ULTRA-VIOLET EMISSION FROM THE CHROMOSPHERE

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Summary

A model of the quiet chromosphere is constructed which gives as good a fit as found possible to data from eclipse observations and to observations of solar radio noise and of the ionosphere, everything being taken at minimum solar disturbance. The model has spherical symmetry and a single value of the kinetic temperature at every height. In the model there is a sharp division between the lower chromosphere, at heights below 6000 km., in which the temperature is 5040 deg., and the upper chromosphere in which the temperature ascends at first very rapidly. It is estimated that the chromosphere emits about $7 \times 10^{14}$ quanta capable of ionizing terrestrial gases per sq. cm. per sec., of which $6 \times 10^{14}$ are emitted in line spectra of ions such as O VI, O V, N V, etc., and the remainder in the Lyman continuum of H. About half the quanta have an energy greater than 13.6 volts, and are therefore capable of ionizing O to O$^+$ in the ionosphere. The energy is supplied by conduction inwards from the corona.

1. Introduction.—In a recent paper (1) we calculated the emission spectrum of the corona in the far ultra-violet and drew attention to the part it might play in ionizing terrestrial gases to form the ionosphere. We suggested at that time that an appreciable part of the ionizing radiation received by the Earth's upper atmosphere might come from the chromosphere.

In this paper we attempt to make the same calculations for the chromosphere as we did for the corona: but whereas in the case of the corona values of the kinetic temperature and electron pressure which we felt to be reliable were available, we could not feel satisfied with any account of the temperatures and pressures in the chromosphere which we could find. Since the ultra-violet emission is very sensitive to both temperature and pressure, it has been necessary for us to work out a model chromosphere satisfying as many observational conditions as possible before we could estimate the ultra-violet emission.

2. Conditions to be satisfied by a model quiet chromosphere.—While the temperatures and pressures in the reversing layer and corona are reasonably well established, there are difficulties associated with attempts to deduce these quantities in the layer between them, the chromosphere. It has not yet been possible to set up a completely satisfactory system of differential equations representing heat transfer and mechanical equilibrium; moreover, direct observation shows that the actual chromosphere is torn about by disturbances, so that a model exhibiting spherical symmetry may be quite misleading. Nevertheless the idea of a quiet chromosphere with spherical symmetry may be of some use. It is known that the chromosphere, corona and ionosphere all exist at times of zero sunspot activity, and we set out with the idea of a model quiet chromosphere which is intended to represent minimum activity.
A satisfactory model of the quiet chromosphere should fit the following conditions:

(a) The far ultra-violet emission should equal the amount required to produce the minimum (zero sunspot) ionosphere.

(b) This emission should equal the amount of energy conducted inwards from the corona.

(c) The emission of thermal radio noise should fit quiet day observations on various wave-lengths.

(d) The density scale height of the low chromosphere should fit eclipse observations of decrement of the intensity of low excitation lines with height.

(e) The density scale height should fit eclipse observations of the decrement of strong Balmer lines extending to considerable heights.

(f) Electron densities should agree with eclipse values (1) from the intensity of the Balmer continuum and (2) from the resolution of Balmer lines.

(g) Electron temperatures should agree with eclipse observations of the Balmer continuum.

(h) Kinetic temperature of atoms should agree with eclipse observations of line width.

(i) Eclipse observations show that both high and low excitation lines (e.g. He and Fe) appear to originate at the same level. The model should explain this.

(j) The density scale height should be not less than the gravitational scale height for the kinetic temperature at the level. If it is greater, there should be some reason for it.

(k) The pressure should not increase outwards.

We have not found it possible to satisfy all these conditions with a spherically symmetrical model possessing a definite single value of the temperature at each height. We have however been unwilling to introduce a parameter representing variations of temperature from one part of the chromosphere to another at the same level, largely because this would introduce too much freedom into the model. Instead the model uses a mean temperature for each height.

It should be said here, briefly, that we have not taken (e) into account, because of the difficulty of calculating populations of the lower hydrogen excited states; that we have been unable to satisfy (h), which appears irreconcilable with (a); and that we cannot satisfy (i), which seems to demand variations of temperature at the same height.

3. Division of the chromosphere into two regions.—It is convenient to define three definite levels in the chromosphere:—

(a) The base of the chromosphere. Quantities evaluated for this level will be given the subscript $o$.

(b) The top of the chromosphere, which will be a specified level in the corona (subscript $c$).

(c) The level where hydrogen is one-half ionized (subscript $i$).

