HEATING OF CORONAL PLASMA BY ANOMALOUS CURRENT DISSIPATION

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ABSTRACT

We show that the observed high temperature and inhomogeneous structure of the solar corona, as well as the long-term spatial and temporal evolution of coronal features, is economically explained by in situ heating of the coronal plasma via anomalous current dissipation. The basic geometrical structure is a loop configuration heated by nearly field-aligned currents occupying a small fraction of the total loop volume. Energy is transferred from the turbulent convective zone and photosphere, where $\beta \gg 1$, into the low-$\beta$ corona via the magnetic fields which link the two regimes. The coronal currents are generated initially by relaxation of emerging magnetic flux to the nearly force-free configuration, and subsequent quasi-steady deposition of energy is achieved via induction processes arising from the continual transfer of azimuthal flux to the corona and from direct generation of electric fields along the flux tube by subphotospheric changes in flux linkage. Laboratory experiments show that the current filamentation necessary for this model can occur if the effective resistivity and radiative losses are strongly temperature dependent, as is the case in the solar corona. As a result, local temperature increases lead, via a regenerative process, to further temperature enhancement; the relative ineffectiveness of cross-field thermal transport leads to well-localized channels of current flow.

Subject headings: hydrodynamics — plasmas — Sun: corona

I. INTRODUCTION

The advent of high-resolution soft X-ray imaging has in recent years been responsible for a radical readjustment in our understanding of the nature of the inner corona. The early view of the corona as amorphous and generally unstructured, with streamers, coronal condensations, and the like providing only occasional and incidental structuring, has been replaced by a picture of the corona as dominated by a vast hierarchy of structures: active regions consisting of close, compact loops; larger interconnecting loops between active regions, coronal holes, filament cavities, bright points, and many other features (cf. Van Speybroeck, Krieger, and Vaiana 1970; Vaiana, Krieger, and Timothy 1973). Virtually all of these observed structures can be directly linked to solar magnetic fields (Krieger, Vaiana, and Van Speybroeck 1971; Pneuman 1973; McIntosh et al. 1976); the usual interpretation is that the observed highly inhomogeneous nature of the inner corona finds its cause in the inhomogeneous nature of magnetic flux eruption through the solar photosphere.

Coronal heating theory is too extensive to be reviewed here (cf. Athay 1976). Progress in this area has traditionally been inhibited by the lack of sufficiently detailed plasma diagnostics on the observational side. This problem is compounded by the fact that the coronal energy requirement for balancing radiative losses is only a small fraction of the energy budget of the solar atmosphere (Athay 1976); therefore only a very modest and consequently difficult-to-detect contribution from several possible energy sources—such as acoustic noise generated by convective motions or magnetic fields emerging from the solar interior—could account for the coronal energy budget.

These uncertainties have been alleviated to some extent by the advent of spatially resolved diagnostic information, such as that provided by imaging X-ray telescopes (cf. Vaiana et al. 1977). The latter in particular have permitted the determination of, for example, the temporal and spatial distribution of the coronal plasma energy density, temperature, and emission measure along the line of sight (Landini et al. 1975; Davis et al. 1975; Kahler 1976). Consequently there now exist considerably more stringent constraints upon heating models; it is no longer obvious how coronal structures characterized by magnetic field strengths, energy densities, and scale sizes varying over many orders of magnitudes are to be maintained at coronal temperatures.

We intend to show that there exist heating mechanisms which connect the observed radiative properties of the inner corona in a simple way to the underlying
solar magnetic field; these mechanisms all involve the generation and consequent dissipation of coronal currents. We argue that the spatially and temporally inhomogeneous nature of the erupting solar magnetic field is an essential element of coronal heating; unlike heating theories conceived in the context of the "homogeneous" corona, this class of current heating models incorporates the observed stochastic coronal structuring at the onset, and does not view it as a complication of an otherwise straightforward model.

The following discussion is separated into two parts. The first summarizes some observational data concerning coronal activity and discusses the implications for coronal heating; the second outlines the proposed heating models, focusing specifically upon questions relating to (1) the generation of coronal currents, (2) the geometry of these induced currents, and (3) the anomalous dissipation of the currents. We have taken particular care to distinguish the conjectural and speculative elements from those results in which more confidence can be placed because our proposed theory differs radically from traditional coronal heating theories.

II. CORONAL CURRENT HEATING: OBSERVATIONAL SUPPORT

This section is devoted to a summary and discussion of some relevant available observations of the coronal plasma. We wish to establish the framework in which the subsequent theoretical discussion is placed, and to argue the plausibility of viewing the corona as heated by coronal current dissipation. It is not our intention to confront competing heating theories with the observations, but rather only to demonstrate the consistency of our approach. We refer the interested reader to Rosner, Tucker, and Vaiana (1978) for a detailed comparison of various heating models with observations of coronal active regions.

The X-ray corona is observed to consist of bright X-ray emitting loops with footpoints anchored in regions of opposite magnetic polarity. This applies not only to active regions but to nearly all of the stable structures visible in the inner corona, from bright points to coronal-hole boundary structures (McIntosh et al. 1976). As noted by Krieger et al. (1970), the brightest X-ray emission in active regions, corresponding to the hottest and densest coronal plasma, comes from a small, compact set of loops bridging the polarity inversion line (the "neutral line") of the longitudinal component of the magnetic field. Successively larger and fainter loops connect more widely separated portions of the active region, with a corresponding decrease in the strength of the confining magnetic field. McIntosh et al. point out that the same picture holds true for larger-scale coronal structures, comprising what is often referred to as the "quiet corona," as well.

This general picture is confirmed by simple potential extrapolations of the photospheric magnetic fields, which often yield a coronal field geometry similar to that of the observed X-ray structures (Poletto et al. 1975). It is particularly noteworthy that this analysis was found to be most applicable to relatively quiescent, slowly varying features, for which the overall X-ray configuration was well reproduced by the computed fields. These calculations yield estimates for the coronal magnetic field strength in active regions, ranging from ~100 gauss in bright X-ray cores at 10° height to ~1 gauss in the outer extended loop structures at 2°-3° above the solar surface. The total magnetic flux \( |\phi_+| + |\phi_-| \) was typically \( 10^{21}-10^{22} \) Maxwell, in agreement with previous measurements.

These studies must, however, be interpreted with some caution. The existence of nonpotential magnetic fields and associated currents in the corona has been discussed by Levine and Altschuler (1974) and more recently, from an observational point of view, by Krieger, DeFeiter, and Vaiana (1976). The results relevant to the present discussion are: first, that while large observed departures of geometries from potential configurations imply the presence of large coronal currents, the converse is not generally true; second, that such large departures in geometry have in fact been observed. Estimates of current densities based on these observations and upon force-free magnetic field extrapolations must, however, be viewed as setting only lower bounds, because these calculations generally assume the torsion \( |\alpha| \) to be spatially uniform. There is therefore at present no reliable method for determining the magnitude of local coronal current densities, although it seems certain that coronal currents are present.

Figure 1 (Plate 1) illustrates the coronal structure of two typical active regions, shown for the purpose of clarifying the geometry of the plasma and the magnetic fields we will be considering. The figure shows a sequence of X-ray images taken essentially simultaneously (0.3 s between exposures) and differing by successive factors of 4 in exposure duration (i.e., 1, 4, 16, and 64 s). The larger region is one rotation old while the smaller region is 4 days old at the time of these photographs. The boxes are approximately 10' \times 12' in size, or \( 4 \times 10^5 \) km by \( 5 \times 10^5 \) km. Also shown to the same scale is a portion of a KPNO magnetogram taken on the same day, showing the longitudinal component of the underlying photospheric magnetic field. We see immediately the close relationship between the photospheric magnetic field and the X-ray emitting coronal plasma. A direct overlay comparison shows that the footpoints of each coronal loop can be traced down to oppositely directed regions of strong magnetic field.

This qualitative association of coronal radiative losses with the coronal magnetic field can be placed on a more quantitative footing. From a simple examination of this one set of exposures we can obtain an estimate of the relative electron gas pressures in the two active regions. Kahler (1976) has shown (see also Landini et al. 1975) that within the temperature range 1.5-5 \times 10^6 \text{ K} the X-ray flux throughput of a single band-band filter, such as the one used to obtain these photographs, is very nearly proportional to the square of the electron gas pressure. Using this simple technique
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and taking into account the differences in line-of-sight path length (since the corona is optically thin at X-ray wavelengths) by assuming that the regions are nearly hemispherical (Krieger et al. 1972; Parkinson 1973), we find that the coronal pressure, or energy density, in the young region is about 8 times as great as that in the older, well-developed active region. This relationship between the energy density and the size and age of an active region is characteristic of the X-ray observations. If we make the additional reasonable assumption that the coronal plasma energy density is positively correlated with the local mechanical heating rate, we are led to conclude that the heating rate is a function of the developmental phase of the active region; in particular, since the level of photospheric magnetic activity is correlated with the age of the active region, we find that the rates of energy deposition and field activity are closely connected.

