphere (see Fig. 1). This region has typically an electron temperature of 5000 K and a gas density of $10^{17}$ particles cm$^{-3}$, with the most abundant element hydrogen being principally neutral or in the form of the negative hydrogen ion. Here the most important processes for determining the continuum fluxes are the photoionization and free-free rates (and the inverse radiative rates) for hydrogen, H$^-$ and, in the UV region, the more abundant heavier ions such as C I, Si I, etc. Many of the weaker absorption lines due to atomic and molecular species are also produced in the photospheric region, and to predict their strengths it is necessary to have accurate values for their oscillator strengths and their radiative and impact damping constants. For the former, recent advances in experimental techniques mean that for some ions, such as a neutral iron, accuracies of better than 1% are now possible (see, for example, Blackwell et al., 1979). A good description of the variety of data required for predicting the photospheric spectrum is given by Lambert (1978).

The cores of stronger lines, where the line opacity is large, are formed higher in the atmosphere in the chromospheric region (see Fig. 1). Here electron temperatures range from 5000 to 30,000 K, and hydrogen now starts to ionize. Also the population of ionic levels now differs significantly from that predicted by the Boltzmann and Saha equations, and it is necessary to consider the interaction of the radiation field and the ions on a microscopic level. Hence, all atomic processes that populate or depopulate ionic levels such as photoexcitation and ionization and electron collisional excitation and ionization, together with their inverse processes, are potentially important in determining the population of ionic levels. Also when considering the radiation field it is necessary to consider both absorption (including free-free transitions) and scattering processes. For the latter, line scattering can lead to a significant frequency redistribution in the cores of the profiles due to both Doppler effects from the thermal motion of the scattering ion and to the energy uncertainty due to the finite lifetime of the excited state. The extreme complexity of this problem has limited progress, although in the last few years, realistic calculations of the Ca II and Mg II H- and K-line cores have been made by Linsky and his co-workers (see, for example, Basri et al., 1979).

In the outermost layers of the atmosphere, called the corona, extremely high electron temperatures in the range $10^7$ to $5 \times 10^7$ K exist, together with lower electron densities of the order of $10^7$ cm$^{-3}$. Under such circumstances, hydrogen, helium, and the lighter elements are fully ionized with
heavier elements, such as the cosmically abundant element iron, being either fully stripped or in high ionization stages. The transfer from the chromosphere to coronal conditions occurs over a small height range of approximately 100 km, known appropriately as the transition region. Here the degree of ionization of elements heavier than helium (hydrogen and helium are effectively fully ionized) varies markedly with height, and together with the corona produces a rich emission line spectra by electron collisional excitation followed by spontaneous emission. These emission spectra lie predominately in the UV and X-ray regions, with the more highly ionized species producing the lines at the shorter wavelengths.

The above outline assumes that the sun is spatial homogeneous and hence that there is a unique atmospheric structure. Observations show that this is not the case. For example, solar photographs taken in the core of the hydrogen Balmer α line show a complicated network structure associated with the local magnetic fields, while in the transition region there are areas characterized as quiet or active whose emission line intensities can differ by an order of magnitude or more. In the corona, there are also marked differences with regions of low intensity called coronal holes. Additionally there are time variations in the solar atmosphere, the most spectacular of these being associated with flares. These explosive events stretch down from the corona, through the transition region, and into the chromosphere and are associated with active regions. The mechanisms controlling their production and evolution are at present of considerable interest and the Solar Maximum Mission Satellite launched in February 1980 has as its primary aim the study of solar flares from the γ ray to the visible spectral regions.

The availability in the last decade of rockets and satellites as platforms for quantitative spectroscopy at wavelengths of less than 3000 Å has led to extensive observation of the solar emission line spectrum. Therefore, the scope of this review will be limited to the atomic physics relevant to the study of the regions producing this spectra, i.e., the transition region and the corona and transient phenomena such as flares. However, the variety of ionization stages and atomic processes considered is such that many of the data discussed will also be relevant to studies of the solar photosphere and chromosphere and to other astronomical objects, such as flare stars, H II regions, and planetary nebulae, which have emission line spectra. Although the atomic physics data and their application will be discussed generally we will, in many cases, use the four-electron Be-like sequence as a representative example. This sequence is chosen as one both with a rich solar line spectrum and because the relevant atomic data have been intensively studied.
II. Atomic Spectra

When attempting to interpret the UV and X-ray region of the solar spectrum it is essential to have a comprehensive knowledge of the wavelengths of the spectral lines of each ion. Primarily the energy structure is used to identify the rich emission line spectra, and hence it is important to know the energy structure to very high accuracy. As we shall see in the following section, *ab initio* theoretical calculations of atomic energy levels may give wavelengths which are accurate to better than 1%, but in general this is not of sufficient accuracy to be used for line identification. In many cases observations of the radiation from laboratory plasmas, produced, for example, by theta pinches, intense laser beams, or beam foil stripping, can supply the ionic energy levels. However, in other cases, particularly for ions with even numbers of electrons which have low lying metastable levels, laboratory plasmas are inadequate because the high laboratory densities coupled with the low ionic column densities, compared with those observed in the sum, tend to mitigate against spin-forbidden transitions. Hence, for example, the \((2s2p)^3P_0 \rightarrow (2s^2)^1S_0\) transitions, which are strong features for all abundant ions in the sun, are not usually observable in laboratory plasmas. To determine the energy structure of these ions it is necessary to use semiempirical fits to extrapolate along isoelectronic sequences (Edlén, 1979), or indeed to use the sun as a laboratory source. The achievement of high wavelength stability in UV and X-ray spectrographs flown on earth-orbiting satellites has resulted in the latter method being extensively used (Edlén, 1979). A comprehensive review of the classification of the spectra of ionized atoms has been given by Fawcett (1974, 1975), and a useful list of the wavelengths of lines of Be-like ions is given by Edlén (1979).

A knowledge of the ionic structure is not only important for line identification but it is also required as a guide to the atomic processes which may be important in determining the level populations of excited states and hence the strength of the emission line spectra. For example, the energy separation of LS degenerate fine structure levels may well determine whether electron or proton collision rates are sufficient to ensure that the corresponding ionic populations follow Boltzmann statistics. Since a very wide range of ionization stages are observed in the solar spectrum, it is found that there are significant quantitative differences among different ions even if they are in the same isoelectronic sequence. This is illustrated in Fig. 2, which shows the energy diagrams for the 46 lowest levels of three ions in the Be sequence, C III, Si XI, and Fe XXIII. In the figure the energy scale \(E'\) is normalized to the ionization potential of the ion.
Fig. 2. Energy levels for (2I2l') and (2I3l') states in the Be-like ions C III, Si XI, and Fe XXIII. The energies have been normalized to the ionization potential of the relevant ion.
A number of trends are observed as the nuclear charge $Z$ of the ion increases. For C III the $(2l2l')$ states are well separated, but as $Z$ increases, these states become closer together, and a similar pattern is observed for the $(2l3l')$ states. This is due to the fact that as $Z$ increases, the system becomes hydrogenic as the electron–nuclear interaction becomes greater than the electron–electron interaction. The energy difference for $\Delta n \neq 0$ transitions increases as $Z^2$, while the difference for $\Delta n = 0$ transitions increases as $Z$. It is also seen that as $Z$ increases, the spin–orbit interaction increases and the splitting of LS levels into their separate $J$ components becomes more pronounced. For example, in C III the splitting in the $(2s2p)^3P^0_{J=0} \rightarrow (2s2p)^3P^0_{J=2}$ levels is only 0.15% of the $(2s^2)^1S \rightarrow (2s2p)^3P^0$ energy difference, compared with 34% for Fe XXIII.

The energy level diagram also provides a useful guide to the approximations which are appropriate in a particular theoretical calculation. For example, the very large energy differences in the $(2l2l')$ levels in C III would suggest that in C III calculations it is important to take careful account of electron–electron interaction, while the large energy splitting in the $^3P$ levels of Fe XXIII would suggest that it is necessary to include the spin–orbit interaction in Fe XXIII calculations. The diagram also indicates that for C III a close-coupling calculation would give good results by including $(2l2l')$ and $(2s3l)$ states as these states are well separated from the $(2p3l)$ states, but for Fe XXIII the $(2p3l)$ states lie close to the $(2s3l)$ states and would have to be included in a calculation.

III. Bound-State Wave Functions

In order to interpret the solar spectrum it is important to have an accurate knowledge of the basic atomic processes producing the radiation. For example, in the case of emission lines it is usually necessary to know the rates of spontaneous decay and the rate of electron excitation to a state. Unfortunately, it is difficult to study multiply charged ions in laboratory plasmas, and it is often necessary to use ab initio quantum mechanical calculations to obtain estimates for the rates of the basic atomic processes. For such calculations it is necessary to have accurate wave functions for the states which are being considered. Over the last ten years there have been considerable theoretical advances in the calculation of nonrelativistic wave functions and their use in calculating atomic rates (Oksüz and Sinanoglu, 1969; Sims and Whitten, 1973; Eissner et al., 1974; Laughlin and Victor, 1974; Nicolaides and Beck, 1975; Watson and O'Neil, 1975; Nussbaumer and Storey, 1978, 1979b; Hibbert, 1980).
Progress has also been made by Kim and Desclaux (1976) and Armstrong et al. (1976) in calculating atomic data using the relativistic multiconfigurational Hartree–Fock method. The relativistic random-phase approximation has also been used to calculate atomic lifetimes (Lin and Johnson, 1977; Cheng and Johnson, 1977).

In this review we shall consider in detail the configuration interaction wave functions generated by the computer program CIV3 (Hibbert, 1970; Burke et al., 1972; Hibbert, 1975; Glass and Hibbert, 1976, 1978b). These wave functions have been used extensively in both atomic lifetime calculations and electron excitation calculations.

For light atoms relativistic effects are small, and the Hamiltonian for an atom or ion may be written as

$$H_{NR} = \sum_{i=1}^{N} \left( -\frac{1}{2} \nabla_i^2 - Z/r_i \right) + \sum_{i>j}^{N} 1/r_{ij} \tag{1}$$

where \( N \) is the total number of electrons and \( Z \) is the nuclear charge. Since this Hamiltonian commutes with the total spin and orbital angular momentum operators \( S^2 \) and \( L^2 \) and their \( z \) components \( S_z \) and \( L_z \), we seek wave functions of the form \( \Psi(LSM_LM_S) \).

A configuration interaction (CI) atomic state wave function is represented by a configuration interaction expansion of the form

$$\Psi_i(LS) = \sum_{j=1}^{M} a_{ij} |\psi_j(\alpha_iLS)\rangle \tag{2}$$

where \( L \) and \( S \) are the total spin and orbital angular quantum numbers of the state and \( \alpha \) defines the coupling scheme. Each of the configuration wave functions \( \psi_j(\alpha_iLS) \), which is, in effect, a linear combination of determinants, is defined with respect to its subshells:

$$\psi_j(\alpha_iLS) = \{(1s)^{\lambda_1}(\alpha_1L_1S_1)(2s)^{\lambda_2}(\alpha_2L_2S_2) \cdots \alpha L S\} \tag{3}$$

so that each subshell, with \( \lambda_j \) electrons, has a set of quantum numbers \( \alpha_jL_jS_j \) defining the seniority and the orbit and spin angular momenta of the subshell. The subshells are coupled to form configuration wave functions \( \psi_j(\alpha_iLS) \), which are eigenfunctions of \( L^2 \) and \( S^2 \). The subshells are themselves constructed from one electron orbitals, which are products of a radial function, a spherical harmonic, and a spin function:

$$\nu_{n_l m_l} = (1/r) P_{n_l}(r) Y_{l}^{m_l}(\theta, \phi) \chi(m_s) \tag{4}$$

In his work Hibbert writes the radial functions in terms of Slater-type orbitals:

$$P_{n_l}(r) = \sum_{j=1}^{k} c_{jnl} r^{l_{jnl}} e^{-\kappa_{jnl} r} \tag{5}$$
and orthonormalizes his orbitals to satisfy

$$\int_0^\infty P_{nl}(r)P_{n'l}(r) \, dr = \delta_{nn'} \quad \text{for} \quad l + 1 \leq n' \leq n \quad (6)$$

With an initial estimate for the parameters in $P_{nl}(r)$, a Hamiltonian matrix can be constructed with matrix elements given by

$$H_{ij} = \langle \Psi_i | H_{NR} | \Psi_j \rangle \quad (7)$$

This matrix can then be diagonalized. The resulting eigenvalues are upper bounds to the corresponding exact energies, and the associated eigenvectors provide the coefficients $a_{ij}$ in Eq. (2) (Perkins, 1965). A different choice of the initial estimate for the parameters in $P_{nl}$ will, of course, lead to different eigenvalues, and so because of the upper-bound property the eigenvalue can be treated as a function to be minimized with respect to variations of the parameters in $P_{nl}(r)$. This procedure has been computerized and is available in a program called CIV3 (Hibbert, 1970, 1975; Glass and Hibbert, 1978b).

As an example, in calculations of the wave functions of Be-like ions the 1s, 2s, and 2p orbitals are normally taken from Hartree–Fock calculations, these being calculations which only take one term in the expansion of the wave function in Eq. (2), and hence do not take account of configuration interaction. These three orbitals could be used to construct the simplest configuration interaction wave function for the six lowest levels of a Be-like ion in which the $^1S_e$ states would use two configurations, viz., $(1s)^2(2s)^2$, and $(1s)^2(2p)^2$, the other four states being represented by a single configuration of the form $(1s)^2(2s2p)$ or $(1s)^2(2p)^2$. The accuracy of such CI wave functions can be increased by including more one-electron orbitals and hence increasing the number of configurations in Eq. (2). For example, Burke et al. (1972) included 3s, 3p, and 3d orbitals in their calculations on the three lowest levels of Be-like ions Be I to Ne VII. They varied the parameters in the radial functions to optimize the lowest eigenvalue of the Hamiltonian matrix formed from the following seven configurations of the ground state, $^1S_e$:

$$\begin{align*}
& (1s^2)(2s^2), \quad (1s^2)(2s\ 3s), \quad (1s^2)(3s^2), \quad (1s^2)(2p^2) \\
& (1s^2)(2p3p), \quad (1s^2)(3p^2), \quad (1s^2)(3d^2) 
\end{align*} \quad (8)$$

These optimized radial functions were then used to calculate the Hamiltonian matrix, which in turn gave the lowest eigenvalues and corresponding eigenvectors of each of the states $^1S$, $^3P$, and $^1P$. The following six configurations were used for the $^3P$ and $^1P$ states:
(1s²)(2s2p), (1s²)(3s2p), (1s²)(2s3p), (1s²)(3s3p)
(1s²)(3p3d), (1s²)(2p3d)

while the seven configurations listed in Eq. (8) were used for the \(^1\text{S}\) state.

In Table I we list the energies of the ground \((2s^2)\) \(^1\text{S}\) state and lowest \((2s2p)\) \(^1\text{P}\) state of some Be-like ions which were calculated using CI wave functions with an increasing number of configurations. It is clear that for the total energy of the ground state the simplest CI function gives a significant improvement compared to the Hartree–Fock function, but the addition of further CI terms produces only small improvements in the total energy. The Hartree–Fock energies of the excited \(^1\text{P}\) states are more accurate than Hartree–Fock energies of the ground \(^1\text{S}\) states, but the CI functions again produce small improvements. In the two cases for which we have a total energy from experiment, the most elaborate CI functions give energies which are considerably higher than the experimental energy. This is chiefly due to the fact that in these calculations the two inner 1s core electrons are “frozen” and do not interact with the outer electrons. However, when we consider the energy difference between two states, the error introduced by this approximation cancels and we obtain quite reliable energy differences.