The region between (a) and (c) we call the low chromosphere. The flash spectrum is emitted mainly from the low chromosphere, in fact most of the phenomena generally regarded as chromospheric occur in this region. On the other hand, the region between (c) and (b), the upper chromosphere, is almost entirely coronal in character. From it come the radio noise and most of the far ultra-violet emission.
Our reasons for supposing that the division between the two levels is sharp are as follows:—

(1) The equation for the conduction of heat inwards from the corona, which leads to a temperature distribution in good agreement with radio noise observations, demands a very sharp drop in temperature at the bottom of the region controlled by conduction.

(2) A layer of extensive thickness at any temperature sufficient to ionize hydrogen appreciably leads to far too great an emission of Lyman quanta.

(3) The corona only escapes from radiative control, which would pin it to the Sun’s surface temperature, because it is optically thin. A few hundred kilometres of neutral hydrogen (at a density appropriate to the middle chromosphere) destroy the condition of optical thinness, through the heavy absorption in the Lyman continuum. Wherever hydrogen is substantially neutral, the material cannot be far from radiative equilibrium and the temperature cannot be far from the photospheric value.

4. The base of the chromosphere.—For the base of the chromosphere it is reasonable to assume equilibrium conditions at the Sun’s surface temperature. The density scale height (i.e. the height within which the density falls one exponential factor) will then be

\[ H = \frac{kT}{\mu g} = 1.2 \times 10^7 \text{ cm}, \]

where \( k \) is the Boltzmann constant, \( T \) the temperature for which we adopt 5040 deg. or \( \Theta = 5040 / T = 1.0 \), \( \mu \) the mean atomic mass for which we adopt \( 2.1 \times 10^{-24} \text{ g} \), and \( g \) gravity \( 2.74 \times 10^4 \text{ cm sec}^{-2} \).

At this level metals will be almost fully ionized and will provide most of the electrons. If the hydrogen : metal number ratio is \( 6000 : 1 \) (1), we obtain

\[ \log (N_e)_0 = \log (N_H)_0 - 3.8, \]  

where \( N_e \) and \( N_H \) are the numbers of electrons and neutral hydrogen atoms per cm\(^3\), and \((\quad)_0\) signifies the base of the chromosphere.

By definition, see e.g. Wildt (2), the continuous absorption for a transverse beam through the base of the chromosphere integrates to unity. For electron densities of the order of \( 10^{12} \) and \( \Theta = 1 \) the absorption is mostly by negative hydrogen ions (3) and is given as \( 9.32 \times 10^{-28} \text{ per } H \text{ atom and per unit electron pressure. The absorption coefficient is then} \]

\[ 6.5 \times 10^{-38} N_H N_e \text{ cm}^{-1}. \]

The integrated absorption through a transverse beam becomes

\[ 6.5 \times 10^{-38} (N_H N_e)_0 (\frac{1}{2} H 2\pi R_\odot)^{1/2}, \]

where \( R_\odot \) is the Sun’s radius, and \( \frac{1}{2} H \) is the scale height of the product \( N_H N_e \).

Equating this to unity and using (4.1) gives

\[ \log (N_H)_0 = 15.9, \quad \log (N_e)_0 = 12.1, \]

and from Saha’s equation the proton concentration \( N_p \) is

\[ \log (N_p)_0 = 11.2. \]

These values of \( (N_H)_0 \) and \( (N_e)_0 \) are a little higher than those determined by Wildt (2), the reason being that we have assumed the scale height of gravitational
equilibrium, whereas Wildt extrapolates the observed scale height to the base of the chromosphere.

5. *The top of the chromosphere.*—It is necessary to choose a particular level in the corona to represent the top of the chromosphere at a height $h$. Baumbach (4) has given the relation between electron density and height, shown in Table I.

<table>
<thead>
<tr>
<th>Table I</th>
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<tbody>
<tr>
<td><strong>The Lower Corona</strong></td>
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<tr>
<td>Height</td>
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<tr>
<td>Electron density</td>
</tr>
<tr>
<td>&amp; $h'N_e^2$ (unit = $10^{26}$ cm.$^{-5}$)</td>
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<td>0</td>
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The ultra-violet and the radio emission depend on $(h'N_e^2)_o$ and it is seen from Table I that $h'N_e^2$ is fairly independent of $h'$. Hence the particular choice is not important. We assume Baumbach’s heights are measured from $h$, the height of the half-ionized layer, and fit the chromosphere to the corona at the height $h = h + 4 \times 10^6$ cm., which is roughly the height where coronal lines reach their greatest intensity (5). Choosing $10^6$ deg. K. as the corona temperature (1, 6) we have the adopted values for the top of the chromosphere as follows:

$$N_e = (N_e)_o = (N_p)_o = 2.4 \times 10^8 \text{ cm.}^{-3},$$

$$T_o = 10^6 \text{ deg. K.,}$$

$$h = h + 4 \times 10^6 \text{ cm.,}$$

$$(h - h)N_e^2 = 2.3 \times 10^{26} \text{ cm.}^{-5}.$$

The electron densities of the corona have been analysed more recently by van de Hulst (7), who finds a considerable change with the sunspot cycle. His values for sunspot minimum give $(h - h)N_e^2 = 3 \times 10^{26}$, and would cause a considerable change in the values calculated in this paper. However, we prefer to accept Baumbach’s data as a standard and modify the results if found necessary.