This relation is further evidenced in the observed response of the coronal portion of an active region to the dispersal of the photospheric magnetic field. Figure 2 (Plate 2) illustrates this process, showing a long-lived active region which was first observed from ATM when the region was two rotations old (top row). The figure also shows the region’s third rotation (middle) and fifth rotation (bottom). We note that as the magnetic field is dispersed, the X-ray loops are seen to connect more widely separated areas and to become larger and dimmer. On the last rotation the active region is no longer identifiable as a separate entity; rather, it is now identified with the “large-scale structure” in the corona. At the same time the associated Hα plage is diffusing and weakening, become nearly indistinguishable on the last rotation; high-resolution Hα filtergrams will still, however, show structure indicative of a large-scale neutral line (McIntosh et al. 1976). Notice that a newly emerging active region on the last rotation is associated in the corona with a substantially higher energy density and in the chromosphere with a compact region of intense Hα plage.

In the following we intend to show that the evolutionary process illustrated above is economically explained by heating of the coronal plasma in situ as a response to the evolution of the magnetic field, the connection established by anomalous dissipation of coronal currents.

III. CORONAL CURRENT HEATING: THEORY

The previous discussion, which suggests an intimate connection between the magnetic fields presumed to structure coronal loops and the mechanical heating necessary to sustain these loops against radiative and conductive losses, leads directly to the problem of accounting for the generation and dissipation of the coronal currents, which we contend provide the connecting link. Thus, two separate issues must be addressed: First, what are the processes responsible for the generation of nonpotential coronal fields?

Second, what are the mechanisms by which the associated currents are consequently dissipated?

The thermalization of free energy resident in the ambient coronal magnetic field as a mechanism for coronal heating was first explored by Tucker (1973) who, following an earlier suggestion by Gold (1964), hypothesized the existence of thin coronal current sheets generated by turbulent fluid motions at the photospheric level; anomalous dissipation of these currents then provided the required heating input. The thrust of this work was to show that on the basis of simple, straightforward arguments, the heating rate due to anomalous current dissipation could be related to the rate of strain of turbulent photospheric fluid motions, and may suffice to balance radiative and conductive losses calculated for a plane-parallel coronal geometry. Further, the hypothesized connection between coronal heating and coronal magnetic field gradients allowed a natural explanation of observed correlations between the intensity of coronal X-ray emission and regions of field complexity and large photospheric field gradients (Krieger, Vaiana, and Van Speybroeck 1971). Our theoretical discussion below adopts the general point of view espoused by Tucker, but enters into the necessary details of the field emergence and current dissipation not addressed in his paper.

In order to clarify the principal issues involved, we address the following questions:

a) Do plausible mechanisms exist for the generation of coronal currents?

b) Are the derived heating rates compatible with observations, from the view of both the total energetics and temporal development?

c) What is the geometry of the current-carrying layers? How is the thermalization of current sheets within loop structures achieved?

d) What are the allowed modes of current dissipation? Are they compatible with the proposed geometries and required heating rates?

The thrust of the following discussion is to demonstrate that within the context of these issues coronal

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1 The dispersal of surface fields is an observational fact.

2 A detailed model of the current-generating process, using arguments derived from terrestrial magnetospheric physics, was proposed later by Heyvaerts (1974), who emphasized its possible role in solar flares.

3 Levine (1974a, b) has proposed an alternative coronal heating mechanism which, unlike current dissipation, entails the localized production of a small subpopulation of suprathermal ions which are suprathermal with respect to both the local plasma and the ambient corona, and which thermalize outside the acceleration region. The ion acceleration proceeds via an induced electric field generated by collapsing neutral sheets (\( \alpha \partial B/\partial t \)) whose time-dependent dynamics is not investigated. It is therefore unclear whether the heating derives its energy from the current (magnetic field) system (since current dissipation is not specified in the model, although field reconnection presupposes its presence), or from the kinetic energy resident in the fluid flow associated with the neutral sheet collapse. The particular time-dependent field geometry chosen by Levine suggests the latter, as the energy residing in the magnetic field in the vicinity of the neutral sheet is an increasing function of time during the collapse. It is therefore unclear whether this model is a “magnetic field coronal heating model” in the sense of Tucker (1973).
current dissipation is a viable coronal heating mechanism from the theoretical point of view, and consistent in its consequences with the observations.

a) Generation of Coronal Currents

As a first step we examine the physics of coronal current flow—-if extant—and propose several means by which such currents may be induced. The Ohm's law for a magnetized plasma, under the usual assumptions governing MHD fluids (Krall and Trivelpiece 1973; Cowling 1976) is

\[
\frac{1}{\eta} E' = J + \frac{\omega_{pe}}{v_{ei}} \mathbf{J} \times (\mathbf{B}/B),
\]

(3.1)

where \( \eta \) is the classical resistivity, \( E' \) the effective electric field in the fluid rest frame, \( J \) the coronal current density, \( \omega_{pe} \) the electron gyro frequency, \( v_{ei} \) the electron-ion collision frequency, and \( B \) the total coronal magnetic field. This relation is resolved into the two components

\[
\eta J_\parallel = E'_\parallel, \\
\eta' J_\perp = E'_\perp - \frac{\omega_{pe}}{v_{ei}} E'_\parallel \times (\mathbf{B}/B),
\]

(3.2)

where \( \eta' \equiv (1 + \omega_{pe}^2/v_{ei}^2)\eta \) is the effective resistivity for current flow perpendicular to the magnetic field. In the corona \( \omega_{pe}/v_{ei} \approx 2.3 \times 10^6 N_\alpha^{-1} T_0^{-1/2} B_2 \gg 1 \),

\[
\eta' \approx \left( \omega_{pe}^2/v_{ei}^2 \right) \eta.
\]

(3.3)

Thus, although current flow parallel to the magnetic field occurs in spite of collisions (\( \eta \propto v_{ei} \)), current flow in the corona perpendicular to the magnetic field occurs precisely because of collisions (e.g., \( \eta \propto v_{ei}^{-1} \)). Furthermore, the second term in equation (3.2b) then dominates, leading to the simple relation

\[
\eta J_\perp \approx -\frac{v_{ei}}{\omega_{pe}} E'_\perp \times (\mathbf{B}/B).
\]

(3.4)

The magnitude of the cross-field current flow can be estimated by balancing the resulting Lorentz force against the pressure gradients in the plasma (assuming steady flow; cf. n. 10 below), with the result

\[
J_\perp \approx 3 \times 10^7 p B_2^{-1} \Delta R_6^{-1},
\]

(3.5)

where \( \Delta p \sim p \) is the pressure drop across the current layer and \( \Delta R \) the corresponding scale length. If the total coronal magnetic field \( B \) is written as the sum of a “potential” component \( B^{(0)} \), due to current outside the corona, and a “nonpotential” component \( b \), due to coronal currents

\[
B = B^{(0)} + b,
\]

we obtain

\[
b \sim 0.13 p B_2^{-1} \text{ gauss}
\]

(4) In order to facilitate estimation of the magnitudes of quantities of interest, we adopt the convention that any subscripted variable \( x_n \) refers to the quantity \( x/10^n \); thus \( T_6 \) means \( T/10^6 \) K.

in the absence of field-aligned coronal current flow (e.g., \( J_\parallel = 0 \)); \( b \) is clearly a minor perturbation of the coronal magnetic field. This can be confirmed by calculating the corresponding effective electric field from equation (3.4):

\[
E'_\parallel \approx 10^{-6} N_\alpha \Delta R_6^{-1} \approx 3 \times 10^{-4} N_\alpha \Delta R_6^{-1} \text{ volt cm}^{-1}.
\]

We recall that the effective electric field \( E' \) is given by

\[
E' = E + \frac{1}{c} v \times B + \frac{1}{N_e} \nabla p_e
\]

(3.6)

where \( E \) is the effective electric field in the rest frame, \( v \) the flow velocity of the fluid, and \( N_e \) and \( p_e \) the electron density and pressure, respectively. The magnitude of the effective electric field can then be compared with, for example, the electric field associated with plasma drifts across field lines

\[
\frac{1}{c} |v \times B| \approx 3 \times 10^{-8} v_B B_2 \approx 9 \times 10^{-8} v_B B_2 \text{ volt cm}^{-1}.
\]

(3.7)

