ACCELERATION OF THERMAL PARTICLES IN COLLAPSING MAGNETIC REGIONS

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ABSTRACT

The acceleration of thermal particles in the vicinity of a magnetic neutral sheet is discussed. This process is considered as the archetypal low-intensity process at a newly formed neutral sheet. The collapse of the field toward the neutral sheet produces acceleration by secularly increasing the velocity component perpendicular to the sheet. The average energy increase for a thermal distribution of particles is calculated both with and without the inclusion of Coulomb losses by accelerated particles to particles in the accelerating region. Emphasis is placed on the conditions necessary for acceleration in the presence of Coulomb losses. The applicability of this approach to regions which collapse very fast is discussed.

Subject headings: hydromagnetics — solar activity

I. INTRODUCTION

The role of magnetic neutral sheets is becoming increasingly important in astrophysical problems. This is especially true in the Sun, where observations show definite associations of neutral sheets and many interesting phenomena. Neutral sheets have been the subject of a great deal of study (e.g., Sweet 1958; Parker 1963; Petschek 1964; Yeh and Axford 1970), most of it relating to the steady-state collapse of the field and the related particle acceleration as a mechanism for solar flares.

In this paper we deal with the other end of the energy spectrum and with the point in time when a neutral sheet has just formed between the oppositely directed bundles of magnetic field. If neutral sheets are significant factors in solar activity then this will be a primary process with which that activity will begin. What is envisioned is the coming together of oppositely directed bundles of magnetic field and their initial collapse. As the field collapses, particles bouncing off the approaching field lines are accelerated. No steady state is assumed, nor is any assumption made about the behavior of the region subsequent to the initial acceleration of thermal particles. This is then the prototype initial process of solar activity, the acceleration of thermal particles resulting in an intensity enhancement that may be anything from unobservable to a bright knot and which may develop into a more intense region or simply break apart. An example of an application of this process to the problem of the heating of the corona is given in the following paper (Levine 1974).

The acceleration of particles in a region of collapsing magnetic field was studied by Severny and Shabanskii (1961) and for relativistic particles by Burke and Layzer (1969), whose work is considerably more detailed. Here we will deal with nonrelativistic thermal particles, caught between oppositely directed bundles of magnetic field which are collapsing toward a surface of zero magnetic field (magnetic null surface). We will try to specify the least amount in the most general way about the field, leaving aside detailed questions about its stability and structure. Our aim will be to find the energy gain for particles in this type of configuration, building in complexity from the simplest equations (§ II) to the inclusion of Coulomb losses (§ III) and very short collapse times of the field (§ IV).

II. ACCELERATION WITHOUT LOSSES

The field geometry is idealized as a magnetic null plane [(x, z)-plane] with $B = B(y, t)\hat{z}$ a function of y and t only. The equations of motion in the (x, y)-plane (i.e., across the magnetic null surface) then become

$$d^2x/dt^2 = \frac{q}{mc} (v_xB + \int_0^y \partial B/\partial t dy') - \frac{q}{mc} \frac{d}{dt} \int_0^y Bdy';$$

$$d^2y/dt^2 = -(q/mc)Bdx/dt,$$

where the particle has a charge q and velocity $v_x$ (v_y is not affected by the magnetic field). An immediate consequence of equation (1) is

$$dx/dt = (dx/dt)_0 + \frac{(q/mc)}{B(y', t)} \int_0^{y'} B(y', t)dy',$$

so that each time the particle crosses the null plane (y = 0) the x-velocity returns to its initial value. Thus it is through the y-component, the direction in which the field is collapsing, that any secular acceleration occurs. From this we infer that if the energy of the particle is changed each time it crosses the null plane, the change in velocity must be all in $v_x$ and hence the

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1 Electrostatic acceleration is not considered here. It is one of the equivalent ways to describe the acceleration in Sweet's mechanism as analyzed by Parker (1963), an acceleration process which, in general, takes much longer than what is envisioned here.
angle $\theta$ in the $(x, y)$-plane at which the particle crosses the null surface $(\tan \theta = v_y/v_x)$ changes from one crossing to the next. If the energy is increased at each crossing, the angle continually approaches $90^\circ$.

We assume that the general form of the magnetic field is

$$B(y, t) = \pm |K(t)x|^r |B_0|,$$

where all the time development is in the function $K(t)$ and $r$ is a free parameter. The sign of the right side is to be taken the same as the sign of $y$.