6. *The level where hydrogen is half ionized.*—We suppose that the character of the chromosphere changes very rapidly at the level where hydrogen is half ionized. Above this level there are few hydrogen atoms and the chromosphere is optically thin; but below this level the strong Lyman continuum makes the optical depth rise rapidly. The absorption of the $H$ atom beyond the Lyman limit is approximately $10^{-17}$ cm.$^3$, so that, if the neutral hydrogen density $N_H$ were only $10^6$, unit optical thickness in the continuum would be reached in 100 km.

A lower limit to the atom and electron populations at this level may easily be obtained. Although gravity can be reduced by supporting forces (such as radiation pressure) the net resultant force on the materials must be inwards, otherwise the chromosphere would disperse outwards; hence the pressure cannot increase outwards. We must have, therefore,

$$\left( N_H + N_p + N_e \right) T_i \geq (N_p + N_e)_o T_o,$$

or

$$3N_i T_i \geq 2N_e T_o,$$

the subscript $p$ referring to protons, $N_i$ is the number of protons, electrons, or neutral $H$ atoms per cm.$^3$ at the half-ionized level, and $N_e$ the number of protons or electrons per cm.$^3$ at the top of the chromosphere.
After carrying out a number of calculations upon models in which the pressure was allowed to decrease with height, and in which the amount of ultra-violet radiation given by the model was always too high (i.e. predicted too great an ionosphere), the authors felt forced to the conclusion that the decrease of pressure with height in the high chromosphere is very small: the pressure has therefore been taken to be constant, that is,

$$3N_i T_i = 2N_e T_e.$$  \hspace{1cm} (6.2)

As the half-ionized level lies immediately above a layer of unit optical depth in the Lyman continuum and is subjected to some ultra-violet radiation from above, the dilution factor of the Lyman continuum radiation must be close to unity. It can be shown that the effect of collisions upon the ionization balance is small, so that the degree of ionization is given well by Saha's equation: this gives a second relation between \((N_e)_i\) and \(T_i\). From Saha's equation and (6.2) we find

$$N_i = (N_e)_i = (N_p)_i = (N_H)_i = 2.5 \times 10^{10} \text{ cm.}^{-3},$$

$$T_i = 6300 \text{ deg.}$$

7. *The lower chromosphere.*—By lower chromosphere we mean the region between the photosphere and the level where hydrogen is half ionized. Since the lower chromosphere is optically thick just beyond the Lyman limit, hydrogen ionization is adjusted to the Sun's surface temperature, and we expect the kinetic temperature to be close to this temperature.

There is ample evidence that the scale height \(H\) in most of this region is considerably greater than the gravitational value \(1.2 \times 10^7 \text{ cm.}\). For example, Cillié and Menzel (8) give about \(6 \times 10^7 \text{ cm.}\) for metals at about 1100 km. height, and Wildt (2) gives \(2.4 \times 10^7 \text{ cm.}\) at a height of 800 km. All observations show \(H\) to increase with height \(h\). Since we have found the same hydrogen to metal ratio in the corona as obtains in the reversing layer (1), we may assume the scale height to be the same for all atoms (provided that all states of ionization are taken together). We adopt

$$H = 1.2 \times 10^7 + 0.20h,$$

where the heights are in centimetres. This formula is in reasonable agreement with eclipse observations and reduces to the gravitational value when \(h = 0\). As we suppose that the kinetic temperature does not increase much with \(h\) in the lower chromosphere, we must attribute most of the rise in scale height to turbulence of the kind suggested by McCrea (9).

The relation between \(N(=N_H+N_p)\) and \(h\) is

$$\frac{N}{N_0} = \left(1 + \frac{0.2h}{1.2 \times 10^7}\right)^{-1/0^2}.$$

From this formula, and from \(\log N_0 = 15.9\), \(N(=N_H+N_p)\) reaches \(5 \times 10^{10}\) when \(h = 6 \times 10^8 \text{ cm.}\), or \(h = 6 \times 10^8 \text{ cm.}\).