Clearly, \( E'_\parallel \ll c^{-1}|v \times B| \); since the last term of equation (3.6) is of order \( E'_\parallel \) as well, the Ohm's law (eq. [3.2]) reduces to the simpler relations

\[
\eta J_\parallel = E'_\parallel,
\]

(3.8)

\[
E'_\perp \approx \frac{1}{c} v \times B.
\]

(3.9)

The coronal currents are thus field-aligned, or nearly force-free, in most of the corona. This condition may be violated wherever the inequality

\[
\omega_{pe}/v_{ei} \gg 1
\]

(3.10)

fails to be satisfied; this typically can occur at magnetic neutral sheets, where \( B \) may vanish (\( \omega_{pe} \rightarrow 0 \)) while the collision frequency remains finite (cf. Vasyliunas 1973). Although these neutral sheet regions may play a role in impulsive phenomena such as flares (cf. reviews by Parker 1972; Sturrock 1972; Svestka 1976), we do not believe them to be relevant to the quiescent energy balance of the corona, and therefore will disregard them in the following development. We base our decision on two observational facts:

1. Quiescent coronal X-ray structures which are well resolved do not indicate the geometries required for neutral sheet field merging and annihilation. Quiescent active region loops generally appear to connect photospheric regions of opposite polarity (see Fig. 1). There is no evidence, based on either simple matching of the X-ray data with magnetograms or more sophisticated comparisons of X-ray images with calculated field extrapolations, for oppositely directed fields within or adjacent to quiescent loop

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structures; these comparisons instead support the view that any given loop arcade consists of loops which outline magnetic flux tubes with the same general field alignment (Poletto et al. 1975). Whether fine structure below present resolution capabilities, with geometric configurations favoring neutral sheet formation, is present is, of course, completely conjectural.

2. The spatial X-ray bright-point distribution shows no significant surface correlation with field activity. That is, X-ray bright points are found in coronal holes as well as near active regions; moreover, there is no apparent association of their physical characteristics with the large-scale ambient fields (Golub et al. 1974). These observations argue against the possibility that neutral sheets—formed as emerging flux encounters overlying magnetic fields not perfectly aligned with the emerging flux—heat the corona via merging of oppositely directed field components (Uchida and Sakurai 1977). The overlying magnetic field structure in coronal holes is clearly substantially different from that in active regions; it seems difficult to reconcile a mechanism strongly dependent upon the ambient fields with which emerging structures interact with the observed spatial homogeneity in the bright point physical characteristics.

The above considerations lead us to conclude that the appropriate mechanism for converting magnetic energy to thermal energy under quiescent conditions is the dissipation of essentially force-free currents. In order to discuss the generation of these coronal currents, we must take cognizance of the distinct evolutionary states of active regions: (1) initial rapid emergence; (2) gradual growth phase; (3) gradual decay phase. Observations indicate that field emergence occurs as an eruptive process, which may be pictured as the lifting of subphotospheric magnetic flux tubes through the photosphere, and up into the chromosphere and corona. If the time scale of this initial eruption is much shorter than the resistive diffusion time

$$\tau_R = \frac{4\pi L^2}{\eta C^2} \quad (3.11)$$

($L$ the field scale length, $\eta$ an appropriate resistivity), then the newly erupted flux tubes will carry those currents which may have been generated in response to stresses experienced below the photosphere, but which have not had sufficient time to decay during the emergence. On the other hand, flux tubes which have already emerged from the solar interior are stressed further by, for example, photospheric motions at their footpoints. These stresses induce electric field and additional coronal currents. We therefore distinguish between coronal currents which have been convected into the corona (during initial field eruption) and currents which have been induced locally in the corona (during the gradual growth and decay phase).

i) Flux-Tube Emergence

A simplified model of a subphotospheric magnetic flux tube is pictured in Figure 3a. The magnetic field is primarily longitudinal, and the azimuthal current shown is simply derived from the Maxwell relation $\nabla \times B = 4\pi J/c$. The problem of macroscopic stability is not addressed here, but has been well summarized by Parker (Parker 1976, and references cited therein); our present interest is focused simply on the consequences of allowing this model—whose essential features are suggestive of the observed structure of emerged flux tubes in the photosphere—to emerge rapidly from below the photosphere.

The current system and associated magnetic fields pictured in Figure 3a presumably arise from the action of stresses exerted by subphotospheric fluid motions. The emergence process lifts a section of this flux tube into the relatively stress-free environment of the corona; if this emerged section experiences no further stresses (except those exerted by the still-submerged section, which we assume for the present not to change in any manner), we can expect the upward-convected currents to decay and the emerged flux tube to relax into a potential configuration. The transition between the configurations of Figures 3a and 3b represents a decrease in the energy stored in the total current system. The initial heating phase of an emerging loop
structure would thus correspond to the relaxation of convected currents into force-free configurations, and their subsequent decay.\(^6\) The latter process is, in this model, accompanied by strong plasma turbulence and consequently enhanced heating rates. This possibility arises because the critical electric field \(E_c\) (cf. n. 5) is directly proportional to the plasma density, and hence decreases precipitously as the flux-tube plasma density approaches coronal values, even if the plasma temperature remains constant. The heating can, however, be expected to increase the plasma temperature, thus further depressing \(E_c\); the dissipation of the convected currents can therefore be characterized as a true instability process, in which departures from any instantaneous state of the system result in further deviations in the same direction.

ii) Gradual Growth and Decay of Active Regions

Let us now consider the dynamics of magnetic flux tubes after their initial eruption. The crucial element entering into the physics of these flux tubes is that \(\beta\) changes dramatically along their length, in the photosphere satisfying the inequality \(\beta \gg 1\), while in the corona \(\beta \ll 1\). As we have pointed out above, the implication is that coronal dynamics is driven by photospheric dynamics; more specifically, field stresses generated in the high-\(\beta\) regime (photospheric and below) may propagate to low-\(\beta\) regions (the corona) in any given flux tube. This process is schematically depicted in Figure 4. Of particular relevance to the coronal heating problem is our previous observation that the low-\(\beta\) region coincides with that region of the flux tube in which the critical electric field \(E_c\) is minimum, and hence where plasma turbulence and the associated elevated current dissipation rates are most likely to occur. Coronal heating is, therefore, in this picture, a consequence of the tendency of magnetic field lines to distribute stresses along their length, the source of the stresses being convective turbulence in the photosphere and below.

This picture unifies previous theoretical work by Tucker (1973) and Parker (1974, 1975). Tucker (see also Anzer 1968) proposed that photospheric fluid motions provide a steady source of torsion, or twist of magnetic flux tubes, and that relaxation of the twist in the corona provided the required coronal mechanical heat input. In this model, the total coronal energy loss rate is balanced at any instant by the rate at which magnetic energy is supplied to the corona by the steady twisting of flux tube footpoints (Fig. 4a).

Parker, on the other hand, has investigated the transfer of azimuthal flux from a highly twisted section of a flux rope to an adjacent section with less torsion. In his model, a flux tube may have been

\(^6\) For a low-\(\beta\) plasma, the time scale of dynamic readjustment (e.g., such that \(J \times B \rightarrow 0\)) is substantially smaller than the classical ohmic dissipation time scale (in which \(J \rightarrow 0\)). As observed immediately below, the probable presence of plasma turbulence will sharply increase the ohmic dissipation rate; the available energy may therefore be shared between fluid motions generated by Lorentz forces and the internal plasma energy, which is increased by current dissipation.

Fig. 4a.—In steady-state, the stochastic convection of plasma at the footpoints of coronal loops can result in a relative rotation between them. As the plasma in the loop interior is shielded from the external plasma, only a thin layer on the outside of the loop will be sheared; thus, the initial magnetic field \(B_0\) will acquire a significant helical pitch \(\theta_r\) only in a thin sheath surrounding the loop.

Fig. 4b.—As discussed by Parker (1974), torsional equilibrium between the coronal and photospheric segments of magnetic flux tubes demands that the coronal field lines have a substantially greater helical pitch; the resulting shear is concentrated near the edge of the flux tube, and thus yields a coronal field configuration similar to that of Fig. 4a. twisted prior to its emergence; as a consequence of the eruption, the emerged section of the flux tube may no longer be in torsional equilibrium with the submerged portion, and hence a continual transfer of torsion to the emerged section may ensue (Parker 1974; also Frankenthal 1977). Parker (1975) has raised the possibility of kink instability of the emerged flux rope, and thereby attempts to account for the vigorous activity of the newly erupted flux, such as in bright points. However, the concentration of torsion outside the flux tube interior, and hence the generation of associated currents in thin layers in the emerged flux rope, raises the possibility of high locally confined current densities which are unstable to current-driven microinstabilities. The flux tube as a whole may still be stable to macroscopic perturbations because the total torsion of the flux tube is low; the highly twisted thin shell of Parker’s model can be stabilized by the comparatively little-twisted core of the flux rope. The magnetic field energy transferred to the corona would in this case be directly thermalized, rather than converted into macroscopic fluid motions as in Parker’s model (Fig. 4b).