The rate of energy gain by the particle from the magnetic field is

$$\frac{dE}{dt} = qE \cdot v = (q/c) \frac{dv}{dt} \int_0^r \frac{\partial B/\partial t}{\partial y} dy,$$

$$= \frac{qB_0}{c} \frac{r}{r + 1} t_b^{-1} K'(t)|y|^r+1 dx/\partial t,$$

where $\partial$ is the induced electric field and

$$t_b = K(dK/dt)$$

and $t_b > 0$ since the field is collapsing onto the null plane. Using the equations of motion the average rate of change in energy between one crossing of the null surface and the next (one cycle) is then

$$\langle \frac{dE}{dt} \rangle = \frac{r}{r + 1} \frac{m}{t_b} t_{cycle}^{-1} \int_0^{t_{cycle}} \langle dy/\partial t \rangle^2 dt,$$

$$= \frac{r}{r + 1} \frac{m}{t_b} \langle \langle dy/\partial t \rangle^2 \rangle.$$

(Angle brackets: $\langle \rangle$ will denote averages of quantities over one such cycle.) This shows that the energy gain in one cycle is always positive.

The equations of motion can be integrated to find the orbit of a particle [projected onto the $(x, y)$-plane], given the initial velocity and angle in the $(x, y)$-plane of crossing of the null surface (measured from the +-$x$-axis). A particle which initially crosses the null plane at an angle less than $90^\circ$ has a sine-wave-like orbit; the $x$-motion is always towards +$x$. Particles with initial angles greater than $90^\circ$ have loop-like orbits with several extrema in $x$-motions. There are two classes of orbits in this category. Particles with initial angles only slightly greater than $90^\circ$ still have net $x$-motion in the +-$x$-direction, i.e., in the direction of the electric field. As the angle is increased a critical angle (depending on $r$) is reached where the orbit is a figure eight and there is no net $x$-motion from one orbit to the next. At initial crossing angles greater than this critical angle the net $x$-motion is in the −-$x$-direction, i.e., opposed to the electric field.

From this information we can deduce several properties of particles caught in such a collapsing magnetic configuration. Because of the positive energy gain in a cycle the angle at which the particle crosses the null plane moves toward $90^\circ$ in successive crossings. Thus a particle whose initial crossing angle is less than $90^\circ$ will move continually in the +$x$-direction, through slightly less distance in each succeeding cycle. A particle whose initial angle is greater than $90^\circ$ but less than the critical angle will have extrema in its cyclic $x$-motion, but its net $x$-motion will always be toward +$x$, the distance increasing slightly at each cycle as $90^\circ$ is approached. Particles with initial angles greater than the critical angle also have extrema in their cyclic $x$-motion. Their net $x$-motion begins in the −-$x$-direction but as their crossing angle moves toward $90^\circ$ the distance covered in a cycle becomes less and less until at the critical angle they make no net motion in the $x$-direction. As the angle moves nearer to $90^\circ$ for these particles, they begin to have net +$x$-motion, moving back along the $x$-axis in the direction from which they came. Their motion continues in this direction for the rest of their encounter with the magnetic field region. In addition to these motions in the $(x, y)$-plane there is a constant-velocity motion in the $z$-direction.

To calculate the energy gain, let

$$\xi = kx; \quad \eta = ky; \quad t = (mc/qB_0)T$$

and assume $k = k/t_b$ and $\xi_0 = 0$. At $\tau = 0$, $\xi_0^2 + \eta_0^2 = v_0^2$ and define

$$\sigma = \xi_0/\eta_0 \cot \theta,$$

$$\gamma = v_0/\eta_0 = \csc \theta,$$

$$\beta = \eta/\eta_0 = \eta \gamma/v_0.$$

The angle $\theta$ here is the angle in the $(x, y)$-plane at which the particle crosses the null surface and the dot denotes differentiation with respect to $t$. Now if $t \equiv d/dT$ the equations of motion lead to

$$\beta \beta' - 2\beta/\tau_b = \mp |\gamma|^r \mp \frac{r^2}{r^2 + 1} \frac{\gamma^2}{v_0^2 (r + 1)} |\eta|2r+1 - \frac{\gamma}{v_0 \tau_b} \eta.$$  

In the case that we want to consider here, that of collapse time long compared to orbit time, $k = 0$. This requires that we set $\tau_b = \infty$ in this equation. Since $dE/dt \propto \langle \xi^2 \rangle/r(r + 1)$, where the average is over one cycle, define

$$F_r(\theta) = \langle \xi^2 \rangle/v_0 = \sin^2 \theta \langle \beta^2 \rangle.$$