Table I also shows that as the nuclear charge increases, the difference between the experimental and theoretical excitation energy increases considerably. This divergence of theory from experiment can be chiefly attributed to the neglect of relativistic effects in these theoretical calculations. These effects are particularly important in the \((1s^2)\) core.

Full relativistic calculations for Be-like ions have been carried out by several groups. Kim and Desclaux (1976) carried out relativistic Hartree–Fock calculations, and Armstrong et al. (1976) used multiconfiguration Dirac–Hartree–Fock calculations, while Lin and Johnson (1977) and Cheng and Johnson (1977) used the relativistic random-phase approximation. In Fig. 3 we compare the theoretical energy differences of the two lowest transitions \((2s^2) \rightarrow (2s2p) \rightarrow (2s3p)\) and \((2s^2) \rightarrow (2s2p) \rightarrow (2s3p)\) calculated by Armstrong et al. (1976) using both relativistic and nonrelativistic wave functions. In order to make the comparison simpler, we have, in fact, plotted the nuclear charge, \(Z\), against the excitation energy divided by \(Z\). For large \(Z\) the nonrelativistic energy difference for \(\Delta n = 0\) transitions increases as \(Z\), and our plots tend to a constant at large \(Z\). The relativistic calculations of the energy difference increase much more rapidly than \(Z\) for large \(Z\). When \(Z\) is small, relativistic effects are small, for example, for the \(^1\text{S} \rightarrow \(^1\text{P}\) transition in \(F\) \(VI\) relativistic effects increase the energy difference by about 0.5%. However, as \(Z\) increases, the effect is much larger, the increases for \(Ar\) \(XV\) and \(Kr\) \(XXXIII\) being 5 and 50%,
### Table I

**Theoretical Nonrelativistic Energies $E$ and Energy Differences $\Delta E$ (in a.u.) of the $(1s^2)(2s^2)^1S$ and $(1s^2)(2s2p)^1P$ States of C III, Si XI and Fe XXIII Calculated Using Configuration Interaction (CI) Wavefunctions with Different Numbers of Orbitals**

<table>
<thead>
<tr>
<th>Orbitals used in CI expansion</th>
<th>Hartree–Fock $1s, 2s, 2p$</th>
<th>CI $1s, 2s, 2p$</th>
<th>CI: $\frac{1s, 2s, 2p}{3s, 3p, 3d}$</th>
<th>CI: $\frac{1s, 2s, 2p}{3s, 3p, 3d, 4f}$</th>
<th>Experiment</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>C III</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$E(1^S)$</td>
<td>$-36.4087^a$</td>
<td>$-36.4770^a$</td>
<td>$-36.47742^b$</td>
<td>$-36.54747^c$</td>
<td></td>
</tr>
<tr>
<td>$E(1^P)$</td>
<td>$-35.9603^a$</td>
<td>$-35.9603^a$</td>
<td>$-36.00698^b$</td>
<td>$-36.08110^c$</td>
<td></td>
</tr>
<tr>
<td>$\Delta E = E(1^P) - E(1^S)$</td>
<td>$0.4484^a$</td>
<td>$0.5168^a$</td>
<td>$0.47044^b$</td>
<td>$0.46637^c$</td>
<td></td>
</tr>
<tr>
<td><strong>Si XI</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$E(1^S)$</td>
<td>$-223.8149^d$</td>
<td>$-223.9760^d$</td>
<td>$-223.9911^d$</td>
<td>$-223.9911^d$</td>
<td>$-224.56119^c$</td>
</tr>
<tr>
<td>$E(1^P)$</td>
<td>$-222.4599^d$</td>
<td>$-222.4599^d$</td>
<td>$-222.5064^d$</td>
<td>$-222.5105^d$</td>
<td>$-223.0604^c$</td>
</tr>
<tr>
<td>$\Delta E = E(1^P) - E(1^S)$</td>
<td>$1.3550^d$</td>
<td>$1.5161^d$</td>
<td>$1.4847^d$</td>
<td>$1.4806^d$</td>
<td>$1.5008^c$</td>
</tr>
<tr>
<td><strong>Fe XXIII</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$E(1^S)$</td>
<td>$-804.9571^d$</td>
<td>$-805.2595^d$</td>
<td>$-805.2760^d$</td>
<td>$-805.2760^d$</td>
<td></td>
</tr>
<tr>
<td>$E(1^P)$</td>
<td>$-802.2565^d$</td>
<td>$-802.2565^d$</td>
<td>$-802.3032^d$</td>
<td>$-802.3079^d$</td>
<td></td>
</tr>
<tr>
<td>$\Delta E = E(1^P) - E(1^S)$</td>
<td>$2.7006^d$</td>
<td>$3.0030^d$</td>
<td>$2.9728^d$</td>
<td>$2.9681^d$</td>
<td>$3.4302^d$</td>
</tr>
</tbody>
</table>

$^a$ Burke et al. (1972).

$^b$ Berrington et al. (1977).

$^c$ Kelly and Palumbo (1973).

$^d$ Glass (1979a) and private communication.
respectively, while for Be-like Pb, the increase is more than a factor of 10.

There is no doubt that for very heavy ions it is necessary to carry out full relativistic calculations. These calculations are much more difficult to perform than nonrelativistic calculations, and to increase the accuracy of the calculations requires considerable labor. Fortunately, the abundance of heavy ions in the sun is very small, and in general we only need to consider ions up to Fe XXIII. For these ions relativistic effects are important enough to be included in a calculation, but it is not necessary to carry out a full relativistic treatment.

If, for example, we consider the full relativistic calculations of Armstrong et al. (1976), using the equivalent of a 1s, 2s, 2p CI wave function, their relativistic results for the excitation energy of the $^1S \rightarrow ^1P$ transitions in C III and Fe XXIII differ by 12 and 2%, respectively, from the experimental values, the difference being mainly due to the simplicity of the wave function. For the same transitions more complicated non-
relativistic CI wave functions (Table I) gave results which were in error of 0.9% for C III and 15% for Fe XXIII, the difference here being due chiefly to the neglect of relativistic effects. Clearly it is desirable to use a method which will use accurate CI wave functions but which also allow us to include relativistic effects.

One method for doing this has been given by Hibbert and Glass (Glass and Hibbert, 1978a,b). They use the Breit–Pauli approximation, which is valid for low $Z$ ($Z \ll 137$), and write the Hamiltonian in the form

$$H_{BP} = H_{NR} + H_{Rel}$$  \hspace{1cm} (10)

where $H_{NR}$ is the nonrelativistic Hamiltonian and $H_{Rel}$ is the relativistic part of the Hamiltonian, which may be written as

$$H_{Rel} = H_{so} + H_{mass} + H_{D1} + H_{soo} + H_{ss} + H_{oo} + H_{D2} + H_{sse}$$  \hspace{1cm} (11)

$H_{so}$ is the one-body spin–orbit interaction of each electron magnetic moment with the magnetic field which it produces by its motion in the field of the nucleus; $H_{mass}$ is the relativistic mass correction; $H_{D1}$ the one-body Darwin term, the relativistic correction to the potential energy; $H_{soo}$ the spin–other orbit interaction is the sum of the spin–orbit coupling of an electron in the Coulomb field of another electron and the interaction of the spin–magnetic moment of an electron with the orbital current of another electron; $H_{ss}$ is the dipole interaction of the spin–magnetic moments of two electrons; $H_{oo}$ the orbit–orbit interaction; $H_{D2}$ the two-body Darwin term, the relativistic correction to the potential energy; and $H_{sse}$ the electron spin contact term.

It is convenient to rewrite

$$H_{BP} = \text{nonfine structure} + \text{fine structure}$$  \hspace{1cm} (12)

where

$$\text{nonfine structure} = H_{NR} + H_{mass} + H_{D1} + H_{oo} + H_{D2} + H_{sse}$$  \hspace{1cm} (13)

$$\text{fine structure} = H_{so} + H_{soo} + H_{ss}$$  \hspace{1cm} (14)

the nonfine structure terms commute with $L^2$, $S_z$, $L_z$, and $L_z$ and the fine structure terms commute with $J^2$ and $J_z$, where $J$ is the total angular momentum and $J_z$ is its azimuthal component. The $J$-dependent configuration interaction (CI) expansion takes the form

$$\Phi_i(J, M_J) = \sum_{j=1}^{M} b_{ij}\Phi_j(\alpha_jL_jS_jJM_J)$$  \hspace{1cm} (15)

Here we allow for the breakdown of LS coupling by adopting an intermediate coupling scheme in which the sum over $j$ includes all configurations
for which
\[ L_j + S_j = J \] (16)

The configurations are constructed from the one-electron orbitals given by Eqs. (4) and (5). For very heavy systems relativistic effects produce changes in the radial functions, the inner orbitals are contracted, and the valence orbitals are expanded (Burke and Grant, 1967; Grant, 1970, 1979). However, we will assume that these effects are negligible for systems in which the Breit–Pauli Hamiltonian is valid \((Z \ll 137)\), and we therefore determine the parameters in the one-electron orbitals by using only the nonrelativistic Hamiltonian in the optimization process. The coefficients \(\{b_{ij}\}\) are then determined as the eigenvectors of the Hamiltonian matrix with basis \(\{\Phi_i\}\), where the Hamiltonian is now the full nonrelativistic Hamiltonian plus relativistic corrections.

More formally for a given parity \(\pi\) we can write Eq. (15) as
\[
|\Delta_i J_i M_{J_i} \pi_i \rangle = \sum_{C_{j}L_{j}S_{j}} \langle \Delta_i J_i \pi_i | C_{j}L_{j}S_{j} \pi_i \rangle |C_{j}(L_{j}S_{j}) J_i M_{J_i} \pi_i \rangle
\] (17)

where \(|\Delta_i J_i M_{J_i} \pi_i \rangle\) is the atomic state function \(\Phi_i\), \(|C_{j}(L_{j}S_{j}) J_i M_{J_i} \pi_i \rangle\) the single configuration \(\phi_j\), and \(\langle \Delta_i J_i \pi_i | C_{j}L_{j}S_{j} \pi_i \rangle = b_{ij}\) is called the mixing coefficient. \(L_j\) and \(S_j\) are, respectively, the total orbital and spin angular momentum of the configuration, while \(\Delta_i\) distinguishes states with the same \(J_i\) but different energies, and \(C_j\) distinguishes configurations with the same \(L_j S_j\). We can also write Eq. (2) as
\[
|\Gamma_j L_j S_j M_{L_j} M_{S_j} \pi_j \rangle = \sum_{C_j} \langle \Gamma_j L_j S_j \pi_j | C_{j}L_{j}S_{j} \pi_i \rangle |C_{j}L_{j}S_{j} M_{L_j} M_{S_j} \pi_j \rangle
\] (18)

\(\Gamma_j\) distinguishes states with the same \(L_j S_j\) but different energies. As this is a unitary transformation, we may write
\[
|C_{j}L_{j}S_{j} M_{L_j} M_{S_j} \pi_j \rangle = \sum_{\pi_j} \langle C_{j}L_{j}S_{j} \pi_j | \Gamma_j L_j S_j \pi_j \rangle |\Gamma_j L_j S_j M_{L_j} M_{S_j} \pi_j \rangle
\] (19)

which with
\[
|C_{j}(L_{j}S_{j}) J_i M_{J_i} \pi_i \rangle = \sum_{M_{L_j} M_{S_j}} C(L_{j}S_{j}J_i : M_{L_j} M_{S_j} M_{J_i}) |C_{j}L_{j}S_{j} M_{L_j} M_{S_j} \pi_j \rangle
\] (20)
gives Eq. (17) as
\[
|\Delta_i J_i M_{J_i} \pi_i \rangle = \sum_{L_j S_j C_{j} \pi_j} \langle \Delta_i J_i \pi_i | C_{j}L_{j}S_{j} \pi_i \rangle \langle C_{j}L_{j}S_{j} \pi_i | \Gamma_j L_j S_j \pi_j \rangle \times |\Gamma_j(L_j S_j) J_i M_{J_i} \pi_i \rangle
\] (21)

we write
\[
f_{ij} = \langle \Delta_i J_i \pi_i | \Gamma_j L_j S_j \pi_j \rangle = \sum_{C_j} \langle \Delta_i J_i \pi_i | C_{j}L_{j}S_{j} \pi_i \rangle \langle C_{j}L_{j}S_{j} \pi_i | \Gamma_j L_j S_j \pi_j \rangle
\] (22)
This is called a "term-coupling" coefficient (Jones, 1975; Saraph, 1972). The atomic state wave functions \( \{ \Phi_i \} \) may be expanded in terms of the single configuration basis \( \{ \phi_i \} \) and mixing coefficients \( \{ b_{ij} \} \) or in terms of the CI basis \( \{ \Psi_i \} \) and term-coupling coefficients \( \{ f_{ij} \} \).

As an example of a nonrelativistic configuration interaction wave function and a relativistic configuration interaction wave function we give the wave functions for Ca XVII using only 1s, 2s, and 2p orbitals, and with the \( (1s^2) \) core "frozen," we have from Eq. (2)

\[
\begin{align*}
\Psi[(2s^2) \, ^1S^e] &= -0.972\psi[(2s^2) \, ^1S^e] + 0.236\psi[(2p^2) \, ^1S^e] \\
\Psi[(2s2p) \, ^3P^o] &= 1.00\psi[(2s2p) \, ^3P^o] \\
\Psi[(2s2p) \, ^1P^o] &= 1.00\psi[(2s2p) \, ^1P^o] \\
\Psi[(2p^2) \, ^3P^e] &= 1.00\psi[(2p^2) \, ^3P^e] \\
\Psi[(2p^2) \, ^1D^e] &= 1.00\psi[(2p^2) \, ^1D^e] \\
\Psi[(2p^2) \, ^1S^e] &= 0.236\psi[(2s^2) \, ^1S^e] + 0.972\psi[(2p^2) \, ^1S^e]
\end{align*}
\] (23)

If we now take the Breit–Pauli Hamiltonian with the spin–orbit mass correction and Darwin terms in addition to the nonrelativistic operators and diagonalize the Hamiltonian matrix to obtain the mixing coefficients \( \{ b_{ij} \} \), then Eq. (15) becomes

\[
\begin{align*}
\Phi[^1S_0^e] &= -0.975\phi[(2s^2) \, ^1S_0^e] + 0.012\phi[(2p^2) \, ^3P_0^e] + 0.222\phi[(2p^2) \, ^1S_0^e] \\
\Phi[^3P_0^o] &= 1.00\phi[(2s2p) \, ^3P_0^o] \\
\Phi[^3P_0^e] &= 0.997\phi[(2s2p) \, ^3P_0^e] + 0.072\phi[(2s2p) \, ^1P_0^e] \\
\Phi[^3P_0^e] &= 1.00\phi[(2s2p) \, ^3P_0^e] \\
\Phi[^1P_0^e] &= 0.072\phi[(2s2p) \, ^3P_0^e] - 0.997\phi[(2s2p) \, ^1P_0^e] \\
\Phi[^3P_0^e] &= -0.039\phi[(2s^2) \, ^1S_0^e] - 0.992\phi[(2p^2) \, ^3P_0^e] - 0.120\phi[(2p^2) \, ^1S_0^e] \\
\Phi[^3P_1^e] &= 1.00\phi[(2p^2) \, ^3P_1^e] \\
\Phi[^3P_2^e] &= -0.970\phi[(2p^2) \, ^3P_2^e] + 0.241\phi[(2p^2) \, ^1D_2^e] \\
\Phi[^1D_2^e] &= 0.241\phi[(2p^2) \, ^3P_2^e] + 0.970\phi[(2p^2) \, ^1D_2^e] \\
\Phi[^1S_0^e] &= 0.219\phi[(2s^2) \, ^1S_0^e] - 0.126\phi[(2p^2) \, ^3P_0^e] + 0.968\phi[(2p^2) \, ^1S_0^e]
\end{align*}
\] (24)

We note the large mixing of \( ^3P_0^e \) and \( ^1D_2^e \) and also \( ^3P_0^e \) and \( (2p^2) \, ^1S_0^e \). There is also smaller mixing of \( (2s^2) \, ^1S_0^e \) and \( ^3P_0^e \), as well as \( ^3P_1^e \) and \( ^1P_0^e \).