In Table II, where we tabulate various quantities for the entire chromosphere, we suppose \(T\) to remain at 5040 deg. up to \(h = 5 \times 10^8 \text{ cm.}\), and to rise to 6300 deg. at \(h = 6 \times 10^8 \text{ cm.}\). The values of \(N_e\) and \(N_p\) in the lower chromosphere are calculated from Saha's equation.

8. *The upper chromosphere.*—The temperature distribution of the upper chromosphere should be governed mainly by the conduction of energy.
inwards (10). Chapman and Cowling (11) have shown that the thermal conductivity of an electron gas is given by

$$\lambda_i = \frac{75}{16} \left( \frac{k}{3\pi m} \right)^{1/2} h^3 \frac{k^8}{e^2} T^{5/2}/A_2(2),$$

where $A_2(2)$ is $9.2 \log \left(4kT/N_e^{1/2}e^2\right) - 2$ or about 50 for the chromosphere, and the other symbols have their usual meaning. For a mixture of protons and electrons in equal numbers the conductivity will be half as great as this, since for a given electron density the free path of electrons will be halved. The heat conductivity in the upper chromosphere then becomes

$$\lambda = \eta T^{5/2},$$

where

$$\eta = 0.5 \times 10^{-6}.$$

Then the equation of heat conduction is

$$\epsilon = \eta \cos \theta T^{5/2} \frac{dT}{dh}, \quad (8.1)$$

where $\epsilon$ is the rate of heat conduction inwards, and $\theta$ is the angle between the magnetic field and the vertical. The $\cos \theta$ term is introduced because in the rarefied chromosphere the ions will have freedom of movement only in the direction of the magnetic field. The amount of heat conducted will decrease inwards because some of the energy will be radiated away. To allow for this decrease we adopt the relation

$$\epsilon = \epsilon_0 \left( \frac{h - h_f}{h_c - h_f} \right)^n = \epsilon_0 \gamma^n, \quad (8.2)$$
where \( y \) is the height above \( h_i \) in terms of the height of the upper chromosphere \( (h_c - h_i) \) and \( n \) has yet to be determined. Equations (8.1) and (8.2) combine to give

\[
T^\frac{5}{2} \frac{dT}{dy} = \frac{\varepsilon_c(h_c - h_i)}{\eta \cos \theta} \cdot y^n,
\]

with the boundary conditions \( T = T_i \) at \( y = 0 \) and \( T = T_c \) at \( y = 1 \), the extra condition imposing a relation between the constants of the differential equation.

If we neglect \( T_i/T_c \) compared with unity the solution of (8.3) is

\[
(T/T_c)^{7/2} = y^{n+1},
\]

and the relation between the constants is

\[
\varepsilon_c = \frac{2(n + 1) \eta \cos \theta}{7(h_c - h_i)} T_c^{7/2}.
\]

The relation (8.5) gives \( \varepsilon_c \) the amount of heat conducted inwards from the corona. Taking the mean value of \( \cos \theta \) to be \( \frac{1}{3} \) and using \( n = \frac{8}{5} \) to fit the radio observations as explained below, and with \( (h_c - h_i) = 4 \times 10^8 \text{cm} \), \( T_c = 10^8 \) (Section 5), we get from (8.5)

\[
\varepsilon_c = 1.7 \times 10^4 \text{erg cm}^{-2} \text{sec}^{-1}.
\]

This is less than estimates given by Alfvén (12) \( (2.2 \times 10^5) \) and Giovanelli (10) \( (6 \times 10^5) \).

The temperature distribution being given by (8.4), the distribution of the electron population follows from the assumption that the pressure is constant (as in (5.2)), for then

\[
N_e = N_c T/T = N_c y^{(2n + 2)/7},
\]

or with \( n = \frac{8}{5} \),

\[
\frac{N_e}{N_c} = \frac{T}{T_c} = y^{2/5}.
\]

It will be remembered that \( N_c = (N_e)_c \). Values of \( N_e \) and \( T \) as a function of height from this relation are given in Table II. In this table ionization of hydrogen in the upper chromosphere is found from the collision ionization formula (1),

\[
\frac{x}{1-x} = \frac{S}{\alpha},
\]

where \( x \) is the fraction of ionized atoms, \( S \) the ionization collision coefficient (for hydrogen \( 4.3 \times 10^{-10} T^{1/2} \exp (-11600 \chi/T) \), with ionization potential \( \chi \) in volts), and \( \alpha \) the recombination coefficient (for hydrogen \( 1.8 \times 10^{-11} T^{-1/2} \) in the chromosphere).

