MAGNETIC NEUTRAL SHEETS IN EVOLVING FIELDS. I. GENERAL THEORY

E. N. PARKER

Department of Physics and Department of Astronomy and Astrophysics, University of Chicago

Received 1982 January 15; accepted 1982 July 14

ABSTRACT

This paper treats the problem of hydrostatic equilibrium of a large-scale magnetic field embedded in a fluid with infinite electrical conductivity. It was shown some years ago that a necessary condition for static equilibrium is the invariance of the small-scale pattern in the field along the large-scale direction. A varying topological pattern implies that there is no fluid pressure distribution for which the field is everywhere static. Magnetic neutral sheets form, and dynamical reconnection of the field occurs. In the present paper it is shown that the invariance is also a sufficient condition for the existence of a fluid pressure distribution producing static equilibrium. However, even in the simplest cases the requirements on the fluid pressure are extreme and a priori unlikely. We conclude that almost all twisted flux tubes packed together produce dynamical nonequilibrium and dissipation of their twisting. This is the basic effect behind the long-standing conjecture that the shuffling of the footpoints of the bipolar magnetic fields in the Sun is responsible for heating the active corona.

Subject heading: hydromagnetics

I. INTRODUCTION

The idea that the active solar corona is heated largely by dissipation of the convolutions of the coronal magnetic fields has been around for nearly 20 years (see Gold 1964). The idea is simply that the footpoints of the flux tubes of the coronal fields are randomly shuffled about on a small scale by the granules and supergranules at the visible surface, twisting and winding neighboring flux tubes around each other in various ways, causing continual dissipation of magnetic energy in the corona. Tucker (1973), Levine (1974), and Sturrock and Uchida (1981) have given further consideration to the idea, estimating the energy input from the rate of winding and estimating the thinness of the current sheets necessary to produce the necessary dissipation.

This point of view has been strengthened by the recent radio, EUV, and X-ray observations of the corona, whose detailed analysis by a number of authors has led Rosner, Tucker, and Vaiana (1978) to the important assertion that there are few, if any, alternatives to the idea. Only the dissipation in situ of the supporting and confining magnetic field seems to fit the requirements.

With this starting point, then, the problem is to establish the basis for the formation of magnetic neutral sheets to such a degree as to produce the necessary dissipation of magnetic field. The formation of neutral sheets in potential magnetic fields was the earliest aspect of the problem to receive serious attention (see review by Priest 1981, pp. 155–163). The basic fact is that an initial potential field containing one or more X-type neutral points, embedded in a tenuous, infinitely conducting fluid, develops magnetic neutral sheets in place of neutral points when strains are externally imposed upon the field (Sweet 1958; Syrovatsky 1971, 1981; Priest and Raadu 1975; Low 1982). The field, which is momentarily distorted from a potential (force-free) form, quickly achieves a state wherein the field is describable by a potential everywhere except at the thin nonequilibrium magnetic neutral sheets. Syrovatsky (1978; Bobrova and Syrovatsky 1979) has extended the arguments to X-type neutral points in general force-free fields, considering in detail the first-order equations for the changes introduced when the force-free field is subject to small externally imposed deformations. Syrovatsky argues that most externally imposed deformations produce neutral sheets.

Parker (1972, 1979, pp. 359–391; Yu 1973) approached the problem from another direction, considering a magnetic field (in an infinitely conducting fluid), whose flux tubes all extend between anchor points on distant boundaries (say $z = \pm L$). His formal solutions show that there is no static equilibrium throughout the region if there is any variation in the $z$-direction of the pattern of winding or wrapping of flux tubes about their neighbors. That is to say, there is no equilibrium no matter what fluid pressure $p(x, y, \pm L)$ may be applied at the boundaries $z = \pm L$. The nonequilibrium takes the form of magnetic neutral sheets, subject to neutral point reconnection. The effect has been called topological dissipation because of its dependence on variation of the

---

1 This work was supported in part by the National Aeronautics and Space Administration under NASA grant NGL-14-001-001.
topology of the winding pattern along the field. Parker (1972, 1979, p. 517) suggested that topological dissipation is the principal dissipation in any magnetic field carried in a turbulent fluid of high electrical conductivity. Glencross (1975, 1980) proposed that topological dissipation is, in fact, the mechanism responsible for dissipating the wrapping of the magnetic flux tubes in the solar corona, providing the coronal heating.