Combining Eqs. (23) and (24), for example, for \( J = 0 \) even, we obtain the term-coupling coefficients \( f_{ij} \) to give Eq. (21):
(2s²)¹S₀ = 0.9998(2s²)¹S₀ + 0.0118(2p²)³P₀ − 0.0141(2p²)¹S₀

(2p²)³P₀ = 0.0099(2s²)¹S₀ − 0.9920(2p²)³P₀ − 0.1261(2p²)¹S₀

(2p²)¹S₀ = 0.0155(2s²)¹S₀ − 0.1260(2p²)³P₀ − 0.9919(2p²)¹S₀

An interesting set of calculations has been carried out by Glass (1979a) on the Be-like ions using a CI wave function with seven orbitals in the Breit–Pauli approximation, with different terms in Eq. (11) being used in the Breit–Pauli Hamiltonian. Table II is a comparison of his results for the excitation energy of (2s²) ¹S → (2s2p) ¹P for Si XI and Fe XXIII with experimental results, with Hartree–Fock calculations, and with the full relativistic calculations of Armstrong et al. (1976). The terms $H_{so}$, $H_{sos}$, and $H_{ss}$ only change the energy difference slightly; a much more marked effect is produced by the inclusion of the $H_{mass}$ and $H_{D1}$ terms only. The $H_{so}$ term added to these produces a small but significant effect, and adding on the $H_{sos}$ and $H_{ss}$ produces a very small change. From Table II it is clear that by using a CI wave function with the above five terms in the Breit–Pauli Hamiltonian we can produce theoretical energy differences which differ from the experimental value by about 1% for Be-like ions from C III to Fe XXIII, the CI wave function being mainly responsible for the good agreement at low Z and the Breit–Pauli approximation being mainly responsible for the agreement at large values of Z.

IV. Spontaneous Decay of Bound States

Over the last decade considerable advances have been made in the measurement and calculation of lifetimes of excited states of positive ions. Beam foil experiments have provided useful data for a large number of transitions in a variety of ions (see, for example, Pegg et al., 1979, and other papers in that journal). Accurate theoretical calculations by a number of groups have also provided a large amount of useful data (Glass, 1979a; Nussbaumer and Storey, 1979a,b; Laughlin et al., 1978; Victorov and Safronova, 1977; Hibbert, 1980). At present the best theoretical results are in general more reliable than the data obtained from experiment, and we shall concentrate on the theoretical results. If one neglects both the spatial variation of the electric vector of the radiation field in the vicinity of an atom and the interaction between the atom and the magnetic vector of the radiation field, then the electric dipole transition probability of a spontaneous radiative transition from an excited atomic state $a$ to a
<table>
<thead>
<tr>
<th>Orbitals used in CI expansion</th>
<th>Hartree–Fock 1s, 2s, 2p N Rel</th>
<th>CI: 1s, 2s, 2p N Rel</th>
<th>CI: 1s, 2s, 2p, 3s, 3p, 3d, 4f N Rel</th>
<th>Rel$^a$</th>
<th>Rel$^b$</th>
<th>Rel$^c$</th>
<th>Rel$^d$</th>
<th>Rel$^e$</th>
<th>Experiment</th>
</tr>
</thead>
<tbody>
<tr>
<td>C III</td>
<td>0.4484</td>
<td>0.5168</td>
<td>0.5204</td>
<td>0.47044</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>0.46637</td>
</tr>
<tr>
<td>Si XI</td>
<td>1.3550</td>
<td>1.5161</td>
<td>—</td>
<td>1.4806</td>
<td>1.4809</td>
<td>1.5136</td>
<td>1.5141</td>
<td>1.5140</td>
<td>1.5008</td>
</tr>
<tr>
<td>Fe XXIII</td>
<td>2.7006</td>
<td>3.0030</td>
<td>3.504</td>
<td>2.9681</td>
<td>3.0124</td>
<td>3.4251</td>
<td>3.4790</td>
<td>3.4699</td>
<td>3.4302</td>
</tr>
</tbody>
</table>

$^a$ Full relativistic calculations Armstrong et al. (1976).
$^b$ Relativistic calculations of Glass (1979a), with $H_{NR} + H_{so} + H_{soo} + H_{ls}$.
$^c$ With $H_{NR} + H_{mass} + H_{D1}$.
$^d$ With $H_{NR} + H_{so} + H_{mass} + H_{D1}$.
$^e$ With $H_{NR} + H_{so} + H_{soo} + H_{ls} + H_{mass} + H_{D1}$.
lower state \( b \) is

\[
A_{E1}(a \rightarrow b) = (64\pi^4/3h\lambda^3)S(b, a)
\]

(26)

where

\[
S(b, a) = e^2 \left| \sum_{i=1}^{N} (b|\mathbf{r}_i|a) \right|^2
\]

(27)

\[
= S(a, b)
\]

(28)

and the wavelength of the transition

\[
\lambda = c/\nu
\]

(29)

with \( h\nu = (E_b - E_a) \) being the energy difference of the transition. Electric dipole transitions are often called \( E1 \) transitions to distinguish them from electric quadrupole (\( E2 \)) and high-order transitions. If we write the normalized spherical harmonic as \( Y_{1\mu}(\hat{r}) \) and define \( C_{1\mu} = (4\pi/3)^{1/2}Y_{1\mu} \), then it may be shown that

\[
|\langle b|\mathbf{r}_i|a\rangle|^2 = \sum_{\mu} |\langle b|\mathbf{r}_iC_{1\mu}(\hat{r}_i)|a\rangle|^2
\]

(30)

This matrix element may also be written in terms of the oscillator strength for the transition

\[
f_{L}(b, a) = (2m/3\hbar)(E_b - E_a) \left| \sum_{i=1}^{N} (b|\mathbf{r}_i|a) \right|^2
\]

(31)

\[
= (2m/3e^2\hbar^2)(E_b - E_a)S(b, a)
\]

(32)

This is the "length" form of the oscillator strength. If the wave functions used are exact, then the "length" formula is equivalent to the "velocity" form

\[
f_{v}(b, a) = \frac{2}{3} \frac{\hbar}{m} \frac{1}{(E_b - E_a)} \left| \sum_{i} (b|\nabla_i|a) \right|^2
\]

(33)

Hence, for a given calculation the difference between length and velocity results is often a useful indication of the error introduced by the use of approximate wave functions. For transitions between two energy levels \( a \) with \( \omega_a \) states with wave functions \( \psi(a_i) \) and \( b \) with \( \omega_b \) states with wave functions \( \psi(b_j) \), the transition probability is obtained by summing over all final states and averaging over all upper states and is given by

\[
A_{E1}(a \rightarrow b) = (64\pi^4/3h\lambda^3)(1/\omega_a)S(b, a)
\]

(34)

where

\[
S(b, a) = \sum_{ij} S(b_j, a_i)
\]

(35)
For LS coupling we can derive selection rules for allowed transitions. For example, the operator for these electric dipole transitions, \( \sum_i r_i \), is spin independent, and the matrix element is zero if
\[
\Delta S \neq 0
\] (36)
and as the operator is also an odd function the matrix element for electric dipole transitions is also zero if the initial and final states have the same parity. Using similar arguments, we can derive other selection rules for electric dipole radiation. For these transition probabilities to be nonzero we must have
\[
\Delta S = 0; \quad \Delta L = 0, \pm 1 \quad \text{(not } 0 \rightarrow 0) ; \quad \Delta J = 0, \pm 1 \quad \text{(not } 0 \rightarrow 0) 
\] (37)
and the parity of the state must change.

In general the electric dipole transitions give the strongest lines. However, if a transition is forbidden by the electric dipole selection rules, it may not be forbidden if one includes in the radiation field either the variation of the field in the vicinity of the atom or the magnetic vector of the radiation field. The former gives rise to electric quadrupole radiation and the latter to magnetic dipole radiation.

The electric quadrupole transition probability is
\[
A_{E2}(a \rightarrow b) = \left( 32\pi^6/3h^3\lambda^5 \right) S_{E2}(b, a) 
\] (38)
where
\[
S_{E2}(b, a) = e^2Z_3 \sum_{\mu} \left| \sum_i (b|r_i^2C_{2\mu}(\hat{r}_i)|a) \right|^2 
\] (39)
In LS coupling this transition probability is nonzero if
\[
\Delta S = 0; \quad \Delta L = 0, \pm 1, \pm 2 \quad \text{(not } 0 \rightarrow 0, 0 \leftrightarrow 1) 
\] (40)
\[
\Delta J = 0, \pm 1, \pm 2 \quad \text{(not } 0 \rightarrow 0, \frac{1}{2}, \frac{1}{2}, 0 \leftrightarrow 1) 
\] (40)
and the parity of the states must not change. Electric quadrupole transitions are often called \( E2 \) transitions.

The magnetic dipole transition probability is
\[
A_{M1}(a \rightarrow b) = \left( 64\pi^4/3h^3\lambda^5 \right) S_{M1}(b, a) 
\] (41)
where
\[
S_{M1}(b, a) = |(-e/2mc)(b|L + 2S|a)|^2 
\] (42)
For this transition probability to be nonzero in LS coupling, we must have
\[
\Delta S = 0; \quad \Delta L = 0; \quad \Delta J = 0, \pm 1 \quad \text{(not } 0 \rightarrow 0) 
\] (43)
and the parity of the states must not change. Magnetic dipole transitions are often called \( M1 \) transitions. In general electric quadrupole and magne-
### TABLE III

**Nonrelativistic Calculations of the Dipole Length and Velocity Formulas, Eqs. (31) and (33), for the Oscillator Strength of the (2s2p)\(^1P_1\) → (2s\(^3\)\(^1S_0\) Transition in C III, Si XI, and Fe XXIII Obtained Using Configuration Interaction (CI) Wave Functions with Different Numbers of Orbitals**

<table>
<thead>
<tr>
<th>Orbitals used in CI expansion</th>
<th>Hartree–Fock 1s, 2s, 2p</th>
<th>CI 1s, 2s, 2p</th>
<th>CI: 1s, 2s, 2p (3s, 3p, 3d)</th>
<th>CI: 1s, 3s, 2p (3s, 3p, 3d, 4f)</th>
</tr>
</thead>
<tbody>
<tr>
<td>C III</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Length</td>
<td>1.0711(^a)</td>
<td>0.7967(^a)</td>
<td>0.7772(^b)</td>
<td></td>
</tr>
<tr>
<td>Velocity</td>
<td>0.5634(^a)</td>
<td>0.6140(^a)</td>
<td>0.7838(^b)</td>
<td></td>
</tr>
<tr>
<td>Si XI</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Length</td>
<td>0.265(^c)</td>
<td>0.2652(^c)</td>
<td>0.2607(^c)</td>
<td>0.2599(^c)</td>
</tr>
<tr>
<td>Velocity</td>
<td>0.161(^c)</td>
<td>0.1583(^c)</td>
<td>0.2590(^c)</td>
<td>0.2597(^c)</td>
</tr>
<tr>
<td>Fe XXIII</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Length</td>
<td>0.132(^c)</td>
<td>0.1323(^c)</td>
<td>0.1310(^c)</td>
<td>0.1308(^c)</td>
</tr>
<tr>
<td>Velocity</td>
<td>0.080(^c)</td>
<td>0.0813(^c)</td>
<td>0.1328(^c)</td>
<td>0.1330(^c)</td>
</tr>
</tbody>
</table>

\(^{a}\) Burke et al. (1972).
\(^{b}\) Berrington et al. (1977).
\(^{c}\) Glass (1979a), and private communication.

tic dipole transition probabilities are of the same order of magnitude, but they are both about \(10^{-8}\) smaller than electric dipole transition probabilities. However, if the electric dipole transition is zero, then the electric quadrupole and magnetic dipole transitions are important.

As an example, transitions between the \((2s^2)\), \((2s2p)\), and \((2p^2)\) configurations of the Be-like ions produce the strongest lines for these ions in the solar spectrum. We shall consider first the recent advances which have been made in these transitions before considering \(\Delta n \neq 0\) transitions.

The oscillator strengths of the electric dipole transitions between the \(n = 2\) levels of the Be-like ions have been calculated using the non-relativistic CI wave functions discussed in the previous section (Burke et al., 1972; Berrington et al., 1977, Glass, 1979a). Table III is a comparison of the electric dipole oscillator strength of the \((2s2p)\(^1P_1\) → \((2s\(^3\)\(^1S_0\) transition in C III, Si XI, and Fe XXIII, calculated using the length and velocity formulas, Eqs. (31) and (33), with a variety of CI wave functions. For low values of \(Z\) and the length formula, elaborate CI wave functions are required to give an accurate result, but for large \(Z\) quite accurate results are obtained using simple CI functions or even Hartree–Fock functions. To obtain accurate results with the velocity formula it is necessary to use elaborate CI wave functions for all values of \(Z\).
In the previous section we have seen that relativistic effects are important in calculating atomic energy levels for large $Z$, and they are also important in calculating oscillator strengths for large $Z$. Using a relativistic, multiconfiguration, Dirac–Hartree–Fock treatment, Armstrong et al. (1976) have calculated the electric dipole oscillator strengths for transitions between the $(2s^2)$ and $(2s2p)$ levels of ions in the Be-like series. These are plotted in Fig. 4, where they are compared with their nonrelativistic calculations. For large $Z$ a full relativistic treatment is essential, but for ions up to about Fe XXIII in the Be sequence, relativistic
<table>
<thead>
<tr>
<th>Orbitals used in CI expansion</th>
<th>Hartree-Fock</th>
<th>CI: 1s, 2s, 2p</th>
<th>CI: 1s, 2s, 2p, 3s, 3p, 3d, 4f</th>
</tr>
</thead>
<tbody>
<tr>
<td>N Rel</td>
<td></td>
<td>N Rel  Rel</td>
<td>N Rel  Rel  Rel  Rel  Rel  Rel</td>
</tr>
<tr>
<td>(2s2p)1P1 → (2s2)1S0</td>
<td></td>
<td>0.311 0.209</td>
<td>0.311 0.316 0.316 0.316 0.316</td>
</tr>
<tr>
<td>Mg IX</td>
<td>0.319</td>
<td>0.198 0.209</td>
<td>0.196 0.207 0.207 0.206 0.207</td>
</tr>
<tr>
<td>Ar XV</td>
<td>0.199</td>
<td>0.132 0.156</td>
<td>0.131 0.154 0.154 0.154 0.154</td>
</tr>
<tr>
<td>Fe XXIII</td>
<td>0.132</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(2s2p)3P1 → (2s2)1S0</td>
<td></td>
<td></td>
<td>1.94 (-5) 3.17 (-5) 2.02 (-5)</td>
</tr>
<tr>
<td>Mg IX</td>
<td>—</td>
<td>— 2.2 (-4)*</td>
<td>— 1.97 (-4) 2.98 (-4) 2.29 (-4)</td>
</tr>
<tr>
<td>Ar XV</td>
<td>—</td>
<td>— 1.5 (-3)</td>
<td>— 1.02 (-3) 1.87 (-3) 1.64 (-3)</td>
</tr>
<tr>
<td>Fe XXIII</td>
<td>—</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

* Full relativistic calculations of Armstrong et al. (1976).