These two mechanisms are thus seen to account for coronal currents in precisely the same way—by transferring azimuthal field upward along flux tubes. The
source of the induced coronal currents is somewhat different: in the first case, torsion is steadily generated at the photospheric footpoints of loops by turbulent fluid motions; in the second case, the still-submerged portion of the flux tube acts as a reservoir of previously accumulated stresses.

The generation of coronal currents can also be understood in terms of a conceptually somewhat simpler scheme which avoids the complexity of the twisting processes. Consider the schematic picture of a flux loop shown in Figure 5. We assume for the moment that the closed loop pictured is threaded by other flux tubes lying beneath the photosphere. Any variation of the flux $\Phi$ that is linked with the considered loop will induce an EMF along the length of the loop, given by

$$ V = \frac{1}{c} \frac{d\Phi}{dt} \quad (3.12) $$

Such variations can be induced either by relative slippage between the subphotospheric magnetic fields or by a variation in the relative strengths of the linked fields.

The corresponding electric field has a component parallel to the magnetic flux defining the loop, and will drive an axial current. Since we expect $\beta \ll 1$ in the corona, the current will be force-free (see above and Appendix); the relatively low value for the critical electric field again allows plasma turbulence to occur. This "transformer" mechanism, in which emphasis is placed upon the current induction process rather than upon flux tube twisting mechanisms, is in fact precisely how longitudinal (field-aligned) currents are produced in tokamaks (cf. Coppi and Rem 1972), as shown in Figure 5b.

From this perspective we therefore propose the following unified coronal heating model:

1. The essential element of the energetics is the interaction between the large-\(\beta\) plasma in the turbulent convective zone and photosphere, and the small-\(\beta\) plasma of the corona. The magnetic field, which permeates both, is the agent of energy transmission and ultimate release.

2. The deposition of energy occurs via the anomalous ohmic dissipation of coronal currents, which have been either convected into the corona or locally induced. The crucial factor is the combination of low coronal plasma density and high coronal plasma temperature, which permits the criterion for onset of plasma turbulence, $E > E_c$, to be relatively easily met in the corona.

3. The initial heating stage of an emerging active region may correspond to the dissipation of non-force-free currents, as emerging flux relaxes to a force-free configuration in the corona. This process can be envisioned as the upward motion of flux tubes together with entrained plasma; this material is strongly heated as the magnetic field enters the small-\(\beta\) regime, where $E$ may be substantially larger than the critical electric field $E_c$, and hence plasma turbulence results.

4. The quasi-steady deposition of energy in active

![Fig. 5.—Coronal currents may also be induced by changes in the magnetic flux linking the field lines which define the loop; this process is schematically depicted in (a). The analogous situation prevails in the tokamak (b), a low-\(\beta\) laboratory plasma containment device (Coppi and Rem 1972). In this case the flux linkage is mechanically achieved via the iron core of an external transformer.](https://example.com/image)

regions is due to the in situ induction of coronal (force-free) currents and their consequent anomalous ohmic dissipation. These currents are induced along magnetic fields already present in the corona, which are presumably related to subphotospheric currents and hence have been termed "potential." The induction process can be understood as arising either from the continual transfer of azimuthal flux to the corona, the source being accumulated or continually generated stresses at the photospheric level of the flux tubes, or from the direct generation of electric fields along the flux tubes by subphotospheric changes in the flux linked with these flux tubes.

Although the present simplified analysis assumes the presence of only one current sheath, this is physically not realistic since it ignores the susceptibility of the current-carrying layer—as opposed to the loop as a whole—to tearing-mode instabilities (Galeev et al. 1978). The destruction of the current sheath by such instabilities represents a relaxation of field stress in the corona. Since the applied stresses at the photospheric level continue unabated, maintenance of torsional equilibrium demands a continual generation of new current sheaths in the corona. We envisage the actual

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situation as consisting of a multitude of current sheaths, all in various stages of evolution; the net effect, however, will be that the rate of stress transmission from the photosphere to the corona will be balanced by the total dissipation of this ensemble of current sheaths.

b) Energetics of Current Dissipation

A well-known aspect of the coronal heating problem is the relatively small amount of energy required to maintain the corona in steady state (cf. Athay 1976). In consequence, a large variety of proposed heating mechanisms can be shown to supply the necessary energy input, and it is therefore difficult to distinguish between these mechanisms on purely energetic grounds. It will, therefore, not come as too great a surprise to the reader that current dissipation can indeed be shown to balance the coronal energy losses. This can be demonstrated by considering the energetics of a single loop of magnetic flux protruding into the solar corona as a rough first approximation of the complexities of a detailed model.

In this spirit we examine the energy balance of the following schematic model: a loop structure characterized by a uniform internal temperature $T$, density $N$, and cross-sectional radius $R$ is assumed to lose energy primarily via radiation; this assumption sets a lower bound on the energy required to be supplied by current dissipation. If the length of the loop (as measured along the magnetic field) is $L$, then the total radiative losses are

$$U_R \approx 7 \times 10^{-27} T^{-1} N^2 \pi R^2 L \text{ ergs s}^{-1}. \quad (3.13)$$

If the heating due to coronal current dissipation is confined to a sheath of thickness $\Delta R$, then the total heat input is given by

$$U_H = (E \cdot J) \cdot 2 \pi R \Delta R \cdot L$$

$$\approx 3.6 \times 10^{16} \pi \eta (R/\Delta R)^{-1} (\Delta b)^2 L \text{ ergs s}^{-1}, \quad (3.14)$$

where $\eta$ is an appropriate resistivity, $J$ is the coronal current density, and where we have used the relations $J = c|\nabla \times B|/4\pi \approx c\Delta b/4\pi \Delta R$ and $E = \eta J$. For the moment we assume that a mechanism sufficient to distribute the energy throughout the required loop volume exists; this crucial issue is discussed below in § IIIc. If we assume energy balance ($U_R \approx U_H$), we can derive a relation between the total current sheath thickness and the total change in the non-potential component of the magnetic field across the sheath:

$$\Delta R \approx 1.6 \times 10^{16} \pi N^{-2} T R^{-1} (\Delta b)^2 \text{ cm}. \quad (3.15)$$

Because the current heating rate is inversely proportional to the current sheath thickness, and since the inclusion of additional coronal loop energy losses (such as conductive loss to the chromosphere) increases the demands placed upon the (nonradiative) energy source, equation (3.15) must be interpreted as placing an upper bound on $\Delta R$.

In order to obtain a numerical estimate for $\Delta R_{\text{max}}$, we must assume a current dissipation mechanism; if we choose to maximize the heating efficiency by requiring the currents to dissipate anomalously via ion-acoustic turbulence, we set $\eta$ equal to

$$\eta \approx 0.13 \xi \omega_{pe}^{-1} \approx 2 \times 10^{-8} N^{-1/2} \text{ s}, \quad (3.16)$$

where $\xi$ is an efficiency factor, here set equal to $10^{-2}$ (see discussion in § IIId); combining equations (3.15) and (3.16), we obtain

$$\Delta R \approx 10^3 N^{-5/2} T R^{-1} (\Delta b)^2$$

$$\approx 5 \times 10^9 \text{ cm} \quad (3.17)$$

for typical coronal parameters $N \approx 2 \times 10^9 \text{ cm}^{-3}$, $T \approx 2.5 \times 10^6 \text{ K}$, $R \approx 10^8 \text{ cm}$, and $\Delta b \approx 10$ gauss. If the classical resistivity

$$\eta_c \approx 10^{-7} T^{-9/2} \frac{\ln \Lambda}{20} \text{ s} \quad (3.18)$$

(Spitzer 1962) is used instead, we find that the corresponding current sheath thickness is substantially smaller; thus

$$\Delta R \approx 1.6 \times 10^{-2} N^{-2} T^{-1/2} R^{-1} (\Delta b)^2$$

$$\approx 0.25 \text{ cm} \quad (3.19)$$

for the same parameters as above. This length scale is of the order of the gyroradius of a thermal electron moving in a magnetic field strength of $\approx 10^2$ gauss; decreasing the current dissipation efficiency to "classical" levels thus leads to unacceptably thin current sheaths if the coronal energetic requirements are to be met. We emphasize again that equation (3.19) is merely an upper bound on $\Delta R$. The inclusion of thermal conductive losses along the field, which are known to be substantial (see discussion by Tucker 1973), will decrease $\Delta R$ further; current dissipation is therefore a viable coronal heating mechanism only in the presence of plasma turbulence.