Parker (1981a, b) showed an additional effect, viz., that any flux tube displaced into a path (between anchor points) that is misaligned with respect to the neighboring flux tubes forms magnetic neutral sheets and is quickly dissipated. Figure 1 shows a sketch of a typical bipolar field, with a misaligned tube of flux indicated by the dashed lines. He suggested that the dissipation of displaced flux tubes provides additional heating of the corona.

The problem before us now is to establish an overall picture of the major theoretical possibilities for producing magnetic neutral sheets, i.e., nonequilibrium current sheets, whose thickness progressively declines toward zero with the passage of time. The dynamical theory of the magnetic neutral sheet has had a long history of development starting with Sweet (1958, 1969; Parker 1957, 1963, 1979, pp. 392–436; Petschek 1964; Vasyliunas 1975; Syrovatsky 1981; Priest 1981, pp. 1–46, 139–215) and need not be repeated here.

The present paper begins with a demonstration that topological invariance of the winding pattern along the mean field direction is a sufficient condition for the existence of static equilibrium if a suitable fluid pressure distribution is applied at the boundary (previous calculations [Parker 1972, 1979, pp. 359–390] showed invariance to be a necessary condition). In the process of establishing sufficiency, however, we discover that the pressure distribution for static equilibrium has special features that make it a priori unlikely to occur in nature, forcing us to the conclusion that dynamical nonequilibrium and the formation of magnetic neutral sheets is unavoidable among most close-packed, twisted flux tubes. This conclusion is the final step in understanding the magnetic dissipation in the highly conducting solar corona. Together with the nonequilibrium of varying winding patterns and of displaced flux tubes, it establishes that all components of the transverse fields produced by the rotation and shuffling of the footpoints of a large-scale field are subject to neutral point reconnection and rapid dissipation. It is the final step in declaring the general nonequilibrium and rapid dissipation of all magnetic fields extending outward from a dense convecting body into a tenuous, but highly conducting, atmosphere. Essentially all of the work done by the fluid on the footpoints goes directly into heating the atmosphere above, no matter how high the electrical conductivity of the atmosphere.

II. STATIC EQUILIBRIUM

To develop a picture for the sufficient conditions for static equilibrium of the invariant magnetic field configuration consider the simple context of a magnetic field that is initially uniform, extending in the $z$-direction from $z = 0$ to some large distance $z = +L$. The space $0 < z < L$ is filled with an infinitely conducting ideal fluid and extends to $x, y = \pm \infty$ so that there are no lateral boundary effects. The stresses are carried by the tension in the lines of force, extending in the $z$-direction from $z = 0$ to $L$, and by the fluid pressure, applied at $z = 0, L$. 

© American Astronomical Society • Provided by the NASA Astrophysics Data System
It is obvious that the lines of force of the initial uniform field can be wrapped about each other in arbitrary patterns by suitable motion of the fluid. Figure 2a is a sketch of some of the individual close-packed flux tubes of which the initial uniform field is composed. Figure 2b is a sketch of three of the flux tubes after being braided by the fluid motions. The essential point here is that in the braiding process each tube is wrapped first one way and then the other about its companions, so that the winding pattern is not invariant to continuous displacement along the direction of the mean field. Figure 2c is a sketch of some of the tubes that are twisted but otherwise undistorted, so that the winding pattern is invariant along the mean field direction. In the situation illustrated in Figure 2 it is readily shown (Parker 1972, 1979, p. 363) that a necessary condition for the existence of magnetostatic equilibrium is the invariance of the winding pattern along the field (Fig. 2c). If the pattern lacks invariance, as in Figure 2b, then there is no equilibrium no matter what fluid pressure \( p(x, y) \) is applied at \( z = 0, L \). Somewhere within the field with varying topology there develops one or more shear planes (a neutral sheet when projected onto the plane lying across the mean field direction) across which the scale of variation of field declines asymptotically to zero with the passage of time. The decline of \( l \) is limited only by diffusion.