* Relativistic calculations of Glass (1979a, b) with $H_{NR} + H_{so} + H_{soo} + H_{ss}$.

* With $H_{NR} + H_{mass} + H_{D1}$.

* With $H_{NR} + H_{so} + H_{mass} + H_{D1}$.

* With $H_{NR} + H_{so} + H_{soo} + H_{ss} + H_{mass} + H_{D1}$.

* The number in brackets denotes the power of ten by which the number should be multiplied.
effects do not dominate, and it is appropriate to use the Breit–Pauli approximation (Glass and Hibbert, 1978b) discussed in the previous section. Table IV gives the results for the Breit–Pauli approximation for the (2s2p) $^1P_1 \rightarrow (2s^2) ^1S_0$ transition in Mg IX, Ar XV, and Fe XXIII obtained using the CI wave function with 1s, 2s, 2p, 3s, 3p, 3d, 4f orbitals and including different relativistic terms in the Hamiltonian (Glass, 1979a). The inclusion of the five relativistic terms $H_{so}, H_{soo}, H_{ss}, H_{mass},$ and $H_{D1}$ in Eq. (11) increases this electric dipole oscillator strength by 2, 6, and 18% for Mg IX, Ar XV, and Fe XXIII, respectively. The major increase is brought about by the inclusion of the non-fine-structure terms $H_{mass}$ and $H_{D1}$, the fine structure terms $H_{so} + H_{soo} + H_{ss}$ having little effect. Figure 5 is a comparison of the nonrelativistic and relativistic calculations of Glass (1979a) for this electric dipole oscillator strength for other ions in the Be-like sequence. As the nonrelativistic oscillator strength decreases as $1/Z$ for large $Z$, it is convenient to plot the product $Zf$ against $Z$. The small difference between the relativistic calculations of Armstrong et al. (1976) and those of Glass is due to the more elaborate CI wave function used by Glass. Also plotted in the figure are the results of Nussbaumer and Storey (1978, 1979a, b). The agreement between the three calculations is excellent, but they do not agree with the beam foil measurements, which are also plotted (Pegg et al., 1977, 1979).

**Fig. 5.** Product of the nuclear charge $Z$ of the ion and the oscillator strength $f$ for the (2s2p)$^1P_1 \rightarrow (2s^2)^1S_0$ transition for Be-like ions. Theoretical results: (—) relativistic; (---) nonrelativistic results (Glass, 1979a, Hibbert, 1974); (○) relativistic results (Armstrong et al., 1976); (+) relativistic results (Nussbaumer and Storey, 1978, 1979a, b). Beam foil measurements: (△) (Pegg et al., 1977); (○) (Pegg et al., 1979).
Calculations have also been carried out on the other electric dipole transitions between the \( n = 2 \) states of the Be-like ions (Nussbaumer and Storey, 1978, 1979a,b; Hibbert, 1974; Glass, 1979a). The agreement between the results obtained by these two groups is very satisfactory.

In the relativistic multiconfiguration Dirac–Hartree–Fock treatment of transitions between \((2s^2)\) and \((2s2p)\) states, Armstrong et al. (1976) automatically obtained a nonzero electric dipole oscillator strength for the transition \((2s2p)^3P_1 \rightarrow (2s^2)^1S_0\). As this transition involves a change of spin, it is strictly forbidden in LS coupling. However, in a full relativistic treatment this transition is allowed; it also becomes allowed in the Breit–Pauli approximation, where the wave functions for the \((2s^2)^1S_0\) and \((2s2p)^3P_1\) states are written [Eq. (24)] as

\[
\Phi[(2s^2)^1S_0] = b_{1,1}(2s^2)^1S_0 + b_{1,0}(2p^2)^3P_0 + b_{1,10}(2p^2)^1S_0
\]

\[
\Phi[(2s2p)^3P_1] = b_{3,3}(2s2p)^3P_1 + b_{3,3'}(2s2p)^1P_1
\]

The products of the singlet terms in the two equations, together with the product of the triplet terms, give rise to a nonzero result for the electric dipole oscillator strength. Table IV gives the results in the Breit–Pauli approximation for the electric dipole oscillator strength for the \((2s2p)^3P_1 \rightarrow (2s^2)^1S_0\) transition in Mg IX, Ar XV, and Fe XXIII obtained by Glass (1979b). Clearly both the fine structure terms \(H_{3s} + H_{3p0} + H_{ss}\) and the non-fine-structure terms \(H_{mass} + H_{D1}\) are important for this type of transition. The Breit–Pauli results of Glass are in good agreement with the relativistic calculation of Armstrong et al. (1976).

Table V is a comparison of the Breit–Pauli approximation results for the electric dipole probabilities for the transition \((2s2p)^3P_1 \rightarrow (2s^2)^1S_0\) obtained by Glass and Hibbert (1978a) and Glass (1979b), with results obtained by Nussbaumer and Storey (1978, 1979a,b) and Laughlin et al.

### Table V

<table>
<thead>
<tr>
<th>Transition Probabilities (sec⁻¹) of the Electric Dipole, (E1), Spin-Forbidden Transition ((2s2p)^3P_1 \rightarrow (2s^2)^1S_0) in the Be-Like Ions</th>
</tr>
</thead>
<tbody>
<tr>
<td>C III</td>
</tr>
<tr>
<td>-------</td>
</tr>
<tr>
<td>9.59¹</td>
</tr>
</tbody>
</table>

ᵃ Superscript denotes the power of ten by which the number should be multiplied.
(1978). The calculations are in quite good agreement; however, we would expect that the results of Nussbaumer and Storey to be somewhat more accurate than the other calculations because of the complexity of their wave functions.

For the range of nuclear charge, \( Z \), considered in Table IV, the spin-forbidden electron dipole transition probabilities increase approximately as \( Z^4 \). This very rapid increase is to be compared with a linear increase in the spin allowed electric dipole transitions probabilities for \( \Delta n = 0 \) transitions.

A study of the wave function in Eq. (24) shows that spin-forbidden electric dipole \((E1)\) transitions also take place to \((2s2p)\) \( ^3\!P_0^0 \) from \((2p^2)\) \( ^3\!P_{0,1,2}^0 \), to \((2s2p)\) \( ^3\!P_{1,2}^0 \) from \((2p^2)\) \( ^1\!D_2 \), and to \((2s2p)\) \( ^3\!P_0^0 \) from \((2p^2)\) \( ^1\!S_0 \). The transition probabilities for these transitions in the Be-like ions have been calculated by Nussbaumer and Storey (1978, 1979a,b). They all increase very rapidly as \( Z \) increases.

Magnetic dipole \((M1)\) transition probabilities have also been calculated for the spin-allowed transitions, \((2s2p)\) \( ^3\!P_0^0 \) to \((2s2p)\) \( ^3\!P_0^0 \) and \((2s2p)\) \( ^3\!P_0^0 \) to \((2s2p)\) \( ^3\!P_1^0 \) (Nussbaumer and Storey, 1978, 1979a,b; Tunnell and Bhalla, 1979; Naqvi, 1951). These are much smaller than the magnetic dipole \((M1)\) transition probabilities for the spin-forbidden transitions from \((2s2p)\) \( ^1\!P_1^0 \) to \((2s2p)\) \( ^3\!P_{0,1,2}^0 \) and \((2p^2)\) \( ^3\!P_1^0 \) to \((2s^2)\) \( ^1\!S_0 \) (Nussbaumer and Storey, 1978, 1979a,b; Naqvi, 1951).

Electric quadrupole \((E2)\) transition probabilities have been calculated for the spin-allowed transitions \((2p^2)\) \( ^1\!D_2 \) to \((2s)\) \( ^1\!S_0 \) and \((2p^2)\) \( ^1\!S_0 \) to \((2p^2)\) \( ^1\!D_2 \) and the spin-forbidden transition \((2p^2)\) \( ^3\!P_2^0 \) to \((2s^2)\) \( ^1\!S_0 \) (Nussbaumer and Storey, 1978, 1979a,b).

The magnetic quadrupole \((M2)\) transition \((2s2p)\) \( ^3\!P_2^0 \) to \((2s^2)\) \( ^1\!S_0 \) has been considered recently by several groups (Nussbaumer and Storey, 1978, 1979a,b; Shorer and Lin, 1977; C. D. Lin et al., 1978; D. L. Lin et al., 1978). There is good agreement among all the recent calculations, but they all differ significantly from earlier calculations of Osterbrock (1970). For \( N IV \) the transition probability of this magnetic quadrupole transition is approximately \( 5 \times 10^4 \) times smaller than the transition probability for the \((2s2p)\) \( ^3\!P_0^0 \) to \((2s^2)\) \( ^1\!S_0 \) transition and may be detectable in the solar spectrum. The \( M2 \) transition probability for the transition \((2p^2)\) \( ^1\!S_0 \rightarrow (2s2p)\) \( ^3\!P_2^0 \) has also been calculated by Shorer and Lin (1977).

Table VI is a summary of the theoretical results which are available for transitions between \((2s^2)\), \((2s2p)\), and \((2p^2)\) configurations in C III and Fe XXIII. In general, calculations which have been carried out for C III have also been carried out for all low \( Z \) values up to Ne VII; for larger values of \( Z \) much less effort has been put into calculating \( E2, M1, \) and \( M2 \) transitions. For low values of \( Z \) the spin-allowed electric dipole transition prob-
<table>
<thead>
<tr>
<th>Upper state</th>
<th>Lower state</th>
<th>( (2s^2)^1S )</th>
<th>( (2s^2)^3P^0 )</th>
<th>( (2s^2p)^3P^0 )</th>
<th>( (2s^2p)^3P^0 )</th>
<th>( (2s^2p)^1P^0 )</th>
<th>( (2p^2)^3D )</th>
</tr>
</thead>
<tbody>
<tr>
<td>((2s^2p)^3P^0)</td>
<td>0</td>
<td>( 5^{+11e,d} )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>((2s^2p)^3P^0)</td>
<td>1</td>
<td>( 9.592^1 )</td>
<td>( 2.39^{-7} ) (M 1)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>((2s^2p)^3P^0)</td>
<td>2</td>
<td>( 5.19^{-3} ) (M 2)</td>
<td></td>
<td>( 2.41^{-6} ) (M 1)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>((2s^2p)^1P^0)</td>
<td>1</td>
<td>( 1.788^9 )</td>
<td>( 1.45^3 ) (M 1)</td>
<td>( 1.09^{-3} ) (M 1)</td>
<td>( 1.81^{-3} ) (M 1)</td>
<td>( 1.330^9 )</td>
<td>( 2.439^2 )</td>
</tr>
<tr>
<td>((2p^2)^3P)</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>((2p^2)^3P)</td>
<td>1</td>
<td>( 9.23^{-4} ) (M 1)</td>
<td>( 4.443^8 )</td>
<td>( 3.329^8 )</td>
<td>( 5.537^8 )</td>
<td>( 1.512^1 )</td>
<td>( 1.512^1 )</td>
</tr>
<tr>
<td>((2p^2)^3P)</td>
<td>2</td>
<td>( 3.83^{-2} ) (E 2)</td>
<td>( 3.336^9 )</td>
<td>( 9.985^8 )</td>
<td>( 1.318^3 )</td>
<td>( 1.318^3 )</td>
<td>( 1.318^3 )</td>
</tr>
<tr>
<td>((2p^2)^1D)</td>
<td>2</td>
<td>( 2.37^3 ) (E 2)</td>
<td>( 4.608^3 )</td>
<td>( 2.927^4 )</td>
<td>( 1.407^8 )</td>
<td>( 1.407^8 )</td>
<td>( 1.407^8 )</td>
</tr>
<tr>
<td>((2p^2)^1S)</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
<td>( 2.037^3 )</td>
<td>( 4.57^{-2e} ) (M 2)</td>
<td>( 2.114^9 )</td>
</tr>
</tbody>
</table>

**Fe XXIII**

<table>
<thead>
<tr>
<th>Upper state</th>
<th>Lower state</th>
<th>( (2s^2p)^3P^0)</th>
<th>( (2s^2p)^1P^0)</th>
</tr>
</thead>
<tbody>
<tr>
<td>((2s^2p)^3P^0)</td>
<td>0</td>
<td>( 3^{-5e} )</td>
<td>( 1.225^{10} )</td>
</tr>
<tr>
<td>((2s^2p)^3P^0)</td>
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<td>( 5.014^7 )</td>
<td>( 4.097^9 )</td>
</tr>
<tr>
<td>((2s^2p)^3P^0)</td>
<td>2</td>
<td>( 7.51 ) (M 2)</td>
<td>( 6.50^{-1} ) (E 2)</td>
</tr>
<tr>
<td>((2s^2p)^1P^0)</td>
<td>1</td>
<td>( 1.974^{10} )</td>
<td>( 5.206^9 )</td>
</tr>
<tr>
<td>((2p^2)^3P)</td>
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<td>( 6.432^9 )</td>
<td>( 4.840^8 )</td>
</tr>
<tr>
<td>((2p^2)^3P)</td>
<td>1</td>
<td>( 1.864^{10} )</td>
<td>( 4.733^9 )</td>
</tr>
<tr>
<td>((2p^2)^1D)</td>
<td>2</td>
<td>( 4.840^8 )</td>
<td>( 4.733^9 )</td>
</tr>
<tr>
<td>((2p^2)^1S)</td>
<td>0</td>
<td>( 4.57^{-2e} ) (M 2)</td>
<td>( 2.114^9 )</td>
</tr>
</tbody>
</table>

---

* The results tabulated are mainly taken from the work of Nussbaumer and Storey (1978, 1979a, b).

* The calculations are all electric dipole transitions (E 1) except those denoted by (E 2) electric quadrupole transitions, (M 1) magnetic dipole transitions, and (M 2) magnetic quadrupole transitions.

* Estimate of two-photon (E 1 + M 1) transition rate (Laughlin, 1980).

* The superscript denotes the power of ten by which the number should be multiplied.

* Shorer and Lin (1977).
abilities are very much larger than all other transition probabilities, but for large \( Z \) the spin-forbidden \( E1 \) transition probabilities are comparable with the spin-allowed probabilities. The \( E2, M1, \) and \( M2 \) transition probabilities all increase rapidly with \( Z \), but even for Fe XXIII they are much smaller than the \( E1 \) probabilities.

Most of the spectral lines of the Be-like ions which have been observed in the solar spectrum have been from \( \Delta n = 0 \) transitions. A small number of lines have been seen from transitions between states with \((2s^2), (2s2p), (2p^3), \) and \((2s3s), (2s3p), (2s3d), (2p3s), (2p3p), \) and \((2p3d)\) configurations (Malinovsky, 1975; K. G Widing, personal communication, 1980). A very extensive series of CI calculations has been carried out by Hibbert (1979, 1980) on the transition probabilities between all states of O V having the form \( 1s^2nln'\), \( n = 2, n' = 2, 3 \). In all, approximately 400 electric dipole spin-allowed and spin-forbidden transitions were considered, and about 50 electric quadrupole spin-allowed transitions were also included. The lifetimes of the states were compared with beam foil experiments, and, in general, theory and experiment were in good agreement (Martinson et al., 1971; Irwin et al., 1973; Knystautas and Drouin, 1975; Buchet et al., 1976, Engström et al., 1979). \( M1 \) and \( M2 \) transitions were also considered, but they did not affect the lifetimes significantly.