The total energy input by current dissipation is most readily calculated by considering the rate of magnetic flux emergence and dispersion. Typically, the initial growth rate of the magnetic flux in an active region is $\Phi \approx 10^{10} \text{ Mx s}^{-1}$ (Cowling 1946). During the decay phase, when the field is diffusing, we take the decrease in magnetic flux of a sunspot as a measure of the flux change in the core of the active region; this rate is typically 3-4 times smaller than the growth rate, or $\Phi \approx 3 \times 10^{15} \text{ Mx s}^{-1}$. These flux changes give rise to induced electric fields and possibly coronal currents,
as proposed in §IIIa above. The induced EMF is given by

\[ \frac{1}{c} \frac{d\Phi}{dt} \approx 10^8 \text{ volts: growth phase} \]

\[ \approx 3 \times 10^7 \text{ volts: decay phase.} \quad (3.20) \]

If we adopt the simple toroidal loop model discussed above, whose major radius is \( R \approx 2 \times 10^9 \text{ cm} \) during the growth phase and \( R \approx 10^{10} \text{ cm} \) during the decay phase (Krieger et al. 1972), we obtain for the average electric field strength \( E_{\text{growth}} \approx 8 \times 10^{-4} \text{V cm}^{-1} \) and \( E_{\text{decay}} \approx 5 \times 10^{-4} \text{V cm}^{-1} \). Since these field strengths are substantially larger than the Dreicer field \( E_D \), the currents driven by these induced electric fields will be strongly turbulent. Again assuming ion-acoustic turbulence (see §III.d below), we can calculate the total energy deposited by current dissipation by integrating the average local deposition rate \( E \cdot J \) over the volume of the current sheath. Using equation (3.14), the total heat input is

\[ U_H < 1.5 \times 10^{26} R_0 V_7 B_2 \text{ ergs s}^{-1}, \quad (3.21) \]

where \( V_7 \) is the induced EMF in volts, and where we have assumed the maximum current sheath thickness, obtained by equating \( \Delta b \) with the local potential coronal magnetic field. Thus, during the growth phase, we obtain \( U_H(\text{growth}) < 10^9 \text{ ergs s}^{-1} \), while during the decay phase \( U_H(\text{decay}) < 1.2 \times 10^9 \text{ ergs s}^{-1} \); in addition to the voltage values previously quoted, we have used the values \( R(\text{growth}) \approx 7 \times 10^6 \text{ cm} \), \( R(\text{decay}) \approx 1.4 \times 10^9 \text{ cm} \) (see Fig. 1), \( B(\text{growth}) \approx 10^2 \text{ gauss} \), and \( B(\text{decay}) \approx 20 \text{ gauss} \) (Poletto et al. 1975).

These values obtained for the total heating rate are well in excess of the requirement imposed by coronal radiative losses, as calculated from equation (3.13):

\[ U_R \approx 2.2 \times 10^{24} N_0^2 T_6^{-1} R_8^{-2} L_{10} \text{ ergs s}^{-1}. \quad (3.22) \]

This is reassuring, since our energy balance considerations have not included thermal conductive losses, which are likely to be significant; thus, if the conductive loss at \( 10^6 \text{ K} \) is taken to be as large as \( 10^6 \text{ ergs s}^{-1} \text{ cm}^{-2} \), and the cross-sectional area at \( 10^6 \text{ K} \) is the same as throughout the rest of the coronal section of the loop, we find the total conductive loss to be

\[ U_C \approx 6 \times 10^{24} F_6 R_9^{-2} \text{ ergs s}^{-1}, \quad (3.23) \]

where \( F \) is the thermal conductive flux. We see that the maximally allowed current heating rate is two orders of magnitude greater than that necessary to balance radiative and conductive losses.

c) Current Sheath Geometry and Heat Transport

The discussion of §III.b above has suggested that coronal currents are confined within relatively small volumes if they are to heat the coronal plasma. This observation raises the question whether more precise bounds can be placed on the current sheath geometry, and poses the problem of accounting for the energy transport from the compact current layers to the ambient corona.

The first issue can be addressed by considering the constraint imposed by the maximally allowed coronal magnetic field strength change. For simplicity, consider the coronal loops to be the coronal segments of toroidal flux ropes, in which the toroidal magnetic field is due mostly to currents flowing outside the plasma under study (e.g., in the convection zone and possibly below).\(^\ast\) The poloidal (azimuthal) field is instead produced by induced currents flowing in the coronal plasma itself. This geometry has been well-studied in the laboratory and occurs in devices such as tokamaks.

If \( R_0 \) and \( R_1 \) denote the minor and major radius of the torus, the magnetic surfaces having circular cross sections, then the poloidal field is known to be stable against macroscopic instabilities if, roughly,

\[ B_{\text{poloidal}} < \frac{R_0}{R_1} \frac{1}{q_0} B_{\text{toroidal}}, \quad (3.24) \]

where \( R_1/R_0 \) is the aspect ratio (assumed to be relatively large) of the toroid and \( q_0 \) the so-called safety factor (see Appendix). In terms of our previous notation, we write instead

\[ b < \frac{R_0}{R_1} B_0 \frac{1}{q_0}. \quad (3.25) \]

The change in \( b \), the field due to coronal current flow, is therefore bounded by this relation; if we adopt the parameters \( R_0 \approx 10^9 \text{ cm} \), \( R_1 \approx 5 \times 10^9 \text{ cm} \), and \( B_0 \approx 10^3 \text{ gauss} \), we find that \( \Delta b \sim (20/q_0) \text{ gauss} \), with \( 1 < q_0 < 2.5 \) dependent on the detailed current distribution. Now consider a single isolated current sheet; using the Maxwell equation curl \( B = (4\pi/c)J \), we have the order of magnitude relation

\[ \Delta R \sim \frac{c}{4\pi} J^{-1} \Delta b, \quad (3.26) \]

where \( \Delta R \) is the current sheath thickness, and \( \Delta b \) the change in the nonpotential component of the coronal magnetic field across the current sheath. Since we require efficient heating, the electron drift speed must exceed the ion sound speed in order to maintain the turbulent plasma state (§III.d below); we therefore set \( J = N_0 e V_{ni} \sim 7.6 \times 10^{-6} NT_1^{1/2} \), and obtain

\[ \Delta R \approx 3.15 \times 10^{14} N^{-1} T^{-1/2} \Delta b \]

\[ \approx 8.7 \times 10^{-3} p^{-1} T^{1/2} \Delta b, \quad (3.27) \]

with \( p \) the plasma pressure. For typical values of the coronal plasma density and temperature (\( N \sim 2 \times 10^9 \text{ cm}^{-3}, T \sim 2.5 \times 10^6 \text{ K} \), we obtain \( \Delta R \sim 10^5 \text{ cm} \), where we have used \( \Delta b \sim 10 \text{ gauss} \). Since energy deposition occurs only within the current sheath, we

\(^\ast\) The toroidal field is thus a potential field as far as the corona is concerned.
expect the relevant plasma temperature to be somewhat higher there; hence, if $p$ is uniform, $\Delta R$ may be somewhat larger. In any case these values for $\Delta R$ are upper bounds (since $J$ has been minimized to the lowest value compatible with plasma turbulence) still consistent with loop stability, and agree fairly well with the current sheath thickness required by the energetics (§IIIb above).

Although we have argued in the previous section that current dissipation can provide sufficient energy to account for coronal heating, it remains to be shown that this energy can be distributed effectively. As pointed out above, current heating implies quite localized energy deposition; we now present an argument to show that this energy is available to heat plasma outside the current sheaths as well.

Thermal energy transport results from both thermal diffusion and convective transport. In the low-collision frequency regime applicable to the corona, the ratio of the thermal conductivities along and perpendicular to the magnetic field is given by

$$\kappa_\parallel/\kappa_\perp \approx 3.1 \times 10^{13} N_e^{-2} B_2^{-2} (20/\ln \Lambda)^2$$  \hspace{1cm} (3.28)

(Spitzer 1962); this ratio is so large that even if the possibly large dissipation between the surface areas across which conduction takes place is taken into account (recall that current sheaths are thin but extended along the magnetic field), thermal conduction parallel to the magnetic field still dominates. Enhancing cross-field conduction by invoking Bohm diffusion (cf. Tucker 1973) will not materially affect this conclusion since the required wave-particle interactions are mostly confined to the magnetic field set up; two examples are shown in Figure 6. The drift rate corresponding to this $E$ can be calculated using our previous estimates for the rate of change of magnetic flux in active regions: we obtain

$$v_\perp (\text{diffusion}) \approx 4.5 \times 10^{-5} pT_\perp^{-1} B_2^{-2} \Delta R_\perp^{-1} \text{cm s}^{-1}$$  \hspace{1cm} (3.31)

where we have used the classical resistivity $\eta_\perp \approx 5 \times 10^{-10} T_\perp^{-3/2} \ln \Lambda/20$ s (Spitzer 1962); under coronal conditions this diffusion rate is negligible. This result is not unexpected, since this diffusion process is intimately connected with cross-field thermal diffusion, which we have already shown to be ineffective.