We have now to justify our choice of \( n \). This has been taken to give the best fit of quiet day solar noise intensities on different wave-lengths. The effective temperature at the centre of the disk \( T_E \) is related to the kinetic temperature \( T \) at various levels by

\[
T_E = \int_{0}^{\infty} T e^{-\tau} d\tau,
\]

where \( \tau \) is the radio optical depth, equal to \( \int_{h}^{\infty} \kappa dh \) at height \( h \), \( \kappa \) being the absorption coefficient per cm. Martyn (13) has given a convenient expression for
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this (which can be deduced either from the Kramers-Gaunt formulae or from the Lorentz-Chapman formulae), namely,

$$\kappa = \frac{4}{3} \left( \frac{2}{\pi} \right)^{1/2} \frac{e^6}{c(mk)^{3/2}} \frac{N_0^2}{f^2T^{3/2}} \ln (3kT/2\hbar)$$

$$= 1.08 \times 10^{-23} N_0^2 \lambda^2 T^{-3/2} \ln (3kT\lambda/2\hbar).$$

Using the value $13$ for the slowly varying term this reduces to

$$\kappa = a N_0^2 \lambda^2 T^{-3/2},$$

where $a = 1.4 \times 10^{-22}$.

Now

$$\tau = \int^\infty_h \kappa dh = a\lambda^2 \int^\infty_h N_0^2 T^{-3/2} dh = a\lambda^2 N_0^2 T^{-3/2} \int^1_0 T^{-7/2} dy,$$

and since $T^{-7/2} = T^{-7/2} \cdot y^{n+1}$ the integration can be easily carried out, giving

$$\tau = (h_c - h_i) N_0^2 a T^{-3/2} \lambda^2 (y^{-n} - 1)n^{-1}.$$

If we now put $m = (2n + 2)/7n$, $C = (h_c - h_i) N_0^2 a T^{-3/2} (\approx 3.2 \times 10^{-5})$ and $z = C\lambda^2/n$, then

$$\tau = z(y^{-n} - 1),$$

from which

$$y^n = \frac{z}{z + \tau}.$$

Now by (8.4) $T/T_c = y^{2(n+1)/7} = y^{nm}$, so that $T/T_c = (z/z + \tau)^m$. We can now integrate (8.7), the result being

$$T_B = T_c z^m \int^\infty_0 (z + \tau)^{-m} e^{-\tau} d\tau$$

$$= T_c z^m e^z E_i(m(z)).$$

Here

$$E_i(z) = E_i(z) = \int^\infty_z x^{-1} e^{-x} dx,$$

$$E_i^2(z) = \frac{1}{z} e^{-z} - E_i(z),$$

$$2E_i^2(z) = \frac{1}{z^2} e^{-z} - E_i^2(z),$$

$$(m - 1)! E_i^m(z) = z^{-m-1} e^{-z} - E_i^{m-1}(z).$$

(The function $E_i(z)$ is given by Jahnke and Emde (14) and called $-E_i(-z)$. It is much more convenient to tabulate $e^z E_i(z)$ which varies comparatively slowly with $z$. This has been done in typescript tables used in the present work. Where $m$ is not a whole number, the Incomplete $\Gamma$ function can be used.)

For $n = 0$, $m$ becomes infinite and the solution of (8.7) is

$$T_B = T_c \frac{7CA^2}{7CA^2 + 2}.$$

For the case which we actually adopted, $n = \frac{3}{8}$ and $m = 1$, giving

$$T_B = T_c z^e E_i(z).$$
The calculated values of $T_E$ for $n = 0$, $m = \infty$; $n = \frac{1}{3}$, $m = 2$; and $n = \frac{2}{3}$, $m = 1$ are shown in Fig. 1. In the same diagram we show various estimates of $T_E$ obtained from observations. Original observations give values of $T_A$, the apparent temperature of the whole Sun, which as a result of limb brightening and corona spreading is greater than $T_E$. Recent calculations by Nicolet (15) can be interpreted to show that $T_E/T_A \approx 0.5$ and is almost independent of frequency in the range 100 to 3000 Mc./s. Eclipse observations (16) on 600 Mc./s. yield a similar factor. However, for the shortest wave-lengths we must expect

$$T_E \approx T_A \approx 5000 \text{ deg. K.},$$

hence we obtain an estimate of $T_E$ from the formula

$$T_E = \frac{3}{5}(T_A + 5000 \text{ deg.}).$$

Values of $T_A$ for quiet Sun are obtained from (a) a list prepared by Pawsey and Yabsley (17) (containing observations by various authorities); (b) low values from

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Fig. 1.—Relation between frequency and effective temperature of the centre of the Sun’s disk.