Accurate CI results have also been obtained by Glass (1979c) for the transition probabilities for the spin-allowed electric dipole between the configurations \((2s^2), (2s2p), (2p^3), (2s3s), (2s3p), \) and \((2s3d)\) in C III, N IV, O V, and Ne VII. His results are in good agreement with beam foil measurements (Buchet-Poulizac and Buchet, 1973; Barette and Drouin, 1974; Buchet and Buchet-Poulizac, 1974).

Finally we note the extensive set of transition probabilities obtained by Czyzak and Krueger (1979) for N IV and O V for transition of the form \( n_1l_1n_2l_2 \rightarrow n'_1l'_1n'_2l'_2 \), where \( n_1l_1 \) and \( n'_1l'_1 \) were either 2s or 2p and \( n_2l_2 \) went up to 6p and \( n'_2l'_2 \) went up to 7f.

V. Electron Excitation

In this review we have attempted to discuss recent advances in calculations for atomic processes which are important in the sun. We have considered the Be-like ions in detail because they have many lines in the solar spectrum and also because the theoretical methods which have been used for Be-like ions can be readily extended to other isoelectronic series. The theoretical results which we obtained for the excitation energies and transition probabilities in the Be-like ions are typical of the results that we would expect from any ion which is observed in the sun.
Recent calculations on the electron excitation of Si III (Baluja et al., 1980) and S IV (Dufton and Kingston, 1980) suggest that for electron excitation, ions in the third row of the periodic table are quite different from ions in the second row. Hence, in this section we first consider in detail electron excitation of Be-like ions and then consider electron excitation of Si III and S IV. The results for the Be-like ions are typical of the results that we would expect for ions in the first row of the periodic table. It is not yet known if the results for Si III and S IV are representative of heavier ions.

A. Electron Excitation of Be-Like Ions

Sections II and IV show that for Be-like ions the observed energy levels of ions are in general more reliable than the calculated values, and, although the measured transition probabilities are obtained quite accurately at present, the theoretical values are normally to be preferred.

Some work has been carried out on the measurement of electron excitation rates for ions using theta pinches. Excitation rates have been obtained for transitions in Ne VII by Tondello and McWhirter (1971) and for transitions in N IV, O V, Ne VII, and Si XI by Johnston and Kunze (1971). However, chiefly because of the uncertainty of determining the populations of the ground \( (2s^2) \, ^1S_0 \) state and the metastable \( (2s2p) \, ^3P_1 \) state these rate coefficients were estimated to have uncertainties of a factor of three.

Because of this large uncertainty in the measured electron excitation rate coefficients, it is important to obtain accurate theoretical rate coefficients for ions which are important in the sun. When electron excitation processes for a particular ion are important, the thermal energy of the free electron is usually approximately equal to or less than the electron excitation energies of the ion. Hence, in our theoretical calculations we will be chiefly interested in the excitation cross section at low energies, i.e., at electron energies up to approximately ten times the excitation energy.

The nonrelativistic Hamiltonian for the \((N + 1)\) electron system, comprising an ion with \(N\) electrons and a free electron, is

\[
H^{N+1} = \sum_{i=1}^{N+1} \left( -\frac{1}{2} \nabla_i^2 - Z/r_i \right) + \sum_{i>j=1}^{N+1} \frac{1}{r_{ij}} \tag{46}
\]

where \(Z\) is the nuclear charge of the ion. With this Hamiltonian the total orbital, \(L\), and spin, \(S\), angular momentum of the electron–ion system are conserved as well as their azimuthal components \(M_L\) and \(M_S\). Because of this we construct the total wave function for the system in terms of channel functions, which are eigenfunctions of \(L^2\), \(S^2\), \(L_z\), and \(S_z\). We consider
a given target state which has quantum numbers $L_i$, $S_i$, $M_{L_i}$, $M_{S_i}$, and which is distinguished by $\Gamma_i$ from other states with the same quantum numbers. Denote the wave function for this target state by

$$\Psi_{M_{L_i}M_{S_i}}^{L_iS_i}(X_1, X_2, \ldots, X_N) \quad (47)$$

where $X$ denotes the $r$ radial, $\hat{r}$ angular, and $\sigma$ spin variables. The free electron is denoted by quantum numbers $l_i$, $s_i$, $m_{l_i}$, $m_{s_i}$, and also $k_i$, where $\frac{1}{2}k_i^2$ is the energy, in atomic units, of the free electron incident on the state $\Gamma_i$. The total energy $E$ is given by $E = \frac{1}{2}k_i^2 + E_i$, where $E_i$ is the total energy of State $\Gamma_i$. The free electron is described by the expression

$$Y_{l_i}^{m_{l_i}}(\theta, \phi)\chi_{s_i}^{m_{s_i}}(\sigma)[F_i(r)/r] \quad (48)$$

and we now couple Eqs. (47) and (48) to form a channel function, which represents an atomic eigenstate, coupled with the spin and angular functions of the incident electron to form an eigenstate of the total orbital, $L$, and total spin $S$ angular momenta and their $z$ components. In this representation we will use $i$ to denote the quantum numbers

$$\Gamma_i, L_i, S_i, l_i, s_i, k_i, L, S, M_L, M_S, \text{ and the parity } p \quad (49)$$

and the channel functions are

$$\Phi_i(X_1, X_2, \ldots, X_N, \hat{r}_{N+1}, \sigma_{N+1}) = \sum_{M_{L_i}M_{S_i}} C(L_iL; L_{M_{L_i}M_{L_i}})C(S_iS; M_{S_i}M_{S}) \times \Phi_{M_{L_i}M_{S_i}}^{L_iS_i}(X_1, X_2, \ldots, X_N)Y_{l_i}^{m_{l_i}}(\theta_{N+1}, \phi_{N+1})X_{s_i}^{m_{s_i}}(\sigma_{N+1}) \quad (50)$$

where the $C$'s are Clebsch–Gordan coefficients.

For each $L$, $S$, and parity $p$ (odd or even) we write the total wave function for a particular energy $E$ as

$$\Psi_E(X_1, X_2, \ldots, X_{N+1}) = \mathcal{A} \sum_i \Phi_i[F_i(r_{N+1})/r_{N+1}] + \sum_j d_j \phi_j \quad (51)$$

where the operator $\mathcal{A}$ ensures that the wave function is antisymmetric in all the coordinates of the $(N + 1)$ electrons. The functions $\phi_j$ are $(N + 1)$ electron functions, which are constructed from one-electron orbitals to have the same parity $p$ and orbital, $L$, and spin, $S$, angular momenta as $\Psi_E$. They are included partially to remove constraints on the total wave function which are imposed by orthogonalizing the one-electron orbitals and partially to take account of short-range electron correlation.

Substituting $\Psi_E$ into the Kohn variational principle (Kohn, 1948) and taking arbitrary variations of the functions $F_i$ and coefficients $d_j$, we
obtain

\begin{align}
\langle \Phi_i | H^{N+1} - E | \Psi_E \rangle &= 0, \quad i = 1, \ldots, n \tag{52a} \\
\langle \phi_j | H^{N+1} - E | \Psi_E \rangle &= 0, \quad j = 1, \ldots, m \tag{52b}
\end{align}

where \( n \) is the number of channel functions retained in the expansion and \( m \) is the total number of \((N + 1)\) electron bound state functions in Eq. (51). In Eq. (52b) the integration is over all electronic coordinates, and in Eq. (52a) the integration is over all electronic coordinates except the radial coordinates of the scattered electron.

The resulting equations are satisfied subject to the boundary conditions

\begin{align}
F_{ij} \sim k_i^{-1/2}(\delta_{ij} \sin \theta_i + R_{ij} \cos \theta_i), \quad i, j = 1, \ldots, n \tag{53}
\end{align}

where

\begin{align}
\theta_i &= k_i r - \frac{1}{2} i \pi - \eta_i \ln(2 k_i r) + \sigma_{li} \\
\eta_i &= -(Z - N)/k_i, \quad \sigma_{li} = \arg \Gamma(l_i + 1 + i \eta_i) \tag{54}
\end{align}

The second subscript \( j \) on \( F_{ij} \) is introduced to label the \( n \) independent solutions. For a given \( E, L, S, \) and \( p \) the \( R_{ij} \) matrix is a function of the incident channel \( \Gamma_i L_i S_i l_i s_i \) and the outgoing channel \( \Gamma'_j L'_j S'_j l'_j s'_j \), and we write

\begin{align}
R_{ij} &= R^{\nu}(\Gamma_i L_i S_i l_i s_i LS k_i; \Gamma'_j L'_j S'_j l'_j s'_j LS k'_j) \tag{55}
\end{align}

or in a more compact form

\begin{align}
R_{ij} &= R^{LS\nu}(\Gamma_i L_i S_i l s; \Gamma'_j L'_j S'_j l' s'; k^2) \tag{56}
\end{align}

When we have calculated the \( R^{LS\nu}(\Gamma_i L_i S_i l s; \Gamma'_j L'_j S'_j l' s'; k^2) \) matrix for a given incident electron energy relative to the ground state of \( \frac{1}{2} k^2 \), and for all values of \( L \) and \( S \) and parity \( p \) (odd or even), we can obtain the total cross section (in \( a_0^2 \)) for a transition from one state \( \Gamma_i L_i S_i \) to another \( \Gamma'_j L'_j S'_j \), using

\begin{align}
Q(\Gamma_i L_i S_i \rightarrow \Gamma'_j L'_j S'_j) &= \frac{\pi}{k^2} \sum_{L S p} \frac{(2L + 1)(2S + 1)}{2(2L_i + 1)(2S_i + 1)} |T^{LS\nu}(\Gamma_i L_i S_i l s; \Gamma'_j L'_j S'_j l' s'; k^2)|^2 \tag{57}
\end{align}

where the \( T^{LS\nu}(\Gamma_i L_i S_i l s; \Gamma'_j L'_j S'_j l' s'; k^2) \) matrix in Eq. (57) was obtained from the \( R^{LS\nu}(\Gamma_i L_i S_i l s; \Gamma'_j L'_j S'_j l' s'; k^2) \) matrix using the relation

\begin{align}
T &= -2 i R/(1 - i R) \tag{58}
\end{align}

The total collision strength is
ATOMIC PROCESSES IN THE SUN

\[ \Omega(\Gamma_i L_i S_i \rightarrow \Gamma'_i L'_i S'_i) \]
\[ = \frac{1}{2} \sum_{L^S, L'} (2L + 1)(2S + 1)|T^{LS}(\Gamma_i L_i S_i l s; \Gamma'_i L'_i S'_i l' s'; k_i^2)|^2 \]  
\[ (59) \]

and hence

\[ Q(\Gamma_i L_i S_i \rightarrow \Gamma'_i L'_i S'_i) = \frac{\Omega(\Gamma_i L_i S_i \rightarrow \Gamma'_i L'_i S'_i)}{(2L_i + 1)(2S_i + 1)} \]  
\[ (60) \]

For an atom in which the term splitting is small compared with the term separation, we can use the \( R^{LS}(\Gamma_i L_i S_i l s; \Gamma'_i L'_i S'_i l' s'; k_i) \) matrices to obtain transitions between different fine structure levels by transforming to pair coupling, where

\[ L_i + S_i = J_i, \quad J_i + l = K, \quad \text{and} \quad K + s = J \]  
\[ (61) \]

Using Racah recoupling coefficients (Racah, 1943), Saraph (1972, 1978) has shown that for a given \( J \) and parity, \( p \),

\[ R^{Jp}(\Gamma_i L_i S_i J_i, lK; \Gamma'_i L'_i S'_i J'_i l' K'; k_i) \]
\[ = \sum_{L^S} C(LSJ, L_i S_i J_i, lK)R^{LS}(\Gamma_i L_i S_i l s; \Gamma'_i L'_i S'_i l' s'; k_i) \]
\[ \times C(LSJ, L'_i S'_i J'_i, l' K') \]  
\[ (62) \]

where the \( C \)'s are products of two Racah coefficients. Collision strengths between the fine structure levels \( \Gamma_i L_i S_i J_i \rightarrow \Gamma'_i L'_i S'_i J'_i \) are then given by

\[ \Omega(\Gamma_i L_i S_i J_i \rightarrow \Gamma'_i L'_i S'_i J'_i) \]
\[ = \frac{1}{2} \sum_{l'k'} (2J + 1)|T^{Jp}(\Gamma_i L_i S_i J_i lK; \Gamma'_i L'_i S'_i J'_i l' K'; k_i)|^2 \]  
\[ (63) \]

where the matrix \( T^{Jp} \) is obtained by substituting \( R^{Jp} \) into Eq. \((58)\).

The accuracy of any electron-scattering calculations depends on at least two things, namely, the accuracy of the atomic target states used in the expansion of the total \((N + 1)\) electron wave function \([Eq. \((51)\)]\) and the number of target states used in this expansion. If we use a large number of target states in Eq. \((51)\) which have poor wave functions or only a small number of accurate target states in Eq. \((51)\), we obtain inaccurate cross sections.

Over the last ten years several groups have produced useful computer packages for obtaining accurate electron excitation cross sections for ions. A method which has been used extensively for the electron excitation of Be-like ions is the \( R \)-matrix method. This method was developed by Burke and his co-workers (Burke, 1973; Burke and Robb, 1975; Barrington \textit{et al.}, 1974, 1978) and uses the CI wave functions discussed in the
previous sections. In the $R$-matrix method the ion is enclosed in a sphere of radius $a$. In the internal region $r < a$, exchange effects are important, and the electron–electron interaction is large. Here, the $(N + 1)$ electron wave function is expanded in a finite set of energy-independent basis functions. Logarithmic boundary conditions are imposed on the wave function on the surface of this sphere. In the outer region $r > a$, the interaction is weak, and we can neglect exchange effects and can approximate the direct potentials by a few terms in their asymptotic expansion. For $r > a$, we solve a set of coupled ordinary differential equations to obtain the wave functions, and these are matched to the inner region solutions on the boundary $r = a$ to give the radial functions and the $R$ matrix.

Another powerful computer code, IMPACT, has been developed by Seaton and his co-workers (Eissner and Seaton, 1972; Seaton, 1974a,b; Crees et al., 1978) for electron ion scattering. In this method the coupled integrodifferential equations are solved using radial functions, which are tabulated at a small number of points. As in the $R$-matrix method, these solutions are obtained out to some point $r = a$, for $r > a$ exchange effects are neglected and the direct potentials are represented by their asymptotic forms. Hence, for $r > a$, the asymptotic solutions are obtained by solving a set of coupled ordinary differential equations. Matching these asymptotic solutions to the inner region solutions at $r = a$ gives the radial functions and the $R$ matrix.

Henry and his co-workers (Smith and Henry, 1973; Henry et al., 1981) have also developed a practical method of obtaining accurate electron excitation cross sections. It is called NIEM, noninteractive integral equation method, and is based on the work of Sams and Kouri (1969).