Suppose for the moment that the fluid throughout the entire region is under our control. We choose to hold motionless the fluid at \( z = +L \), so that the field remains fixed there. At \( z = 0 \) we introduce the fluid motion \([v_x(x, y, t), v_y(x, y, t)]\) for the purpose of winding the lines of force about each other throughout \( 0 < z < L \). Fluid is pumped in or out of the region at \( z = 0 \) so that the fluid pressure \( p(x, y) \) may be manipulated to establish magnetohydrostatic equilibrium whenever that is theoretically possible.

So starting with the uniform field (Fig. 2a), imagine that the fluid at \( z = 0 \) is set in motion in some fixed, closed cellular pattern, introducing fields throughout \( 0 < z < L \) that are invariant with respect to \( z \), i.e., \( \partial B_z/\partial z = 0 \). The resulting field consists of cylindrical cells containing twisted fields of various forms, as sketched in Figure 2c. The general invariant cylindrical form may include such complications as two or more twisted ropes of flux contained inside a cylindrical shell of twisted (i.e., helical) field, extending with uniform cross section from \( z = 0 \) to \( z = L \). The necessary topological invariance for equilibrium is fulfilled. It is well known (Dungey 1958) that the equilibrium in this case \( (\partial B_z/\partial z = 0) \) is described in terms of the vector potential by

\[
B_x = + \partial A/\partial y, \quad B_y = - \partial A/\partial x, \quad B_z = B_z(A),
\]  

and \( A \) is described by the quasi-linear field equation

\[
\nabla^2 A + 4\pi P'(A) = 0,
\]

where \( P \) is the total pressure

\[
P(A) = p(A) + B_z^2(A)/8\pi.
\]

The question now is whether there is a solution to equation (2) for every topology of the invariant winding pattern, given some suitable choice of the function \( P(A) \).
The answer is affirmative. In the paragraphs below we construct a scheme by which any field topology can be brought into hydrostatic equilibrium by suitable adjustment of \( p(x, y) \).

Once that point is established, the next question is whether the appropriate \( p(x, y) \) for equilibrium is available in nature, and what happens if it is not.

III. SUFFICIENT CONDITIONS FOR STATIC EQUILIBRIUM

To explore the general principles of static equilibrium, consider the solution to equations (2) and (3) for the simple case that \( B_z = B_0 \) and \( p = k^2 A^2 / 8\pi \) so that equation (2) reduces to

\[
\left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + k^2 \right) A = 0.
\]

(4)

The basic solution that is bounded over the \( xy \)-plane is

\[
A = \exp i \left( k_x x + k_y y \right)
\]

for the real wave numbers satisfying \( k_x^2 + k_y^2 = k^2 \). The special case

\[
A = C \sin k_x x \sin k_y y
\]

(5)

is sketched in Figure 3. The solution represents close packed, twisted flux tubes, each tube squashed in its own rectangular cell by the pressure of its four neighbors, the basic cell being \( 0 < x < \pi / k_x, \ 0 < y < \pi / k_y \). Adjacent cells have opposite twist, as indicated by the arrows in Figure 3.

Consider, then, what happens if adjacent cells have same rotation or circulation, resulting in \( A \) of the form sketched in Figure 4. There is a fundamental difference between the potentials sketched in Figures 3 and 4. In Figure 3 the surface consists of alternate sinkholes and peaks. The level routes across the region (i.e., the cell boundaries, \( A = 0 \)) are the straight lines \( k_x x = \pm n\pi, \ k_y y = \pm m\pi \) across the saddle points at the common vertices of the cells. The magnetic field vanishes nowhere along the level routes except at the saddle points (which are \( X \)-type neutral points). On the other hand, in Figure 4 the same level routes (\( A = 0 \)) represent the center line of level valleys running between rows of peaks. The field vanishes everywhere along the level routes. The level routes have the one common feature in both cases that the fluid pressure is constant along them, because they lie along a common "line of force," \( A = 0 \).

The impossibility of equilibrium of the potential distribution sketched in Figure 4 is immediately evident. Each rectangular cell \((a)\) represents a twisted flux tube that has been squashed into a rectangular shape to fit into the cell, while \((b)\) each cell has a uniform total pressure \( P(0) \) exerted on its boundaries. But a twisted tube surrounded by uniform pressure has a circular, rather than rectangular, form, so \((a)\) and \((b)\) are mutually exclusive.