The function $F_r(\theta)$ and other important quantities defined below can be put into a recognizable analytic form. This is done in detail in Burke and Layzer (1969). The result is

$$F_r(\theta) = \sin^2 \theta I(\theta)/I(\theta),$$

and $I$ and $J$ involve Gauss’s hypergeometric functions $F(a, b; \gamma; z)$:

$$I_r(\theta) = S(p, \frac{3}{2}) \sec \frac{1}{2} \theta F(-\frac{1}{2}, 0; 0; p; \sin^2 \frac{1}{2} \theta),$$

$$J_r(\theta) = S(p, \frac{1}{2}) \cos \frac{1}{2} \theta F(0, 0; \frac{1}{2}; p; \sin^2 \frac{1}{2} \theta).$$
where \( p = r/(r + 1) \) and

\[
S(x, y) = \int_0^1 t^{x-1}(1 - t)^{y-1} dt,
\]

\[
= \Gamma(x) \Gamma(y)/\Gamma(x + y), \quad \text{for } x, y > 0. \quad (17)
\]

These hypergeometric functions converge for all \( \theta \) and all \( r(\frac{1}{2} \leq p \leq 1) \). For \( r = \infty \)

\[
F_\omega(\theta) = \sin^2 \theta. \quad (18)
\]

The average energy gain of a particle in one cycle is now seen to be

\[
\langle dE/dt \rangle = [mr(r + 1)](v^2/I_0)F_\omega(\theta) \cos^2 \phi, \quad (19)
\]

where \( \phi \) is the angle the total velocity makes with the \((x, y)\)-plane, \( \cos \phi = (v_x^2 + v_y^2)^{1/2}/v_0 \). This is the average energy gain in one cycle as a function of the initial velocity for that cycle. It can be interpreted as a formula for the increase in the initial crossing velocity \( v_0 \) or as a formula for the increase in the average velocity of a particle over several cycles, as long as the changes in \( \theta \) and \( \phi \) are taken into account.

The average \( x \)-motion of a particle depends on its initial angle \( \theta \) of crossing of the null plane. We will define this in terms of

\[
\frac{\delta x}{\delta t} = \frac{qB_0}{mcK_0} \delta \xi \equiv \frac{qB_0}{mcK_0} v_0 G_r(\theta) \equiv v_0 G_r(\theta) \cos \phi \quad (20)
\]

where \( \delta \) refers to the change in a quantity over one cycle. An analysis similar to the above for \( F_r \) leads to

\[
G_r(\theta) = \cos \theta + (1 - \cos \theta)K_r(\theta)/J_r(\theta), \quad (21)
\]

where

\[
K_r(\theta) = B(1 + p, \frac{1}{2}) \cos \frac{1}{2} \theta F_r(\frac{1}{2}, \frac{3}{2}; p; \sin^2 \frac{1}{2} \theta).
\]

For \( r = \infty \),

\[
G_\omega(\theta) = \cos \theta. \quad (22)
\]

The functions \( F_r \) and \( G_r \) are shown in figure 1 for \( r = 1 \). The energy per unit time gained by a particle in a cycle is zero for particles whose initial angle \( \theta \) is 0° or 180°, i.e., particles traveling in the neutral plane. Particles near 0° still have negligible energy gain whereas particles close to 180° gain much more energy per unit time in one cycle because \( F_r \) grows rapidly as \( \theta \) departs from 180°. The maximum power gain occurs for particles at an angle near 108° (for \( r = 1 \)). This maximum shifts to the left (lower angles) as \( r \) increases until for \( r = \infty \) the maximum is at 90° and particles near 0° and 180° behave the same. The critical angle where \( G_r = 0 \) (132° for \( r = 1 \)) corresponds to the figure-eight mentioned above. This critical angle also moves to lower values as \( r \) increases, until for \( r = \infty \) the orbit with no net \( x \)-motion is at an initial crossing angle of 90°.

Since the change in \( v_x \) over a cycle is 0, the change in \( v_x \) over the cycle determines the variation in the crossing angle and the energy gain

\[
\delta E = \frac{1}{2} m \delta (v^2) = m v_x \delta v_x
\]

\[
= m v_x \cos^2 \theta \delta \theta = m v_x \tan \delta \theta. \quad (24)
\]

If the change in angle in one cycle is small this implies

\[
E/E_0 = \cos^2 \theta_0/\cos^2 \theta, \quad (25)
\]

where \( E_0 \) is the initial energy and \( E \) the energy at the end of the cycle. In an encounter with the magnetic configuration consisting of several cycles the energy \( E \) at the end of the first cycle becomes the \( E_0 \) for the second, and so on. So when several ratios are multiplied together all except the first and last cycle terms cancel out and equation (25) is valid for the energy and crossing angle at the end of the entire encounter. The problem of finding the energy gain therefore reduces to that of finding the change in the angle in the \((x, y)\)-plane at which the particle crosses the null surface. Of course for particles whose initial angle is 90° the initial and final angles are equal and the energy gain has to be determined another way. This will be done below. Also, since \((\cos 90°)^{-2} \) is infinite we expect that the crossing angle of 90° is never actually reached, but only approached. This will be clear below.