The three methods discussed above all solve the scattering problem using a moderate number of target states in the expansion of the total wave function Eq. (51). Mott and Massey (1933) suggested that if the coupling between the initial and final states is weak and the coupling to all other states is weak, then we could use only two target states, the initial state and the final state, in Eq. (51). They also assumed that the coupling potential between the two states could be ignored in the incoming channel. The solutions of the two integrodifferential equations allowed for the distortion of the incident and outgoing waves of the electrons by the target atoms. The method was called the distorted wave approximation. Since that time there have been many variants on the distorted wave approximation. At least six of these variants have been discussed by Henry (1981) in a recent review. As the distorted wave methods are normally only approximations to the collision problem with two target states in $\Psi_E$, results from these methods should not be as accurate as $R$-matrix calculations, which include two or more target states.
Another approximate method which has been used extensively at large electron impact energies or large values of $L$ is the Coulomb–Born approximation. In this approximation exchange effects are ignored and the initial, i, and final, f, state wave functions are taken to have the form

$$\Psi_i(X_1, \ldots, X_{N+1}) = \Phi_i(X_1, \ldots, X_N, r_{N+1}, \sigma_{N+1})(k_ir_{N+1})^{-1}F_i(k_ir_{N+1})$$

(64)

$$\Psi_f(X_1, \ldots, X_{N+1}) = \Phi_f(X_1, \ldots, X_N, r_{N+1}, \sigma_{N+1})(k_ir_{N+1})^{-1}F_f(k_ir_{N+1})$$

respectively, where the channel quantum numbers of $\Phi_i$ are

$$\Gamma_iL_iS_iL_SM_LM_S,$$

and the parity $p$

and $\Phi_f$ are

$$\Gamma_fL_fS_fL_SM_LM_S,$$

and the parity $p$

In Eq. (64) $F_i(kr)$ is the regular Coulomb function for charge $z = Z - N$, which satisfies the boundary condition

$$F_i(kr) \sim \sin(kr - \frac{1}{2}l\pi) - \eta \ln(2kr) + \sigma_i$$

(65)

with

$$\eta = -(Z - N)/k \quad \text{and} \quad \sigma_l = \arg \Gamma(l + 1 + i\eta)$$

(66)

The $R$ matrix is then given by

$$R^\text{CB}_{if} = -2(k_i k_f)^{1/2} \langle \Psi_i | V | \Psi_f \rangle$$

(67)

where

$$V = \sum_{l=1}^{N} \frac{1}{r_{l,N+1}} - \frac{N}{r_{N+1}}$$

(68)

Using CI bound state wave functions [Eq. (2)], this reduces to

$$R^\text{CB}_{if} = -2(k_i k_f)^{-1/2} \int_0^\infty V_{fi}(r)F_i(k_ir)F_i(k_ir) \, dr$$

(69)

where $V_{fi}$ is the direct potential between the initial and final channel.

As an example of recent calculations on electron excitation of ions we will first consider the Be-like ion C III. There have been many theoretical estimates of the electron excitation collision strength for the transition from the ground $(2s^2)^1S$ state to the lowest metastable level $(2s2p)^3P$ of C III. This transition is not only important in solar applications, but is also important theoretically as a test calculation for spin-forbidden transitions.

Calculations have been carried out by Osterbrock (1970), Eissner (1972), Flower and Launay (1973), Hershkowitz and Seaton (1973), Berlington et al. (1977, 1980), Mann (1980), and Robb (1980). Some of these
calculations are compared in Fig. 6. The most accurate calculations are the $R$-matrix results of Berrington et al. (1977) (see also Berrington et al., 1981). They used the six lowest states $(2s^2)\, ^1S$, $(2s2p)\, ^3P$, and $(2p^2)\, ^3P$, $^1D$, and $^3S$, of the ion in their expansion of the total wave function, and in the CI wave functions for the target states they employed seven one-electron orbitals. Their calculations show that at low electron energies the collision strength is dominated by the series of resonances which converge to the excitation thresholds of the other four states. A more detailed graph of this resonance structure is given in Fig. 7. Since this collision strength decreases rapidly as the energy increases, this resonance region makes a significant contribution to the electron excitation rate coefficient at low temperatures.

In order to judge the accuracy of their calculations Berrington et al. (1977) also carried out $R$-matrix calculations in which they also used six target states, which were obtained with only three one-electron orbitals, $1s$, $2s$, and $2p$. The accuracy of these wave functions was discussed in Sections III and IV. At an excitation energy of 10 Ry these less accurate
calculations were about 20% too small, but at 2 Ry they were 20% too large. These 1s, 2s, 2p results are in good agreement with the close-coupling results of Robb (1980).

Calculations were also carried out using only the three lowest target states constructed from the seven one-electron orbitals discussed above. The results of these calculations, for one partial wave, are compared with the six-state, seven-orbital calculations in the resonance region (Fig. 8). It is clear that the introduction of the three higher states in the six-state calculation lowers the position of the resonances and also alters their structure.

Also plotted in Fig. 6 are the distorted wave results of Flower and
Launay (1973); their results are in quite good agreement with the three-orbital, six-state results of Berrington et al. (1977). These three-orbital, six-state $R$-matrix results are also in good agreement with the distorted wave calculations of Mann (1980) at energies above about 2.5 Ry and with the distorted wave results of Eissner (1972).

The distorted wave calculations of Mann (1980) do not give any resonance structure, and in Fig. 7 we compare his results in the resonance region with the accurate results of Berrington et al. (1977). We shall show in the case of O V that this resonance structure leads to a large increase in the electron excitation rate coefficient.

The electron excitation transition from the ground $(2s^2)^1S$ state of C III to the $(2s2p)^1P$ state is important for solar studies. It is also important theoretically as it is typical of a spin-allowed optical transition. The most accurate calculations available at present are again the six-state, seven-orbital $R$-matrix results of Berrington et al. (1977). In these calculations the first eight partial waves were calculated using the $R$-matrix method, and the remaining partial waves were obtained using the Coulomb–Born
approximation with the same seven-orbital target states. Figure 9 presents the total collision strength as well as the contributions from $L \leq 7$ and from $L > 7$. At low energies there is a small amount of structure in the collision strength; this structure is shown in more detail in Fig. 7. Since the collision strength for optically allowed transitions increases with increasing energy, the contribution from the resonance region to the electron excitation rate coefficient does not dominate, in contrast to the spin-forbidden transition $\left(2s^2\right)^1S \rightarrow (2s2p)^3P$.

$R$-matrix calculations were also carried out, with six target states generated from only three orbitals, for electron incident energies above the resonance region. These results are only about 5% lower than the accurate results and are close to the $R$-matrix calculations of Robb (1980). The close-coupling results of Osterbrock (1970) are, however, about 50% higher than the accurate results at low energies, while the close-coupling results of Flower and Launay are about 20% lower. The distorted wave results of Mann (1980) are in good agreement with the accurate $R$-matrix
calculations; only in the resonance region do the distorted wave results differ by more than 10% from the $R$-matrix results. Also plotted in Fig. 9 are the Coulomb–Born results obtained with the seven-orbital CI wave functions. These Coulomb–Born results are much closer to the accurate results than the Coulomb–Born results of Nakazaki and Hashino (1977). However, the Coulomb–Born results obtained from the seven-orbital CI wave functions are in good agreement with the distorted wave results of Mann (1980), except in the resonance region, where they are 20% higher.

This reasonable agreement between the $R$-matrix, distorted wave, and Coulomb–Born results is to be expected for an optically allowed transition. For these transitions high partial waves are important, and these partial waves are insensitive to the procedure which is adopted to calculate them.

Six-state, seven-orbital $R$-matrix calculations have also been carried out for O V (Berrington et al., 1977). The results of these calculations for transitions from the ground to the $(2s2p)^3P$ and $(2s2p)^1P$ states are given in Figs. 10 and 11, where they are compared with other calculations. As in

![Fig. 10. Collision strength for the $(2s^2)^1S \rightarrow (2s2p)^3P$ transition in O V. Theoretical results: (---) six-state, $R$-matrix calculations (Berrington et al., 1979); (+) distorted wave calculations (Mann, 1980); (O) distorted wave calculations (Malinovsky, 1975); (□) distorted wave calculations (Saraph, 1972).](image-url)
the case of C III the distorted wave results for O V (Saraph, 1972; Malinovsky, 1975; Mann, 1980) are all in reasonable agreement with each other, and they are also in agreement with the more accurate $R$-matrix results, except in the resonance region, where the $R$-matrix results are much larger for the $(2s^2)^1S \rightarrow (2s2p)^3P$ transition.

In their calculations of the electron excitation of C III, Berrington et al. (1977) used the six target states $(2s^2)^1S$, $(2s2p)^3P$, $^1P$, and $(2p^2)^3P$, $^1D$, $^1S$, and seven orbitals. These calculations automatically give the excitation cross sections for all of the transitions between these six states. It is difficult to compare these cross sections directly; it is much more convenient and useful to compare the electron excitation rates. For a transition from a state $i$ to $f$ with cross section $Q(i \rightarrow f)$ (in $a_0^2$) the rate coefficient $C(i \rightarrow f)$ in units of cm$^3$ sec$^{-1}$ at a temperature of $T$ K is

$$C(i \rightarrow f) = \frac{1.735}{T^{3/2}} \int_{\Delta E_{if}}^{\infty} EQ(i \rightarrow f) \exp[-(3.158 \times 10^5 E)/T] \, dE$$ (70)
where $E$ is the energy of the incident electron and $\Delta E_{if}$ is the excitation energy in atomic units.

Figure 12 is a plot of the electron excitation rate coefficients for excitation of the ground state, $(2s^2)^1S$, of C III to the first five excited states. At low temperatures the excitation to the $(2s2p)^3P$ state is the dominant process, but as the temperature is increased, excitation to the $(2s2p)^1P$ state becomes important. Excitation to the $(2p^2)^1D$ state is always at least an order of magnitude slower than excitation to the $(2s2p)^1P$ state, and excitation to the $(2p^2)^3P$ or the $(2p^2)^1S$ states is about two orders of magnitude slower.

As the $(2s2p)^3P$ states are metastable, their populations may be quite large, and electron excitation from them is important. We compare in Fig.
Fig. 13. Rates for electron induced transitions from the metastable (2s2p)3P_0 state of C III to the (2s^2)^1S, (2s2p)^3P_{1,2}, (2s2p)^1P, (2p^2)^3P_{0,1,2} (2p^2)^1D, and (2p^2)^1S states (Dufton et al., 1978). Also plotted on the graph as a dashed line is the rate for the proton induced fine structure transitions (2s2p)^3P_0 \rightarrow (2s2p)^3P_1 + ^3P_2 (Doyle et al., 1980).

The rate coefficients for electron excitation and deexcitation from the (2s2p)^3P_0 fine structure level. For low temperatures the electrons are excited to the other (2s2p)^3P_{1,2} fine structure levels, and deexcitation to the ground (2s^2)^1S state is also important. However at 200,000 K excitation to the (2p^2)^3P_{0,1,2} levels is as important as excitation to the (2s2p)^3P_{1,2} levels, with excitation to the (2s2p)^1P and (2p^2)^1D levels being an order of magnitude smaller and excitation to the (2p^2)^1S level two orders of magnitude smaller.

A comprehensive tabulation of electron excitation rates for other transitions between the six lowest levels of C III has been given by Dufton et al. (1978). Rate coefficients for transitions between the six lowest levels of N
IV, O V, and Ne VII are also available (Dufton et al., 1978, 1979). As the nuclear charge increases, the rate coefficient for a given transition decreases rapidly. If we consider an optically allowed transition, the cross section (in $\pi a_0^2$) varies for large values of $E$ as

$$\frac{(2.303 f_{it}/\Delta E_{it})(\log E)/E}{(71)}$$

where $f_{it}$ is the optical oscillator strength for the transition $i \rightarrow f$. For $\Delta n = 0$ transitions, the oscillator strength for a given transition varies approximately $f_{it} = C/\Delta E_{it}$ as we go along an isoelectronic series. Hence, at high energies we expect the cross section for a particular optically allowed $\Delta n = 0$ transition to vary as $\Delta E_{it}^{-2}$ in an isoelectronic series. It is found that the low-energy cross section also varies as $\Delta E_{it}^{-2}$. Changing the variable in Eq. (70) to $X = E/\Delta E$, it is found that the rate coefficient, written as a function of $T/\Delta E$, varies as $\Delta E^{-2.5}$ for a particular optically allowed $\Delta n = 0$ transition in an isoelectronic series.

As an example of this variation we plot in Fig. 14 the electron excitation rate coefficient for the transition $(2s^2)^1S_0 \rightarrow (2s2p)^1P_1$, in C III, O V, and Ne VII. It would appear that this procedure may provide a reliable way of obtaining rate coefficients for higher members of the isoelectronic series.

**Fig. 14.** Electron excitation rates for the $(2s^2)^1S_0 \rightarrow (2s2p)^3P_{0,1,2}$ and $(2s^2)^1S_0 \rightarrow (2s2p)^1P_1$ transitions in C III, O V, and Ne VII (Dufton et al., 1978, 1979). The rate for the $(2s^2)^1S_0 \rightarrow (2s2p)^3P_{0,1,2}$ transition has been multiplied by $(\Delta E)^{3.5}$ and the rate for the $(2s^2)^1S_0 \rightarrow (2s2p)^1P_1$ transition has been multiplied by $(\Delta E)^{2.5}$. 
Because of the complex resonance structure which exists in the cross section for the \((2s^2)^1S \rightarrow (2s2p)^3P\) transition it is difficult to judge how this cross section varies as we move up the isoelectronic series. Empirically we find that if we again plot the temperature in terms of \(T/\Delta E\), then the rate coefficient appears to vary as \(\Delta E^{-3.5}\). The plot of the rate coefficient multiplied by \(\Delta E^{3.5}\) is given in Fig. 14.

To test the accuracy of their six-state \(R\)-matrix calculations for electron excitation of O V, Berrington et al. (1979) repeated their calculations with 12 target state functions consisting of the six \(n = 2\) states with configurations \((2s^2)\), \((2s2p)\), and \((2p^2)\) and also the six \(n = 3\) states with configurations \((2s3s)\), \((2s3p)\), and \((2s3d)\). The collision strength which they obtained for the \((2s^2)^1S \rightarrow (2s2p)^3P\) transition is given in Fig. 15. It is seen that the introduction of the six new \((2s3s)^1S\), \((2s3p)^1P\), and \((2s3d)^1D\) states introduces new series of resonances converging to their thresholds. Similar resonance series were found in the \((2s^2)^1S \rightarrow (2s2p)^1P\) collision strength.

The effect of these resonances on the rate coefficient for the \((2s^2)^1S \rightarrow (2s2p)^3P\) transition is large [unlike the \((2s^2)^1S^e \rightarrow (2s2p)^1P\) transition, where it is less than 2%] and is demonstrated in Fig. 16. As the new resonance structure occurs at incident electron energies between 3 and 6 Ry, it is found that at the temperature of maximum abundance of O
Fig. 16. Electron excitation rate for the \( (2s^2)^1S \rightarrow (2s2p)^3P \) transition in O V. Theoretical results: (—) \( R \)-matrix 12-state calculations (Berrington et al., 1979); (—--) \( R \)-matrix six-state calculations (Berrington et al., 1977); (-----) distorted wave calculations (Mann, 1980).

V in the sun, \( T = 200,000 \) K, the effect of these new resonance structures is about 10%. However, at higher temperatures we would expect a much larger increase in the cross section. An interesting calculation by Cowan (1980) on the rate coefficient for the \( (2s^2)^1S \rightarrow (2s2p)^3P \) transition in O V suggests that for rate coefficient calculations it is possible to include the resonance effects using simple perturbation methods. His calculations also
suggest that in the region of 200,000 K this rate coefficient should not be greatly affected by the inclusion of more target state functions.