During the course of its evolution the magnetic field strength within a coronal flux tube will be changing, and hence an electric field perpendicular to the magnetic field set up; two examples are shown in Figure 6. The drift rate corresponding to this $E$ can be calculated using our previous estimates for the rate of change of magnetic flux in active regions: we obtain

$$v_\perp (\text{diffusion}) \approx (5 \times 10^{10} - 1.5 \times 10^{10}) R_0^{-1} B_2^{-1} \text{cm s}^{-1}$$  \hspace{1cm} (3.32)

where $R_0$ is the radius of the flux tube, the larger drift speed corresponding to an emerging (growing) active region. During the growth phase of an active region, the drift is directed into the interior of the flux tube (Fig. 6a); during the decay phase the drift is reversed (Fig. 6b).

Consider the case of an inward drift. Assume that only one current sheath is present on the exterior of the flux tube and that therefore the plasma is heated upon passing through the sheath and cools radiatively afterwards. If the flux-tube cross-sectional radius is $R$, then the time of passage from the current sheath to the interior is of the order of $\tau_\pi \approx R V_{\text{disk}} \approx 6.7 \times 10^5 R_0$ s; the corresponding cooling time is given by $\tau_\pi \approx 3NkT/T^4 \approx 6 \times 10^5 N_0^{-1} T_\perp^2 s$. Thus, at coronal temperatures ($T > 2 \times 10^6$ K) $\tau_\pi$ and $\tau_\pi$ can be of the same order of magnitude; we therefore find that convective transport may suffice to disperse the thermal energy released in compact current sheets.

We note at this juncture that the above argument is based upon the assumption of only a single current sheath. We have, however, previously noted that the finite lifetime of current layers due to tearing mode instabilities, together with the requirement for torsional equilibrium, make it likely that the actual current distribution consists of an ensemble of current layers at various stages of evolution. The above requirements

\hspace{1cm} 10 Strictly speaking, the hydrostatic pressure law used here is not compatible with the Ohm's law written above since eq. (3.29) assumes nonvanishing plasma motions. For substantially subsonic flow speeds, the approximation used here is correct because the additional terms in the momentum equation are then small; thus $p_v \left( v \cdot \nabla v \right) \approx \rho v^2/\tau \approx |v|^2$ for $v \ll v_{\text{sound}}$ (cf. Shafirovich 1966).
The simple model sketched here, in which only a single current sheath is present, suggests several interesting observational consequences. It predicts that active-region loops during their early evolutionary stage have cores which are relatively cool when compared with the loop exterior. Since the drift reverses direction during the decay phase of an active region, one would not expect such cool cores in well-evolved coronal loops. Furthermore, since the current sheath envelops the loop volume and is quite thin, one would not expect current broad-band spatially resolving instruments such as the X-ray telescopes on board Skylab to resolve them as separately identifiable structures. On the other hand, spectral observations of X-ray line emission may show the excitation lines corresponding to the small high-temperature current-sheath volume. One can therefore take advantage of the fact that broad-band observations with an imaging instrument are dominated by the relatively low mean temperature of the region, whereas spectral line observations can to some extent differentiate between the various temperature regimes. Observational evidence for significant emission measure at high temperatures (>3 × 10^6 K) from spectra, together with the absence of evidence for “hot spots” in broad-band imaged data, would thus point to a corningling of hot and cool plasma difficult to account for by any coronal heating model except local current dissipation. The recently reported results of Pye et al. (1978) may correspond precisely to the model suggested here.

d) Anomalous Current Dissipation

The energy balance of loop structures will, as we have seen, depend strongly upon the details of the turbulent plasma processes leading to enhanced current dissipation rates. We therefore need to determine the sequence of events which may lead to the turbulent state, and to obtain estimates of the dissipation rates once turbulence has been established. The present discussion has focused solely upon plasma processes leading to anomalous resistivity effects. Other means of thermalizing the free energy available in the coronal magnetic fields, particularly via tearing mode instabilities, are discussed by Galeev et al. (1978).

Experimental evidence obtained in laboratory work has shown that magnetic flux surfaces in an essentially classically collisionless plasma may not be equipotentials; thus, it has been demonstrated that electric fields parallel to magnetic field lines can be maintained by virtue of microscopic plasma fluctuations which scatter current carriers, and hence provide an effective resistivity (cf. Hamberger 1975, and references cited therein). The types of instabilities relevant to our present concerns can be broadly categorized as follows:

1. \( v_d/v_e > 1 \); \( v_d/v_e < 1 \);
2. \( T_e/T_i \gg 1 \) (ion-acoustic turbulence);
2. \( v_d/n_e \gg v_d/n_i \geq 1 \);
3. \( T_i/T_e \sim 1 \) (Buneman turbulence),

where \( v_d, v_e, \) and \( v_i \) are the current carrier drift speed,
sound speed, and electron thermal speed, respectively, and \( T_e \) and \( T_i \) the electron and ion temperatures. While the present discussion is limited to these instabilities, we caution that a considerable number of alternative instabilities have been proposed to account for observed enhanced current dissipation rates; the following should, therefore be viewed as a possible scenario and not as a definitive theory.

We first note that each of the above turbulent regimes involves a set of conditions which appears at first sight to be unlikely:

a) Can \( T_e/T_i \approx 1? \) Our previous discussions have shown that the fractional volume occupied by the current sheets required for coronal heating is very small. The contribution from electron bremsstrahlung in these current sheets—assuming \( T_e/T_i \approx 1 \)—to the total coronal X-ray emission is, therefore, correspondingly small. Observations are therefore unlikely to argue against this precondition for strong ion-acoustic turbulence. As discussed in III C, the plasma can drift through the current sheet. It is thus relevant to ask whether electrons, heated on a time scale \( \tau_{\text{err}}^{-1} \) (\( \nu_{\text{eff}} \) the effective electron collision frequency), can effectively thermalize on a time scale \( \tau_{\text{ei}} \equiv (m_e/m_i) \tau_{\text{err}}^{-1} \) (\( \nu_{\text{ei}} \) the electron-ion collision frequency) with ions while residing within the current sheet. Because \( \nu_{\text{ei}}/\nu_{\text{eff}} \approx 10^{-3} N_e^{1/2} T_e^{-3/2} \) for ion-acoustic turbulence \( \ll 1 \) under coronal conditions, the rapid electron temperature increase is followed by a corresponding increase in the ion temperature only if ions remain in the sheath a sufficiently long time, that is, if the ion residence time \( \Delta R/\nu_D \) (cf. eqs. [3.27] and [3.32]) is large when compared with the electron-ion thermalization time. However, \( \tau_{\text{ei}}^{-1} \Delta R/\nu_D \ll 1 \); hence \( T_e/T_i \approx 1 \) is viable on theoretical grounds as well. It remains to be shown, however, how this temperature disparity can be obtained; we discuss this below.

b) Can \( v_d/v_e > 1? \) While we have previously shown that the large electric field strengths necessary to produce this inequality can be produced in the corona, it is quite unclear whether these fields can be maintained. This issue has been explored experimentally by Hamberger, Jancarik, and Sharp (1969), and has been discussed by Kaplan and Tsytovitch (1973); the latter state that the Buneman regime (under which \( v_d \) exceeds \( v_e \)) is likely to be only an initial intermittent phase, leading to ion-acoustic turbulence. Although we shall in the following also assume that steady anomalous current dissipation occurs via ion-acoustic turbulence, we must note that Hamberger et al. (1971) have reported observations of fluctuation spectra consistent with Buneman turbulence well after \( v_d < v_e \), suggesting that a nonthermal component of the electron velocity distribution is responsible.

Given that steady anomalous current dissipation occurs via ion-acoustic turbulence, and since the coronal plasma is quite likely to be isothermal (\( T_e \sim T_i \)) before the onset of turbulence, we require a mechanism which will raise \( T_e \) on a time scale short when compared with the electron-ion collision frequency. Two methods have been proposed to achieve this goal. The first (Coppi and Friedland 1971; Coppi 1975) has the character of a thermal instability; one writes the energy balance equation for electrons

\[
\frac{3}{2} \frac{\partial T_e}{\partial t} = \eta_{\text{classical}} c_P^2/N - 3 v_{ei} \frac{m_e}{m_i} (T_e - T_i) - Q(T_e),
\]

(3.33)

where \( \eta_{\text{classical}} c_P^2 \) is the ohmic heating term,

\[
3 v_{ei} (m_e/m_i)(T_e - T_i)
\]

the collisional loss to ions, and \( Q(T_e) \) all other losses (conduction, radiation) from the electron gas.