It follows, then, that if the potential \( A \) sketched in Figure 4 is produced through suitable motion of the fluid, the resulting twisted flux tubes are not in static equilibrium. The system is in a state of dynamical nonequilibrium for the simple reason that the cells press together more strongly at the middle of each side than at the vertices. This extra pressure at the midpoints is transmitted across the neutral sheet to the next cell by the fluid (since \( B_z \) is uniform and \( B_x = B_y = 0 \) on the boundary). That is to say, the fluid pressure along the boundary is, in fact, higher in the middle of each cell boundary than at the vertices, causing the fluid to flow out from between the cells and to accumulate in the low pressure region around the vertices. This is the usual neutral point reconnection situation. The fluid is squeezed from between the cells, drawing the opposite

---

Fig. 3.—A sketch of the surface \( A(x, y) \) for the cellular pattern described by eq. (9), in which adjacent cells have opposite circulation. The level contours represent the lines of force of the transverse components \( (B_x, B_y) \) of the magnetic field.
fields (either $B_x$ or $B_y$) on either side closer together. The transverse scale (thickness) $l$ of the neutral sheet [initially $O(k_x^{-1}, k_y^{-1})$] goes asymptotically to zero with the passage of time. Reconnection of the lines of force occurs at the X-type neutral point in the middle of each cell boundary when $l$ becomes sufficiently small. Figure 5 is a sketch of the lines of force $A(x, y, t)$ constant at some time $t$ after the onset of reconnection. The figure illustrates how the field topology changes, with the creation of closed lines which circulate around the vertices of the initial cells with the opposite sense to the initial circulation around the center of each cell.

The first question that confronts us, then, is whether the application of some suitable fluid pressure distribution $p(x, y)$ can establish equilibrium and prevent the reconnection. The answer is affirmative. For suppose that we pump the fluid out of the interior of each of the cells of Figure 4 and into the region of the vertices $(k_x, x = \pm n\pi, k_y, y = \pm m\pi)$. The initial X-type neutral point at each vertex is inflated into a column of field-free fluid (extending from $z = 0$ to $L$) at the same time that the twisted flux tube, which originally filled the entire cell, is compressed about its central axis by the diminishing internal fluid pressure and rising external fluid pressure. It is obvious that this procedure takes the configuration of Figure 4 into the configuration of Figure 6, consisting of separated twisted flux tubes, no longer close-packed as in Figure 4. Each has a circular cross section and is surrounded by field-free fluid. There is always an equilibrium of a circular flux tube. Lüst and
Schlüter (1954a, b) showed that the equilibrium equation is satisfied by

\[ p + B_r^2/8\pi = f + \frac{1}{2} \varpi \frac{df}{d\varpi}, \]

\[ B_\varpi^2/8\pi = -\frac{1}{2} \varpi \frac{df}{d\varpi}, \]

where \( f(\varpi) \) is an arbitrary function of the distance \( \varpi \) from the axis of the tube, subject only to the restriction

\[ -2/\varpi \leq d \ln f / d\varpi \leq 0, \]

so that the fluid pressure \( p \) is positive and the fields \( B_r \) and \( B_\varpi \) are real. Any arbitrary (bounded) radial distribution of \( B_\varpi(\varpi) \) is possible.

It follows that any invariant topology, i.e., any cylindrical field form \( A(x, y) \), can be put into static equilibrium by separation into isolated columns and hollow shells, etc., by shrinking troublesome flux tubes and/or inflating shells, etc., until neighboring magnetic structures are surrounded entirely by field-free fluid. There can then be no neutral sheets because neighboring structures do not touch. Indeed, once separated, any degree of shrinking may be carried out, preserving a static equilibrium state.\(^2\) Hence, topological invariance of a field pattern is both a necessary and a sufficient condition that some suitable fluid pressure \( p(x, y) \) can produce static equilibrium.