A small number of calculations have also been carried out in Be-like ions on the electron excitation of transitions in which \( \Delta n \) changes by one or more. Distorted wave results for O V have been reported for a small number of energies by Malinovsky (1975) for the transitions from the \((2s^2)^1S\) and \((2s2p)^3P\) states to the \((2s3s)^1^3S\), \((2s3p)^1^3P\)\((2s3d)^1^3D\) states. These calculations have been corrected and extended to include excitations from the \((2s2p)^1P\) state (W. Eissner, private communication, 1980). Scaled electron impact collision strengths, \(Z^2\Omega,\) for transitions between the \((2s2l)\) states and \((2l3l')\) states \((l = s \) or \(p \) and \(l' = s, p, \) or \(d\) have been obtained by Parks and Sampson (1977) using the method suggested by Burgess et al. (1970). The Born approximation has been used to calculate excitation cross sections for transitions from the ground state of C III, N IV, and O V to the \((2sn)s\) \(^1S\), \((2snp)^1P\), and \((2npd)^1D\) states for \(n = 3, 4,\) and \(5\) (Ganas and Green, 1979). For Si XI nonexchange distorted wave approximation has been used to calculate the electron excitation rate from the ground state to the \((2s4p)^1P, (2s5p)^1P,\) and \((2s4d)^1D\) states (Davis et al., 1977).

We have seen in Sections III and IV that for heavy ions it is important to take account of relativistic effects when calculating atomic energies and oscillator strengths. It is evident that it is also important to consider relativistic effects in the target wave functions in calculations on the electron excitation cross sections of heavy ions. We have already discussed (Section III) a method of obtaining quite reliable CI relativistic wave functions for ions in which relativistic effects were not very large (Glass and Hibbert, 1978b). In this method the parameters of the one-electron orbitals [Eq. (4)] are obtained by diagonalizing the nonrelativistic Hamiltonian matrix, but the coefficients \(b_{ij}\) in the \(J\)-dependent CI wave function expansion [Eq. (15)] are determined by diagonalizing the Breit–Pauli Hamiltonian.

In intermediate coupling the wave function for a fine structure state with a given \(J_i\) and \(M_{J_i}\) and parity \(p\) is [Eq. (21)]

\[
|\Delta_i J_i M_{J_i} p\rangle = \sum_{L_i S_i i} f(\Delta_i J_i, \Gamma_i (L_i S_i p) / \Gamma_i (L_i S_i) J_i M_{J_i} p) \]  

where \(f\) [Eq. (22)] are called term-coupling coefficients (Jones, 1975; Saraph, 1972). Here \(\Delta_i\) distinguishes between two states with the same values of \(J_i\) and \(M_i\) but different energies, and \(\Gamma_i\) distinguishes between two states with the same \(L_i S_i\) but different energies. Using similar arguments (Jones, 1975) we can transform the \(R^{Jp}(\Gamma_i L_i S_i J_i l K; \Gamma'_i L_i' S_i' J'_i l' K'; k\ell)\) matrix to intermediate coupling using
\[ R^{J' \nu}(\Delta_i J_i l' K; \Delta_i' J_i' l' K'; k_i^2) \]
\[ = \sum_{l'_1 l'_2 S_1 S_i} f(\Delta_i J_i, \Gamma_i L_i S_i) R^{J' \nu}(\Gamma_i' L_i' S_i, J_i l' K; \Gamma_i' L_i' S_i, J_i' l' K'; k_i^2) \]
\[ \times f(\Delta_i' J_i', \Gamma_i' L_i' S_i') \]  
(73)

With this term-coupled \( R^{J\nu} \) matrix in Eq. (58) we obtain the term-coupled \( T^{J\nu} \) matrix. This can then be used in

\[ \Omega(\Delta_i J_i \rightarrow \Delta_i' J_i') \]
\[ = \frac{1}{2} \sum_{J_p} (2J + 1) \sum_{l'_1 k k' \nu} |T^{J\nu}(\Delta_i J_i l' K; \Delta_i' J_i' l' K'; k_i^2)|^2 \]  
(74)

to give the collision strength from one fine structure level \( \Delta_i J_i \) to another \( \Delta_i' J_i' \).

The effect of including relativistic effects in electron excitation calculations is most marked in spin forbidden transitions. Some very interesting term-coupling calculations have already been carried out for the electron excitation of Fe XXIII from the \((2s^2)^1 S_0\) state to the \((2s2p)^3 P_1\) state. These are displayed in Fig. 17. The nonrelativistic close-coupling calculations of Henry and Bhadra (1980) only differ by about 20% from the nonrelativistic distorted wave calculations of Mann (1980). The shape of the two collision strengths is also similar; they both decrease rapidly as the incident electron energy increases. This is the typical behavior of a spin-forbidden transition. The relativistic close-coupling (Robb, 1980) and distorted wave calculations (Mann, 1980) are also in close agreement, but they differ significantly from the nonrelativistic results. For example, at the excitation threshold the relativistic calculations are a factor of two larger than the nonrelativistic calculations, and at an energy of one hundred times threshold the factor is greater than ten. The inclusion of relativistic effects in heavy ions makes the collision strengths for spin-forbidden transitions similar in size and shape to the collision strengths for spin-allowed transitions. This is similar to the effect of including relativistic effects in transition probability calculations for heavy ions, where spin-forbidden transitions can have quite large transition probabilities when relativistic effects are included.

For Fe XXIII relativistic effects are also important for the transitions \((2s^2)^1 S \rightarrow (2p^2)^3 P\) and \((2s^2)^1 S \rightarrow (2p^2)^1 D\). Collision strengths for these transitions in Fe XXIII and other transitions between \((2s^2), (2s2p),\) and \((2p^2)\) configurations have been obtained by Mann (1980), Robb (1980), Younger (1980), Feldman et al. (1980), Bhadra and Henry (1980), and Parks and Sampson (1977).
B. Electron Excitation of Many-Electron Systems

In the previous section we have discussed the accuracy of electron impact excitation data for light ions using the Be sequence as a representative example. Here we discuss briefly results for ions in the third row of the periodic table and in particular new results for the S IV and Si III ions, which are both observed in the solar emission line spectrum.

Electron impact collision strengths for S IV have been published by Bhatia et al. (1980) and Bhadra and Henry (1980). The former used a distorted wave approximation and relatively simple configuration interaction target wave functions; the latter chose a close-coupling calculation with more complex wave functions. For the \((3s^33p)^2P^0 \rightarrow (3s3p^3)^1P^e\) transitions (the most important transition for the interpretation of solar UV
observations) the results of Bhadra and Henry are approximately 50% higher than those of Bhatia et al., which Bhadra and Henry attribute principally to the difference in target wave functions. Both sets of authors limit themselves to moderately high electron impact energies away from the region where resonances effects could be important.

Recently Dufton and Kingston (1980) carried out R-matrix calculations using target wave functions similar to but somewhat more complicated than those of Bhadra and Henry. For the common energies points agreement between the two calculations is good, with, for example, the differences for the $^2P^0 \rightarrow ^4P^e$ transition being less than 20%. However, in the low-energy region the collision strengths for many transitions are dominated by the resonance structure. This structure is shown in Fig. 18 for the $^2P^0 \rightarrow ^4P^e$ transition, and is particularly complex due to the many partial waves which contribute to it. The resonances have the effect of increasing the collision rate, and for an electron temperature of 70,000 K (corresponding to the maximum ionization fraction for S IV) the R-matrix results give a rate a factor of three higher than that deduced from the values of Bhadra and Henry.

A similar effect is found for Si III, for which Baluja et al. (1980) pub-
lished results from a major 12-state calculation. Again for many transitions, the low-energy cross sections are dominated by resonances, and in Fig. 19, the structure for the \((3s^2)^1S \rightarrow (3s3p)^3P^0\) transition is shown. This ion has a structure analogous to that of the Be sequence, but with additional closed shells. However, while in the Be sequence the effect of resonances was to increase the \(^1S^e \rightarrow ^3P^0\) collision rate by less than 50%, in the case of Si III, the rate is increased by factors of between two and six compared with the calculations of Nicolas (1977), which did not include resonances.

Hence, there is some evidence that for some transitions in heavier ions, the collision rates are dominated by resonances, and that calculations which fail to take such effects into account will give incorrect results by up to an order of magnitude.

VI. Proton Excitation

For most transitions, excitation by electrons is very much more effective than excitation by protons. However, it was pointed out by Seaton (1964) that excitation by protons can be important for excitation between fine structure levels. If we consider a plasma at a given temperature, then the kinetic energy of the electrons, \(\frac{1}{2}m_e v_e^2\), is equal to the kinetic energy of
the protons, $\frac{1}{2}m_p v_p^2$, where $m_e$ and $v_e$ are the electron mass and velocity and $m_p$ and $v_p$ are the proton mass and velocity, where $m_p/m_e = 1836$. The cross sections for electron and proton excitation are approximately equal when their velocities are equal. As the electron excitation cross section is largest for electron energies from one to about ten times the excitation energy of the transition, the proton excitation cross section will be largest at approximately 2000–20,000 times the excitation energy. Proton collisions will therefore be important for transitions which have excitation energies very much smaller than the thermal energy of the plasma. Excitation by other ions such as He$^+$ will be much less important due to the higher abundance of the protons.

There have been many estimates of the cross section for fine structure transitions induced by proton impact (Seaton, 1964; Bahcall and Wolf, 1968; Reid and Schwartz, 1969; Bely and Faucher, 1970; Masnou-Seeuws and McCarroll, 1972; Sahal-Brechot, 1974; Faucher, 1977; Faucher and Landman, 1977; Kastner, 1977). The close-coupled impact parameter approximation of Reid and Schwartz (1969) and Masnou-Seeuws and McCarroll (1972) has been used by Doyle et al. (1980) to obtain proton excitation rates for the fine structure transitions between the levels $\text{(2s2p)}^3\text{P}_{0,1,2}$ and between the levels $\text{(2p}^2)^3\text{P}_{0,1,2}$ of the Be-like ions C III, O V, and Ne VII. In this approximation the perturbing proton follows a classical Coulombic trajectory and the intermultiplet transitions are caused by the quadrupole component of the electrostatic interaction of the proton with the electrons of the ion. This arises because, in the expansion of the electrostatic interaction in terms of $R$, the distance between the proton and the nucleus, the first nonzero term in this expansion is the quadrupole term $\langle r^2 \rangle / R^3$, where $\langle r^2 \rangle$ is the expectation value of $r^2$ for the proton.

Faucher and Landman (1977) have carried out a more exact quantal treatment of these proton–ion collisions, and they have shown that the use of semiclassical methods does not give rise to significant errors. They suggest that the major error in the use of semiclassical methods arises from an inexact treatment of the penetration by the proton of the electron cloud of the ion. In their work Doyle et al. (1980) took account of this penetration by using several different short-range potentials and concluded that this penetration gave an error of about 1% in the cross sections below 200 eV but gave larger errors at higher energies.

Figure 20 gives the proton excitation rates for the $\text{(2s2p)}^3\text{P}_{1} \rightarrow \text{(2s2p)}^3\text{P}_{2}$ and $\text{(2s2p)}^3\text{P}_{0} \rightarrow \text{(2s2p)}^3\text{P}_{2}$ transitions for C III, O V, and Ne VII, which were calculated by Doyle et al. (1980). It was found convenient to plot the proton excitation rate multiplied by $(Z')^3$ against a reduced temperature $T K$ divided by $(Z')^3$, where $Z'$ is the nuclear charge minus two. The authors quote an accuracy of 1% for these results for temperatures below $10^6 K$. 


Fig. 20. Fine structure proton excitation rate multiplied by \((Z')^2\) for the \((2s2p)^3P_0^0 \rightarrow (2s2p)^3P_0^1\) and \((2s2p)^3P_1^1 \rightarrow (2s2p)^3P_2^0\) transitions in C III, O V, and Ne VII (Doyle et al., 1980).

and an error of 3\% at a temperature of 10^7 K. Their results for the transitions \((2p^2)^3P_1 \rightarrow (2p^2)^3P_2\) and \((2p^2)^3P_0 \rightarrow (2p^2)^3P_2\) are only slightly different from those for the \((2s2p)^3P\) transitions.

The rate for proton excitation of the \((2s2p)^3P_0^0\) state to the \((2s2p)^3P_1^1\) state for C III, O V, and Ne VII was also calculated by Doyle et al. (1980); their results are shown in Fig. 21. They also found that the rate

Fig. 21. Fine structure proton excitation rate multiplied by \((Z')^2\) for the \((2s2p)^3P_0^0 \rightarrow (2s2p)^3P_0^1\) transition in C III, O V, and Ne VII (Doyle et al., 1980).
for the \((2p^2)^3P_0 \rightarrow (2p^2)^3P_1\) transition was close to that for the \((2s2p)^3P_0 \rightarrow (2s2p)^3P_1\) transition.

Similar calculations were carried out by Malinovsky (1975) for O V using the formulas of Sahal-Brechot (1974). For \(J = 0 \rightarrow 2\) and \(J = 1 \rightarrow 2\) transitions, her results differ by less than 20\% from the results of Doyle et al. (1980), but for \(J = 0 \rightarrow 1\) transitions her results differ by about 30\%.

The proton excitation rate for excitation of the \((2s2p)^3P_0\) level to the \((2s2p)^3P_1\) and \(^3P^o_2\) levels in C III is plotted in Fig. 13, where it is compared with the rates of electron excitation of that level. At low temperatures the proton excitation rate is much smaller than the equivalent electron rate, but as the temperature increases, the proton excitation rate increases rapidly while the electron excitation rate decreases rapidly, at higher temperatures the proton rate being larger than the electron rate.

VII. Applications of Atomic Data to Solar Plasmas

There are a number of excellent reviews (Gabriel and Jordan, 1972; Dupree, 1978; Dere et al., 1979; Feldman et al., 1978) on the interpretation of the solar emission line spectra. Hence, here we shall limit ourselves to a description of how the observed emission line spectra can be used to derive the parameters of the emitting plasma. Given the atomic data discussed in the previous sections, it is possible to predict intensities for the solar emission line spectrum. The line strengths will depend upon the population of the ionic upper level of the transitions along the line of sight. In deriving such level populations it is often possible to consider each ionization stage independently. This is equivalent to assuming that the processes between different ionization stages are slow compared with those within each stage. Then for a set of \(n\) levels of a given ion, the change in the population \(N_i\), of level \(i\) is given by

\[
\frac{dN_i}{dt} = N_e \sum_{j=1}^{n} N_j C_{ji} - N_e N_i \sum_{j=1}^{n} C_{ij} + \sum_{j=i}^{n} N_j A_{ji} - N_i \sum_{j=1}^{i} A_{ij}
\]

(75)

where \(C_{ij}\) is the electron collision rate for a transition between levels \(i\) and \(j\) and unit electron density with \(C_{ii} = 0\) and \(A_{ij}\) is the spontaneous radiative deexcitation rate from level \(i\) to level \(j\) with \(A_{ii} = 0\).

The first two terms are the collisional rates into and out of level \(i\), respectively, while the second two terms give the corresponding radiative rates. As only spontaneous radiative deexcitation is considered, the sums
in the latter terms are limited to downward transitions. For some ions, e.g., He-like species, stimulated processes may also be important, and this would lead to additional terms, including the local radiation field in Eq. (75) (see, for example, Doyle, 1980). This formulation also assumes that transitions via ionic impacts are negligible. As discussed above, the solar plasma parameters are such that this process will only be significant for mixing between fine structure levels. These additional rates can be easily included via expressions analogous to the first two terms in Eq. (75).