In equilibrium the left-hand side of this equation vanishes; if departures from equilibrium are such that the ohmic heating term dominates the other terms on the right-hand side, an explosive increase in \( T_e \) results. Since \( \nu_{ei} \propto T_e^{-3/2} \), whereas \( \nu_{\text{ion-ion}} \sim T_e^1 \), the condition of electron runaway can be reached; the "runaway" beam of electrons and the remaining electron distribution interact to produce a two-stream instability. The resulting spectrum of electrostatic fluctuation acts to prevent further electron runaway by increasing the effective collision frequency. The resulting "anomalous" resistivity (Hamerger and Jancarik 1972, and references cited therein)

\[
\eta_l \approx 2 \left( \frac{m_e}{m_i} \right)^{1/3} \omega_{pe}^{-1} \approx 2 \times 10^{-11} N_0^{-1/2} \text{ cm s}^{-1}
\]

(3.34)

acts to heat primarily the electrons; as now \( T_e \gg T_i \), the electron electrostatic turbulence decays to ion-acoustic turbulence (Kaplan and Tsytovitch 1973). In order to maintain the latter turbulence, however, the current drift speed \( v_d \) must satisfy

\[
v_d > v_e \approx (2.8 kT_i/m_i)^{1/2} \sim 1.65 \times 10^5 T_{6\text{(eclonns)}} \text{ cm s}^{-1}.
\]

(3.35)

An alternative sequence to the above requires that the drift speed exceed the electron thermal speed \( v_e \) (cf. Kaplan and Tsytovitch 1973)

\[
v_d > v_e \approx 5.5 \times 10^9 T_{6\text{(eclonns)}}^{1/2}.
\]

As above, the result is strong excitation of the two-stream instability. If the amplitude of the drift speed is maintained, the turbulence will be dominated by electron electrostatic modes, with the temperature-independent resistivity \( \eta_l \) above determining the heating rate; if, on the other hand, \( v_d \) drops so that \( v_e < v_d < v_e \), ion-acoustic modes are excited and will instead determine the dissipation rate. In either case the development of ion-acoustic turbulence leads to an anomalous resistivity\(^{11}\) given by

\[
\eta_l \approx 0.125 \frac{v_d}{v_e} \omega_{pe}^{-1};
\]

(3.36)

\(^{11}\) Quite generally we may write \( \eta = 4 \pi n_{\text{eclonns}} \omega_{pe}^2 \), where \( \nu_{\text{err}} \) is an effective electron collision frequency (with ions or turbulent field fluctuations). Hamberger and Jancarik (1970) and Kaplan and Tsytovitch (1973) quote \( \nu_{\text{err}} \) (ion-sound) \( \approx (\omega_{pe}^2/k v_d^2/v_e^2) \), while Sagdeev (1967) and Gary and Paul (1971) use \( \nu_{\text{err}} \) (ion-sound) \( \approx (\omega_{pe}^2/k v_d^2 c_T/c_F) \); we use the former value here.
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or, in terms of the temperature-independent Buneman resistivity,

\[ \eta_{(\text{ion-acoustic})} \approx 0.77 \xi \eta_{(\text{Buneman})}, \]  

(3.37)

where \( \xi(= v_d/v_e) \) lies between \( 10^{-2} \) and 1.

IV. SUMMARY

In the above discussion we have shown that the observed high temperature, inhomogeneous structure, and detailed spatial and temporal evolution of non-flaring coronal features can be economically explained by in situ heating of the coronal plasma via anomalous current dissipation. The basic geometrical structure is a loop configuration having nearly field-aligned currents which occupy a small fraction of the total loop volume, for reasons described in § IIIa. Energy is transferred from the turbulent convective zone and photosphere, where \( \beta \gg 1 \), into the low-\( \beta \) corona via the magnetic fields which link the two regimes.

The coronal currents are generated via several mechanisms, operating with varying degrees of importance throughout the evolution of a coronal feature. The initial heating stage of emerging magnetic flux consists of a relaxation to the nearly force-free field configuration which must eventually prevail (see Appendix). As the flux tube and entrained plasma rise, they enter the low-\( \beta \) regime where the induced electric field \( E \) may be substantially larger than the Dreicer field \( E_c \), thus leading to plasma turbulence and associated heating.

Following the initial rapid emergence, a quasi-steady deposition of energy is achieved via currents induced along the magnetic field lines already present in the corona. The induction process arises either from the continual transfer of azimuthal magnetic flux to the corona, the source being either previously accumulated or continually generated stresses at the photospheric level, or from the direct generation of electric fields along the flux tube by subphotospheric changes in flux linkage.

The current sheaths are quite thin in comparison to typical observed loop diameters; the ratio of sheath thickness to loop diameter is less than \( 10^{-4} \) in most cases. The thickness has been estimated in two ways: First, an upper bound may be obtained by examination of the energy balance in a loop. The energy losses must be at least as great as those due to radiation alone (eq. [3.13]) and the heating efficiency is maximized by assuming ion-acoustic turbulence (eq. [3.16]). For typical active-region coronal parameters, this approach yields an upper limit for the sheath thickness of \( 5 \times 10^6 \) cm. Use of the classical resistivity yields a thickness of less than 0.25 cm, which is comparable to the electron gyroradius and thus unacceptably thin.

The second way we have estimated the sheath thickness is by requiring the loop to be stable against macroscopic instabilities. A simple inverse inequality exists between the allowed ratio of poloidal and toroidal magnetic field and the aspect ratio of the loop (eq. [3.24]). Again using accepted values for the coronal parameters \( T, N_a \), and \( B \), we obtain an upper limit of \( \sim 10^4 \) cm for the sheath thickness (eq. [3.27]). These calculations are based upon the presumption of anomalous current dissipation. If the thermalization of free energy resident in coronal magnetic fields proceeds via other mechanisms, such as tearing mode instabilities (Galeev et al. 1978), then analogous calculations can be performed to obtain the appropriate current sheath thickness.

The initial formation of these current sheaths has been observed in many laboratory experiments and has received extensive theoretical study. Briefly, current filamentation can occur if the effective resistivity and radiative losses are strongly temperature-dependent. These requirements are met in the corona, with the result that an increase in the local temperature leads, via a regenerative process, to a further increase in temperature. The relative ineffectiveness of cross-field thermal transport leads to well-localized channels of current flow.

The total heat input by ohmic dissipation is found to be a function of the evolutionary history of the region. In particular, a numerical estimate of the heating rate was obtained in terms of the loop size, magnetic field strength, and instantaneous rate of flux emergence. We find that, for a typical active region, the heat input during the growth phase is an order of magnitude larger than in the decay phase, in rough agreement with at least the observed radiative loss rates presented in § II.

Observations indicate that coronal loops emit over volumes large compared with those occupied by the currents discussed in this paper. Although thermal conductivity is insufficient to remove heat from the current sheath (eq. [3.28]), the heated electrons themselves drift out of the sheath at a substantial rate (eq. [3.32]). This drift, moreover, appears to change sign during the development of an emerging flux region so that active region loops during their early stages of evolution may have cool cores relative to the outer layers of the loops. Furthermore, the likely presence of an ensemble of current sheaths, as opposed to the single layer envisaged in the simplified analysis, reduces the severity of the heat redistribution requirements.

The coronal heating model presented here thus accounts for the observed correlation between coronal plasma energy density enhancements and changes in the photospheric magnetic field. The model also explains the basic physical properties of the X-ray corona, including its geometric structure and the energy balance and evolution of these loop structures, and suggests that observations of emission measures at high (> \( 4 \times 10^6 \) K) plasma temperatures in active regions may be due to unresolved regions of current dissipation which are required in this model.

On a more speculative level we note that the connection between coronal heating and photospheric turbulent activity established here suggests that coronal loops connecting interiors of large sunspots—where convective activity is less vigorous than in the surrounding photosphere—should be cooler than loops.
with one or both footpoints in areas of increased (e.g., normal) convection. In addition, the accurate determination of transverse magnetic fields in the photosphere is of substantial interest as our model predicts a detailed correlation between the transfer of "non-potential" flux into the corona and coronal heating. High-resolution (<0.5) diagnostics of coronal loop structures will be required to accurately determine the radial (transverse) temperature and density structure of loops; thus our model predicts that energy deposition within loops is far from homogeneous and is instead concentrated in sheaths concentric with the loop axis.