IV. THE GENERAL OCCURRENCE OF NONEQUILIBRIUM

If it is possible to achieve equilibrium by application of a suitable fluid pressure \( p(x, y) \) that shrinks all the flux tubes into isolation so that each is surrounded by field-free fluid, the next question is what happens if this special fluid pressure \( p(x, y) \) is not applied. After all, there is no theoretical reason to expect the fluid pressure in most natural settings to have this extraordinary property.\(^3\) Indeed in a tenuous plasma such as the solar corona the gas pressure is so small compared to the magnetic pressure that there is no possibility that \( p(x, y) \) can maintain separate flux tubes. Failing such judicious evacuation, adjacent tubes press against each other, squashing each other out of round. In that case nonequilibrium and reconnection can be avoided only if tubes with the same sense of twisting have no common border. The solutions to equation (2) all have this remarkable property, that no tubes with the same twisting have a common border. The example (5), sketched in Figure 3, provides an illustration. Figure 4 sketches the opposite extreme, wherein all tubes have the same twisting, so that all borders are shared by tubes with the same twist. In that case all borders are subject to nonequilibrium and neutral point reconnection, as described above and sketched in Figure 5. The obvious question is whether equal numbers of tubes with opposite twists generally coexist in nature in static equilibrium, along the lines sketched in Figure 3.

The answer appears to be negative, because in nature, first of all, we do not expect the tubes to have the same cross-sectional area and strength, so they generally cannot be packed together to avoid all contact between tubes with the same sense of twist. In any case, even if all the tubes had the same size and strength and were packed into the rectangular array of tubes with alternate signs described by equation (5), they would tend to slip out of the rectangular arrangement into the lower energy state represented by hexagonal close packing. Then every tube is squashed against at least one neighbor with the same sense of twisting. A detailed analysis of the

\(^2\)The equilibria may be unstable in various ways, but that is a separate question.

\(^3\)We should be aware, of course, that it is just exactly this peculiar situation that is observed in the photosphere of the Sun, where the magnetic field is broken into separate intense flux tubes as a consequence of the evacuation of their interiors (Parker 1979, pp. 207–214).
form of close packing of tubes is presented elsewhere (Parker 1982b). The basic point is that while there are infinitely many formal solutions to the equation (2) for static equilibrium, the close packing of parallel twisted flux tubes in nature fails to achieve the perfect balanced symmetry required to avoid broad contact between tubes with the same twist. It is simply not possible in the real world to achieve the neatly alternating array of twisted tubes (Fig. 3) necessary for static equilibrium. Nonequilibrium, and hence reconnection and destruction of the transverse field within each flux tube, is the general rule. That is to say, it appears to be automatic that, when a number of slender twisted flux tubes are packed together, their torsional energy is soon dissipated by neutral point reconnection, no matter how large the electrical conductivity of the fluid in which they are embedded. The phenomenon is, in fact, nothing more than the familiar coalescence of magnetic islands (individual twisted flux tubes) so familiar in the plasma physics laboratory (Dickman, Morse, and Nielson 1969; Finn and Kaw 1977; Biskamp and Welter 1980). The dissipation goes on until a final state is reached in which the initial tubes have coalesced into two large tubes of opposite twist, which may then exist side by side in static equilibrium.

V. IMPLICATIONS

It was pointed out in the introduction that the small-scale convection at the surface of the Sun causes the magnetic fields of bipolar regions to be composed of large numbers of close-packed slender flux tubes twisted and wrapped about each other as a consequence of the stochastic rotations and displacements, respectively, of their footpoints. It was previously shown (Parker 1972, 1979, pp. 359–391, 1981a, b) that the irregular wrapping causes dynamical nonequilibrium and neutral point reconnection, rapidly reducing the tubes to a close-packed array of parallel twisted tubes. The present paper points out that the close-packed parallel twisted tubes are generally subject to dynamical nonequilibrium and reconnection as well, reducing the array toward two large tubes with only slight residual twists of opposite sign. Taken together these two principles imply that all transverse field components are subject to rapid reconnection and dissipation, so that the work done by the stochastic rotation and shuffling of the footpoints of the flux tubes proceeds directly into heat in the atmosphere permeated by the flux tubes. The energy proceeds directly into heat no matter how large the electrical conductivity of the gas in which the tubes are embedded. This general nonequilibrium, and neutral point reconnection, is the basic dynamical effect on which rests the general idea, initiated by Gold nearly 20 years ago, that the active corona is heated by the random shuffling of the footpoints of the magnetic field.

In the next paper (Parker 1982a) we take up the shuffling of footpoints and the coronal heating from this basic dynamical nonequilibrium.

REFERENCES


E. N. PARKER: Laboratory for Astrophysics and Space Research, University of Chicago, Enrico Fermi Institute, 933 East 56th Street, Chicago, IL 60637

© American Astronomical Society • Provided by the NASA Astrophysics Data System