A: Calculated $n = 0$, $m = \infty$.
B: Calculated $n = \frac{1}{3}$, $m = 2$.
C: Calculated $n = \frac{2}{3}$, $m = 1$.
○: Observations listed by Pawsey and Yabsley.
□: Observations from Quarterly Bulletin.
×: Other observations.
daily observations from the *Bulletin of Solar Activity* 1949 (in preparation; containing observations from the Cavendish Laboratory, Cambridge; National Bureau of Standards, Washington; National Research Council, Ottawa; Radiophysics Laboratory, Sydney; and Commonwealth Observatory, Canberra); and (c) recent observations by Minnett and Labrum (18); Piddington and Hindman (19); Laffineur and Houtgast (20); and the S.P.I.M. (21).

In Fig. 1 it is seen that the higher frequencies show best agreement for \( n = 3 \) and \( m = 1 \). It might be noticed also that on the lower frequencies better agreement would have been obtained if a coronal temperature \( T_c = 700,000 \) deg. had been adopted instead of \( 1,000,000 \) deg.

A chromospheric model based on solar noise intensities has been devised by Piddington (22). His analysis is similar in intention to ours but makes very different use of the optical evidence. His chromosphere does not have the sharp dividing level. The fact that both his chromosphere and ours are in agreement with the solar noise observations shows that these observations alone are not sufficient to define the chromospheric model.

9. **Discussion of the model.**—Several attempts have been made to interpret the various observations and set up a model of chromospheric conditions. These of course differ in detail but they fall into two main classes, according to the kinetic temperature given to the optically observable chromosphere, the lower chromosphere of our model. There are two distinct views:

(a) that the temperature of the optical chromosphere is about 30,000 deg., and
(b) that it is about 5000 deg.

The main arguments on each side are

(a) for high temperatures, arguments from

(i) thermal line width measurements (23),
(ii) scale height measurements (2), (8),
(iii) the emission of He lines;

and (b) for the low temperature, arguments from

(i) radio noise measurements (22).
(ii) excitation temperature measurements (24),
(iii) solar ultra-violet emission as indicated by the ionosphere,
(iv) electron temperature (25),
(v) the non-appearance of forbidden lines (26).

In making a decision we were mainly influenced by (b) (iii), as we were forced to adopt the low temperature to bring the Lyman continuum emission down to a level allowed by the ionosphere. On the other hand, a high temperature chromosphere has been chosen by Giovanelli (27) and by Thomas (28).

Once the temperature of the low chromosphere has been decided, the model only depends on the base conditions and scale height. If we compare our model with requirements listed in Section 2 we find (a), (f) and (g) in fair agreement with eclipse observations (2), (8), (29), but (h) and (i) in disagreement. The model offers no reason why the scale height should be greater than the gravitational scale height, but McCrea's suggestion of turbulent motion (9) may be accepted. It is possible that the excitation of He and He\(^+\) lines (i) may be due to a coincidence of a He wave-length with a far ultra-violet coronal emission: van Dijke suggests (30) that the 2s^3S metastable state of He is somehow excited, and this could be due to
resonance with Mg X radiation from the corona. The wave numbers concerned (31) are He I 150850 and Mg X 159929. Since the latter spectrum line has not actually been observed, the coincidence may be closer than this value suggests.

The failures encountered by our model might be surmounted if we abandoned spherical symmetry and postulated a chromosphere made up of pockets of material at very different temperatures. This is of course strongly suggested by the irregular appearance of the chromosphere and by the apparent presence of high and low excitation lines at the same level. We have however, as has been said before, been unwilling to introduce more variables into our model.

A feature of our model is the sharp dividing level between upper and lower chromospheres, already advocated on very different grounds by Wurm (26). In the real chromosphere this division cannot occur everywhere at the same height without distortion. Such distortion may not have much effect on the amounts of far ultra-violet and microwave energy emitted by the centre of the disk, though there may be some reduction of the microwave limb brightening. The emission of ultra-violet radiation is however very greatly influenced by the thinness of the skin separating upper and lower chromospheres. Any smoothing of the (mean) temperature curve at the dividing point greatly increases the emission in the Lyman continuum and other spectra.