For a stationary plasma, the time derivatives will be zero, and Eq. (75) reduces to \( n \) simultaneous equations of the form

\[
N_i = \frac{N_e \sum_{j=1}^{n} N_j C_{ji} + \sum_{j=1}^{n} N_j A_{ji}}{N_e \sum_{j=1}^{n} C_{ij} + \sum_{j=1}^{i} A_{ij}}, \quad i = 1 \text{ to } n
\]  

(76)


together with the constraint that the level populations are related to the volume density of the ionization stage, \( N_{\text{ION}} \), by

\[
\sum_{i=1}^{n} N_i = N_{\text{ION}}
\]

(77)

(the neglect of highly excited levels, which are not included in the rate equations, from this normalization is not likely to be serious as for solar plasma conditions they have minute populations).

The set of equations (76) has two limiting solutions corresponding to low- and high-density regions. For cases with low electron densities, where level \( i \) has an allowed transition to the ground state, the first term in the denominator is negligible. Also under such conditions the values of the excited level populations are very small (by typically eight orders of magnitude for \( N_e = 10^9 \text{ cm}^{-3} \)) compared with the ground state. The first term in the numerator is then the more significant, and the coronal approximation of Elwert (1952) is found, viz.,

\[
N_i = N_e N_1 C_{1i} / A_{i1}
\]

(78)

where the subscript 1 refers to the ground state.

In contrast at high electron densities the radiative terms become insignificant, and Eqs. (76) reduce to

\[
N_i = \frac{\sum_{j=1}^{n} N_j C_{ji}}{\sum_{j=1}^{n} C_{ij}}, \quad i = 1 \text{ to } n
\]

(79)
The relation between inverse collisional rates (see, for example, Mihalas, 1978) then leads directly to the thermodynamic equilibrium population distribution, namely,

$$N_j/N_i = g_j/g_i \exp(-E_{ji}/kT_e)$$  \hspace{1cm} (80)

where $g_j, g_i$ are the level degeneracies and $E_{ji}$ is the energy difference. For E1 transitions with spontaneous deexcitation rates of more than $10^8$ sec$^{-1}$, the high-density limit is only achieved for electron densities of the order of $10^{17}$ cm$^{-3}$, much higher than those found in the solar transition region or corona. However, for groups of levels which have no large allowed transitions (for example, fine structure levels), the high-density limit can often be obtained at solar densities.

The line emissivity, $\epsilon_{ij}$, for a transition between levels $i$ and $j$ will be given by (see, for example, Mihalas, 1978)

$$\epsilon_{ij} = (h\nu/4\pi)N_jA_{ji}$$  \hspace{1cm} (81)

and for an optically thin plasma the observed surface flux will be simply an integral over the line-of-sight region, where the line-emitting ion is abundant

$$F_{ij} = \int \epsilon_{ij} \, dx$$  \hspace{1cm} (82)

In the idealized case of a constant density and temperature with a line-of-sight dimension, $l$, the observed flux will be

$$F_{ij} = (h\nu/4\pi)A_{ji}lN_j$$  \hspace{1cm} (83)

and we note that for the coronal approximation the line strength becomes independent of the Einstein A coefficient and proportional to the collisional excitation rate.

Equations (76) and (82) provide a general solution for a stationary optically thin plasma. In a typical case the lowest 10–20 levels (including each fine structure level separately) would be included in the statistical equilibrium calculation, requiring the solution of the same number of simultaneous equations. This procedure would be undertaken at various positions with different electron densities and temperatures along the line of sight and the line flux calculated by direct numerical integration of Eq. (82). Several programs exist (see, for example, Dufton, 1978) which solve this relatively simple set of equations.

Such calculations are normally used to deduce plasma parameters, such as the electron density, from the observed relative line strengths. As an illustrative example we shall consider an isothermal, constant-density plasma (because a given ion has normally a significant population over only a relative small region of plasma, this approximation is useful for many solar features). Then the electron temperature can be deduced from
the ratio of the intensities of emission line originating from levels with different excitation energies of the same ion. The method is based on the different temperature dependence of the excitation rates populating the two levels. If we consider two levels $i$ and $j$ for which the principal rates are spontaneous radiative deexcitation and electron impact excitation from the ground state (i.e., excitation rates from metastable levels are not significant), then the coronal approximation gives

$$R = \frac{I_{ji}}{I_{it}} = \frac{E_j}{E_i} \frac{C_{ij}}{C_{1i}}$$

(84)

where $I_{ji}$ is the intensity of the transition from level $j$ to the ground state, $E_j$ is the energy of level $j$ relative to the ground state and $C_{ij}$ is the excitation rate to level $j$, which may be written as (see, for example, Gabriel and Jordan, 1972),

$$C_{ij} = \left( \frac{8.65 \times 10^{-6}}{g_1 T^{1/2}} \right) \Omega_{1j} \exp\left( -\frac{E_j}{kT} \right)$$

(85)

where $g_1$ is the degeneracy of the ground state and $\Omega_{1j}$ the effective collision strength is a slowly varying function of temperature.

Then

$$R = \frac{\Omega_{1j}}{\Omega_{1i}} \frac{E_j}{E_i} \exp\left( -\frac{E_j - E_i}{kT} \right)$$

(86)

Hence, given the observed line ratio, $R$, the electron temperature, $T$, can be derived.

Unfortunately for the line ratio to be sensitive to temperature the value of $(E_j - E_i)$ must be large. In such cases the transitions from the two levels to the ground state will be at well-separated wavelengths, leading to problems in determining the observational ratio accurately. For example, in O V the (2s3p)$^1$P $\rightarrow$ (2s$^2$) $^1$S and (2s2p)$^1$P $\rightarrow$ (2s$^2$) $^1$S lines used by Malinovsky (1975) to derive temperatures lie at 172 and 630 Å, respectively. The sensitivity of this ratio to electron temperature is illustrated in Fig. 22.

This problem may be circumvented by considering a transition from level $j$ to some excited level $k$. Then, provided that this spontaneous radiative transition is the main depopulation mechanism for level $j$, Eq. (86) becomes

$$\frac{I_{jk}}{I_{it}} = \frac{\Omega_{1j}}{\Omega_{1i}} \frac{E_j - E_k}{E_i} \exp\left( -\frac{E_j - E_i}{kT} \right)$$

(87)

and the wide wavelength separation need no longer necessarily exist. An example of such a situation for O V is the (2p$^2$)$^1$D$^e$ $\rightarrow$ (2s2p)$^1$P$^o$ transition at 1371 Å, which may be compared with the (2s2p)$^3$P$^o$ $\rightarrow$ (2s$^2$)$^1$S$^e$ transi-
Fig. 22. Temperature-sensitive line ratios in O V. The solid curve is the ratio of the intensities of the $(2s3p)^1P^0 \rightarrow (2s^2)^1S^0$ line at 172 Å to the $(2s2p)^1P^0 \rightarrow (2s^2)^1S^0$ line at 630 Å. The dashed curve is the ratio of the $(2s2p)^3P^0 \rightarrow (2s^2)^1S^0$ line at 1218 Å to the $(2p^2)^1D^0 \rightarrow (2s2p)^1P^0$ line at 1371 Å.

tion at 1218 Å. These lines are at similar wavelengths, and for electron densities less than $3 \times 10^{10}$ cm$^3$ both obey the coronal approximation. The temperature sensitivity of the ratio is again shown in Fig. 22 (Dufton et al., 1978). Although this is not as large as for the other O V ratio, this is probably more than compensated by the greater accuracy with which the observational ratio can be measured.

Although a number of different techniques exist for estimating the electron density, these all ultimately depend on comparing the population of a level whose population mechanisms are dominated by collisional processes with that of a level having a spontaneous deexcitation rate, which is greater than any of the collisional rates. The relative population of the former is given by Eq. (80) and is independent of electron density, while that of the latter is proportional to electron density. Hence, any method measuring the ratio of these populations will yield an estimate of electron density.

One approach consists of comparing the intensity of an intercombination spin-forbidden transition with that of an allowed transition. As an
example, the O IV intercombination lines \((2s2p^2)^4\text{P}^e \rightarrow (2s^22p)^2\text{P}^0\) can be compared with the C IV resonance \((2p)^2\text{P}^0 \rightarrow (2s)^2\text{S}^e\) lines, as has been discussed by Feldman and Doschek (1978). For electron densities \(N_e \geq 3 \times 10^{10} \text{ cm}^{-3}\), the upper levels of the O IV transitions have a Boltzmann population compared with the ground state, while the C IV upper levels are still in the coronal approximation due to their large spontaneous deexcitation rates. Hence, for large electron densities the ratio of the C IV line intensities to those of O IV are proportional to electron density. To use this method to derive absolute densities (compared with density differences between different solar regions) it is necessary to know the carbon-to-oxygen abundance ratio in the emitting plasma (usually assumed to be the photospheric value) and also to choose ions which will have similar spatial distributions. In ionization equilibrium O IV has a maximum ionization fraction at approximately \(1.2 \times 10^5 \text{ K}\) (Jordan, 1969), while C IV is formed at a lower temperature of \(9 \times 10^4 \text{ K}\) in presumably an overlapping but not spatially coincident region of the atmosphere. This difficulty can be partially eliminated by considering a second optically allowed transition in an ion, such as N V, which has a temperature of maximum ionization fraction higher than that of O IV, and hence in effect bracketing the O IV ion. Such methods have been widely used by Doschek, Feldman, and their co-workers (Bhatia et al., 1980; Feldman et al., 1978).

The problem of spatial coincidence can be considerably reduced by using two lines from the same ion. For example, in O V the resonance \((2s2p)^1\text{P}^0 \rightarrow (2s^2)^1\text{S}^e\) transition at 630 Å can be compared with the intercombination \((2s2p)^3\text{P}^0 \rightarrow (2s^2)^1\text{S}^e\) transition at 1371 Å. The sensitivity of this ratio to electron density is shown in Fig. 23 (it also has a small variation with temperature), the diagnostic being useful for \(N_e \geq 3 \times 10^{10} \text{ cm}^{-3}\), where the collision rate for the intercombination line is comparable with or greater than the radiative rate. Although this is theoretically more reliable, the constraint of using only one ion often results in suitable line pairs being widely separated in wavelength, with the observational value of the ratio being therefore more difficult to determine. Additionally, this method requires accurate atomic data. This is illustrated in Fig. 23, where the effect is shown of increasing the \(^1\text{S}^e \rightarrow ^3\text{P}^0\) rate by 25% and decreasing the \(^1\text{S}^e \rightarrow ^1\text{P}^0\) rate by a similar amount.

Other more indirect methods exist for estimating the electron density. For example, in the beryllium sequence the ratio of the two optically allowed transitions, \((2s2p)^1\text{P}^0 \rightarrow (2s^2)^1\text{S}^e\) and \((2p^2)^3\text{P}^e \rightarrow (2s2p)^3\text{P}^0\), has been extensively used (Loulergue and Nussbaumer, 1974, 1976; Jordan, 1974; Dufort et al., 1978; Dupree et al., 1976; Malinovsky, 1975). The intensity of the former depends on the population of the upper level, while
that of the latter is a measure of the populations of both its upper and lower metastable $^3P^0$ levels, because the principal population mechanism of the $^3P^e$ states is electron excitation from the $^3P^0$ states. The density dependence of this ratio is illustrated in Fig. 23 for O V, the results again being taken from Dufton et al. (1978). This indirect measurement of the population of the $^3P^0$ states has the advantage over the more direct measurement discussed above, in that the lines have a smaller wavelength separation (630–760 Å for O V). However, the theoretical results are again sensitive to the atomic data, particularly for the collision rates. This is shown in Fig. 23 by plotting the theoretical ratio with the collision rates changed as discussed above. As can be seen, the electron density deduced from a given ratio is changed by more than a factor of two by these modest changes in the atomic data.

Another method of determining electron densities utilizes lines within the same multiplet, for example, the $(2p^2)^3P^e \rightarrow (2s2p)^3P^0$ components in the beryllium sequence and the $(2s2p^2)^4P^e \rightarrow (2s2p^2)^2P^0$ components in the boron sequence. For the latter, the lines of O IV (Feldman and Dos-
Fig. 24. Ratio of the intensities of the \((J = 3/2, J' = 3/2)\) line to the \((J = 5/2, J' = 3/2)\) line in the S IV \((3s3p^2)^4P_j \rightarrow (3s^23p)^2P_j\) multiplet. The heavily dashed curve uses the collision rate data of Dufton and Kingston (1980), and the lightly dashed curve the data of Bhatia et al. (1980).

Chek, 1979) are of particular interest as they are well separated, while lying at a wavelength (1400 Å) accessible to conventional UV grating spectrographs. They have been observed by many solar experiments, including the recently launched Solar Maximum Mission Satellite. All the lines in the multiplet are spin forbidden and have small Einstein \(A\) coefficients. Hence, as the electron density increases, the population of the upper levels relative to the ground state changes from the coronal approximation to Boltzmann statistics. However, due to variation in the spontaneous radiative rates, this occurs at different electron densities for the different \(^4\)P\(_e\) fine-structure levels, leading to the density sensitivity.

This method has many advantages, including the use of a single-ion, small-wavelength separation of the components and line ratios near to unity, which are important for detectors such as photographic film, which suffer from saturation effects. However, there is again the need for accurate atomic data. This can be seen in the case of the analogous \((3s3p^2)^4P_j \rightarrow (3s^23p)^2P_j\) transitions in S IV. In Fig. 24 the ratio of the intensities of the \(J = 3/2\) to \(J' = 3/2\) transition at 1416.9 Å to the \(J = 5/2\) to \(J' = 3/2\) transition at 1406.0 Å is plotted as a function of electron density.
The dashed curve refers to the atomic data of Bhatia et al. (1980) based on relative simple S IV wave functions and ignoring the effect of resonances in the electron excitation rates. The heavily dashed curve incorporates more sophisticated wave functions for both the spontaneous radiative and $R$-matrix collision strength calculations and explicitly includes the effect of resonances (Dufton and Kingston, 1980). The Einstein $A$ coefficients are changed by typically 10–20%, but the collision rates are increased by factors of up to five (see the section on electron excitation calculations), and this, in turn, leads to major changes in the electron densities deduced from a given ratio.

Besides being used as probes of electron density and temperature, line ratios can provide information of the dimensions of the emitting plasma through optical depth effects. The above discussion has assumed that the plasma is optically thin, i.e., any photon emitted has a negligible probability of being reabsorbed. In practice for strong allowed transitions there is a significant chance that a photon will be lost by the inverse process; the greater the extent or density of the plasma, the greater is the probability. Hence, by comparing the intensity of different lines, usually in the same multiplet, it is possible to deduce information about the extent of plasma where the relevant ion has a significant population. Although atomic physics data are required for such analyses, they do not have the crucial importance as, for example, in electron density diagnostics. For further details the reader is referred to Doyle and McWhirter (1980) and Nicolas (1977).

VIII. Conclusions

In this review we have discussed recent advances which have been made in theoretical calculations of basic atomic data which are relevant in the analysis of solar emission line spectra. The best theoretical calculations are now estimated to be accurate to better than 5%, although such results are only available at present for a small number of ions. It is envisaged that in the future precise theoretical data for many other ions will become available and will be used in interpreting the large amount of observational data obtained from rocket- and satellite-borne solar instruments. Additionally, high-quality observations of the solar emission spectrum should provide some of the most rigorous checks on the accuracy of the theoretical calculations.
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