Finally, we observe that the photospheric motions ultimately driving the heating process are chaotic. The resulting fluctuations in the rate of stress to which individual loops are exposed therefore lead, in consequence of our model, to large time-dependent variations in the loop heating rate, and hence in the emissivity of coronal loops. While available data for X-ray loops are suggestive of this process (cf. Rosner, Tucker, and Vaiana 1978), detailed correlated observations of photospheric motions and fields, and coronal EUV and X-ray emission from coronal loops, would be highly desirable.

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APPENDIX

AN AXISYMMETRIC TOROIDAL LOOP

A model that can be treated analytically and should illuminate the considerations we have made in the previous sections on loop configurations is that of a toroidal, magnetically confined plasma column. The toroid is assumed to be axisymmetric; we refer to cylindrical coordinates \( \hat{R}, \phi, z \) in which \( \hat{R} \) indicates the distance from the symmetry axis, \( \phi \) the toroidal (azimuthal) angle, and \( z \) the distance from the equatorial plane. We introduce the variable \( \psi \), such that

\[
B \cdot \nabla \psi = 0 ,
\]

which labels a family of nested toroidal magnetic surfaces. Given the symmetry of the problem, we must have

\[
\psi = \psi(R, z) .
\]

We also assume, for simplicity, that no external force is applied in the toroidal \( (e_\theta) \) directions. Then the relevant magnetic configuration can be represented in terms of the variable \( \psi \) and \( R \) as

\[
B = \frac{R_0}{R} \left[ B_{\theta 0 } \alpha (\nabla \psi \times e_\theta) + B_\phi (\psi) e_\theta \right] .
\]

Here \( R = R_0 \) is the distance of the (toroidal) magnetic axis, \( B_\theta \) is the toroidal magnetic field value on this axis, and \( B_{\theta 0} \) is a measure of this poloidal magnetic field at the edge of the plasma column that is assumed to have a characteristic minor radius \( a \) (see Fig. 5b).

The corresponding expression for the current density, as derived from \( J = (c/4\pi) \nabla \times B \), is

\[
J = \frac{c}{4\pi} \frac{R_0}{R} \left\{ B_{\theta 0} \frac{d\phi}{d\psi} \frac{\nabla \psi \times e_\theta}{e_\psi} - B_{\phi 0} \alpha \left[ \frac{d^2 \psi}{dz^2} + R \frac{\partial}{\partial R} \left( \frac{1}{R \frac{\partial}{\partial R}} \right) \right] e_\theta \right\} .
\]

Therefore,

\[
\frac{1}{c} J \times B = -\frac{1}{4\pi} \left( \nabla \psi \right) \left( \frac{R_0}{R} \right)^2 \left\{ B_{\phi 0} \alpha \left[ \frac{d^2 \psi}{dz^2} + R \frac{\partial}{\partial R} \left( \frac{1}{R \frac{\partial}{\partial R}} \right) \right] + \frac{1}{2} B_{\theta 0} \frac{d\phi}{d\psi} \right\} .
\]

Notice that the poloidal current \( J_p \) can be written as

\[
J_p = \alpha (\psi) B_\theta ,
\]

where

\[
\alpha (\psi) = \frac{c}{4\pi \alpha B_{\theta 0} \frac{d\phi}{d\psi}} .
\]

and in the case of a force-free field configuration where

\[
J = \alpha (\psi) B ,
\]
equation (A5) leads to the following equation for $\psi$:

$$\frac{d^2\psi}{dz^2} + R \frac{\partial}{\partial R} \left( \frac{1}{R} \frac{\partial \psi}{\partial R} \right) = -\frac{1}{2a^2} \left( \frac{B_{t0}}{B_{p0}} \right)^2 \frac{d\psi}{d\psi} \left( \frac{B_{t0}}{B_{p0}} \right)^2 \frac{d\phi}{d\phi}.$$

(A9)

If we consider the effects of an isotropic plasma pressure, we have

$$\frac{1}{c} \mathbf{J} \times \mathbf{B} = \nabla p,$$

implying that

$$p = p(\psi).$$

(A10)

Thus, recalling equation (A5), we have

$$\frac{\partial^2\psi}{\partial z^2} + R \frac{\partial}{\partial R} \left( \frac{1}{R} \frac{\partial \psi}{\partial R} \right) = -\frac{1}{2a^2} \left[ \left( \frac{B_{t0}}{B_{p0}} \right)^2 \frac{d\phi}{d\phi} + \left( \frac{R}{R_0} \right)^2 \frac{8\pi}{\beta_p} \right].$$

(A12)

We define $\bar{p}$ as the average pressure over the plasma column, the parameter

$$\beta_p = \frac{8\pi\bar{p}}{B_{p0}^2},$$

(A13)

and the function $\bar{p}(\psi) = p(\psi)$ such that the equilibrium equation (A12) reduces to

$$\frac{\partial^2\psi}{\partial z^2} + R \frac{\partial}{\partial R} \left( \frac{1}{R} \frac{\partial \psi}{\partial R} \right) = -\frac{1}{2a^2} \left[ \left( \frac{B_{t0}}{B_{p0}} \right)^2 \frac{d\phi}{d\phi} + \beta_p \left( \frac{R}{R_0} \right)^2 \frac{d\bar{p}}{d\phi} \right].$$

(A14)

The limit in which the force-free configuration is recovered, so that equation (A14) reduces to equation (A9), corresponds to

$$\beta_p \ll 1.$$  

(A15)

In dealing with confinement configurations for plasmas of thermonuclear interest it is customary to assume reasonable profiles for $\bar{p}(\psi)$ and $J_\phi$ or $\psi(\phi)$ and to proceed to evaluate $\psi = \psi(R, z)$ by equation (A14). This surface procedure gives then a first approximate description of the isobaric surfaces that can be reasonably expected in a given confinement experiment.

Finally, we consider for simplicity a toroidal configuration in which the toroidal magnetic field is mostly due to currents flowing outside the plasma, in a set of conducting coils. The poloidal field is instead produced by an induced current within the plasma. In this manner we schematically model a coronal loop, whose defining longitudinal (≡ toroidal) magnetic field is “potential” and hence due to noncoronal, subphotospheric currents. If we consider configurations with a relatively large aspect ratio $R_0/a$ and magnetic surfaces having circular cross section, the poloidal field is limited by consideration of macroscopic instabilities. This limitation is expressed by the Kruskal-Shafranov criterion

$$q_0 > 1,$$

(A16)

where $q_0$ is the so-called safety factor; it is defined by the relation

$$q(r) = \frac{r}{R_0} \frac{B_i}{B_p}.$$  

(A17)

with $r$ the distance from the toroidal axis $R = R_0$, and $q_0 = q(a)$. Note that $q(r)$ is a measure of the helical pitch of field lines lying in the flux surface of cross-sectional radius $r$. Equation (A16) thus translates into an upper limit upon the poloidal magnetic field strength given by

$$B_{p0} \leq \frac{a}{R_0} B_{t0}.$$  

The poloidal field is therefore constrained by the requirements for both stability and (in the simplest model) force-free geometry to lie between the limits

$$\left(8\pi\bar{p}\right)^{1/2} \leq B_{p0} \leq \frac{a}{R_0} B_{t0}.$$  

(A18)
Note that for coronal loops, $\bar{p}$ refers to the average pressure difference between the loop interior and the ambient plasma. Typical values for the relevant parameters ($\bar{p} \sim 0.2$ dyn cm$^{-2}$, $a/R_0 \sim \frac{1}{10}$, $B_{p0} \sim 10^2 - 3 \times 10^4$ gauss) lead to the inequality
\[ 2 \text{ gauss} \ll B_{p0} < 10 - 10^8 \text{ gauss}. \]  
(A19)

Equations (A18) and (A19) are noteworthy in that they show the plasma $\beta$ defined by the poloidal field component—not the toroidal component—to significantly constrain the possibilities for a force-free geometry.

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Fig. 1.—Four X-ray photographs of a pair of active regions; the larger is one rotation old, and the smaller is 4 days old at the time of these photographs (1973 June 20, 0900 UT). X-ray exposure durations increase by successive factors of 4 from top left to bottom right, showing successively larger and fainter loop structures surrounding the bright cores. Note also that the energy density in the younger region is more than an order of magnitude greater than that of the older region (see text).

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Fig. 2.—Three rotations of a long-lived active region, illustrating the coronal response to diffusion of the photospheric magnetic fields. Overlay comparisons show that the X-ray loops are anchored in regions of opposite polarity longitudinal field and correspond to the chromospheric brightenings seen in lines such as Hα; the X-ray intensity is closely correlated with the intensity of chromospheric emission. Photographs taken 1973 June 2 (top), June 29 (middle), and August 27 (bottom).

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