10. Ultra-violet emission.—The study of the chromosphere just described was initiated for the prime purpose of determining the emission of ultra-violet radiation. An approximate estimate of the number of ultra-violet quanta emitted from the Sun’s surface at sunspot minimum may be obtained from ionospheric studies (32), (33), giving for the E region $2 \times 10^{13}$ quanta cm.$^{-2}$ sec.$^{-1}$, and for both the F$_1$ and F$_2$ region $10^{14}$ quanta cm.$^{-3}$ sec.$^{-1}$. The wave-lengths of these quanta are not known but there is good reason to suppose that they are below the Lyman limit at 912 A., and we give our attention to these wave-lengths, i.e. to excitation energies greater than 13.6 volts. All early attempts to construct a model chromosphere gave far too many quanta and this has been taken into consideration in choosing the model we have described.

The black-body emission above 13.6 volts from the low chromosphere at 5040 deg. is $2 \times 10^{11}$ quanta cm.$^{-2}$ sec.$^{-1}$ and negligible. However, near the half-ionized level the temperature has risen sharply to 6300 deg., for which the black-body emission is $1.4 \times 10^{14}$ quanta cm.$^{-2}$ sec.$^{-1}$. On account of the rapid temperature change the low chromosphere emission will be less than this, probably about half, but it will be of the same order as the emission required to produce one of the ionospheric layers.

The upper chromosphere is optically thin in the Lyman continuum, so that the quanta emitted can be determined simply by counting the recombinations. If $\alpha$ is the recombination coefficient, the total emission per unit time from a column of unit cross-section is

$$Q = \int_{h_i}^{h_c} N_e N_p \alpha \, dh = \int_{h_i}^{h_c} N_e^2 \alpha \, dh.$$  

Putting

$$\alpha = bT^{-1/2} \quad (\text{where } b = 1.8 \times 10^{-11})$$

we have generally

$$Q = (h_c - h_i) N_e^2 b T_c^{-1/2} \int_0^1 y^{8(n+1)/7} \, dy$$

$$= \frac{7b(h_c - h_i)N_e^2 T_c^{-1/2}}{2 - 5n},$$
but this formula breaks down if \( n > \frac{3}{8} \), in which case it is necessary to set the lower boundary not at \( y = 0 \) but an appropriate value \( y = y_1 \), giving the lower temperature \( T_1 = 6300 \text{ deg.} \) instead of \( T_i \) zero. For \( n = \frac{3}{8} \),

\[
Q = (h_e - h_i)N_e^2 b_T c^{-1/2} \ln \left( \frac{1}{y_1} \right),
\]

and \( T_1 = 6300 \text{ deg.} \) gives \( \frac{1}{y_1} = 316,000. \) Then

\[
Q = 5.2 \times 10^{18} \text{ quanta cm}^{-2} \text{ sec}^{-1}.
\]

These quanta are emitted in all directions, of which half are outwards, so that the outward emission is

\[
2.6 \times 10^{18} \text{ cm}^{-2} \text{ sec}^{-1},
\]

which is not great enough to influence materially the total Lyman emission from upper and lower chromosphere: we have already estimated about \( 10^{14} \text{ cm}^{-2} \text{ sec}^{-1} \) from the lower chromosphere.

It should be noticed that this result has only been achieved by admitting the sharp rise of temperature above the half-ionized layer. A chromosphere at 30,000 deg. would give far too much radiation to be allowed by the ionosphere, for, if the chromosphere were neutral, it would give 30,000 deg. black-body radiation in the Lyman continuum, which is too intense; and if it were ionized it would have to have a sharp temperature boundary at its base to keep the recombination radiation down. The boundary in our model is given by the conductivity equation, but there seems no reason for another sharp boundary at a lower level below a 30,000 deg. plateau as implied by Giovanelli (27).

In order to compute the line emission from the upper chromosphere we use the expression derived in our earlier paper (1). The number of quanta emitted per unit volume and time is taken to be equal to the number of atoms excited to the upper state by electron collisions, that is equal to

\[
N_a N_e \frac{12\pi e^2}{(2\pi m k T)^{1/2}} \frac{I}{X} \left\{ \exp \left( -\chi \right) - \chi \epsilon \left( \chi \right) \right\},
\]

(10.1)

where \( N_a \) is the concentration of atoms in the particular state of ionization being considered, \( \chi' \) is the excitation potential (in ergs) of the strongest transitions, \( f \) the oscillator strength, and \( y = \chi'/kT \). To simplify the calculations we assume that each atom is in one particular state of ionization in a certain range of chromospheric height. The level dividing each ion from the next higher ion is obtained by putting \( x/(1-x) = \chi \), where \( x \) is the fraction ionized. We have the relation

\[
\frac{x}{1-x} = \frac{S}{\alpha},
\]

where \( S \) is the collision ionization coefficient and \( \alpha \) the recombination coefficient. We use values that are generalized from our earlier paper as follows:

\[
S = 3 \times 10^{-8} T^{1/2} \chi^{-2} \exp \left( -11600 \chi/T \right) \quad \text{(with } \chi \text{ in volts)}
\]

\[
\alpha = 1.5 \times 10^{-11} \chi^2 T^{-1/2},
\]

from which we find that the temperature of the dividing level is given by the solution of

\[
T = 5 \times 10^{-4} \chi^2 \exp \left( 11600 \chi/T \right) \quad \text{(with } \chi \text{ in volts)}.
\]

From the model, the degree of ionization \( Z \), and the ionization potential \( \chi \), it is possible to determine the height range occupied by each ion. From (10.1) we determine the emission per unit volume at the top and the bottom of the range.
### Table III

<table>
<thead>
<tr>
<th>Ion</th>
<th>II</th>
<th>III</th>
<th>IV</th>
<th>V</th>
<th>VI</th>
<th>VII</th>
<th>VIII</th>
<th>IX</th>
<th>X</th>
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<th>XIII</th>
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<th>XV</th>
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<tr>
<td>C</td>
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<td>9'6</td>
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<tr>
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<tr>
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<td>12'5</td>
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</table>

Excitation potentials in volts $\chi'$, and rate of quantum line emission per cm.$^2$ of chromosphere per sec. $Q'$. 

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If these agree within about a factor of 3 we take a mean and multiply by the height to obtain the total emission. If they differ we use the difference to determine a scale height of the quantity and multiply this by the higher value. In order to apply the procedure we require values of the element/hydrogen ratios, ionization potentials $\chi$, and excitation potentials $\chi'$ and oscillator strengths $f$ for the strongest lines. The element/hydrogen ratios are from our earlier paper (1), and both excitation and ionization potentials from Mrs Moore Sitterly's recent tables (34). It was assumed that most of the energy in each spectrum was emitted in the transition from the lowest configuration to the next configuration of opposite parity. For neutral atoms it would be appropriate to take $f = 1$, but when the two configurations have the same total quantum number, as is usually the case, the higher ions become more hydrogen-like and $f$ tends to zero for this transition (35). Thus for $\text{Mg}^X$ the $f$-values, which may be determined from the tabulation of Bates and Damgaard (36), are as follows:

<table>
<thead>
<tr>
<th>Transition</th>
<th>$f$</th>
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</thead>
<tbody>
<tr>
<td>$2s - 2p$</td>
<td>0.12</td>
</tr>
<tr>
<td>$2s - 3p$</td>
<td>0.33</td>
</tr>
<tr>
<td>$2s - 4p$</td>
<td>0.08</td>
</tr>
</tbody>
</table>

We adopt the value of 0.2 for $f$ throughout, and the emission is divided by 2 to give $Q'$ the outward flux per cm.$^2$. Values obtained for the more abundant ions are shown in Table III. The total outward emission is about $3 \times 10^{14}$ quanta cm.$^{-2}$ sec.$^{-1}$ for radiation above 13.6 volts (and therefore capable of ionizing $O$ to $O^+$ in the Earth's atmosphere) and another $3 \times 10^{14}$ quanta between 10 and 13.6 volts. These values are reasonably close to the requirements of the F regions of the ionosphere, namely $10^{14}$ quanta cm.$^{-2}$ sec.$^{-1}$. According to our calculations, line emission from the chromosphere is the most important source of solar far ultraviolet emission, the chromosphere being more important than the corona.

We find that both continuous Lyman emission and emission in the lines of ionized atoms are of the right order of magnitude to produce an ionospheric layer; but our calculations are not accurate enough to justify an attempt to identify any particular ionospheric layer with a particular solar emission.

It may be noticed that the calculated solar emission, which is rather high for the ionospheric requirement, would be reduced if we were to use van de Hulst's electron densities (7) for the corona, instead of Baumbach's (4).

It remains to be shown that the calculated emission of quanta does not require more energy than is conducted inwards from the corona. The various emissions considered have an average energy of about 13 volts, or $2 \times 10^{11}$ ergs, so that the emission of about $7 \times 10^{14}$ quanta requires $14,000$ ergs cm.$^{-2}$ sec.$^{-1}$, compared with $17,000$ ergs cm.$^{-2}$ sec.$^{-1}$ conducted inwards according to equation (8.5). The model is therefore consistent in this respect, and it is unnecessary to postulate a source of energy in the chromosphere as distinct from the corona.

Commonwealth Observatory,
Mount Stromlo, Canberra:
1950 February 28.

References

(22) J. H. Piddington, unpublished material.
(35) This was pointed out in correspondence with Mr D. R. Bates.