From equations (19) and (24) the change in the angle \( \theta \) per unit time in one cycle is

\[
\frac{\delta \theta}{\delta t} = \frac{r}{t_\theta(r + 1)} \cot \theta F_r(\theta) \cos^2 \phi. \quad (26)
\]

Equation (20) then gives

\[
\frac{\delta x}{\delta \theta} = v_0 t_\theta \frac{r + 1 \tan \theta}{r \cos \phi} G_r(\theta)/F_r(\theta). \quad (27)
\]

The angle in the \((x, y)\)-plane at which the particle crosses the null plane moves closer to 90° in successive cycles. There are two main reasons why this approach...
will actually stop short of 90°. The first is that the (x, z)-motion of the particle may take it out of the region. (Another reason, not major, is that the y-motion of the particle could conceivably take it out of the region.) The second reason, and the only alternative if the particle is to have a noninfinite gain in energy, is that the particle stays within the confines of the region long enough that the region eventually weakens or reaches a steady state and ceases to accelerate the particle in this manner.

The total x-motion of a particle over several cycles is obtained by integrating equation (27). The total change of x over an encounter (several cycles) is thus (changes over an encounter will be denoted by Δ)

\[ \Delta x/v_0 f_b = \frac{r + 1}{R} \int_{\theta_0}^{\theta} \cos \phi G_\phi(\theta)/F_\phi(\theta) d\theta. \]  

(28)

The time for an encounter is obtained in a similar way from equation (26) and the change in z is determined by this time, the constant \(\epsilon_z\), and \(\phi\). The change in \(\phi\) is related to the change in \(\theta\) by \(\cos \phi d\phi = -\tan \theta d\theta\).

If we take the limiting time beyond which the region ceases to accelerate as \(t_b\) and if the particle escapes from the region in the critical distances \(x_c\) and \(z_c\) in the x- and z-directions, respectively, the final angle \(\theta_f\) at which the particle crosses the null surface is that for which

\[ \epsilon_z = x_c/v_0 f_b = \frac{r + 1}{R} \int_{\theta_0}^{\theta_f} \cos \phi G_\phi(\theta)/F_\phi(\theta) d\theta, \]  

(29a)

\[ \epsilon_z = z_c/v_0 f_b = \frac{r + 1}{R} \int_{\theta_0}^{\theta_f} \tan \phi \tan \theta/\cos \theta F_\phi(\theta) d\theta, \]  

(29b)

or

\[ \frac{t}{t_b} = 1 = \frac{r + 1}{R} \int_{\theta_0}^{\theta_f} \tan \theta/\cos \phi \sin^2 \phi F_\phi(\theta) d\theta. \]  

(30)

first holds. Equation (25) can then be used to find the energy gain over the encounter. Since each integral approaches infinity from both sides at 90°, a particle will never actually get to a 90° crossing angle, no matter how close to 90° it starts. Particles starting at exactly 90° will be dealt with below.

In the nonrelativistic case we are considering the change in \(\theta\) over an encounter need not be small and equations (29) and (30) cannot be approximated, as is done in the relativistic case by Burke and Layzer.

The energy increase, calculated as outlined above, is shown in figure 2 for \(r = 1\), \(v_x = 0\) (cos \(\phi = 1\)), and for two values of \(\epsilon_z\). The quantity plotted is

\[ \alpha \equiv \Delta E/E = (E_f - E_0)/E_0. \]  

(31)

For \(\epsilon_z > 0.5\) the time cutoff predominates. Actually, values of \(\epsilon_z\) greater than 1 do not produce different acceleration curves (unless the time cutoff is changed)

\[ *\] Naturally, this is somewhat arbitrary. Larger times used for this purpose will produce larger values of the energy gain in the succeeding analysis wherever a time cutoff is used.

because \(\epsilon_z = 1\) means that \(x_c = v_0 t_b\) so that in a time \(t_b\) only particles with initial crossing angles 0° and 180° will have (barely) escaped. All other particles will still be in the region and subject to the time cutoff. The rapid drop in the lower \(\epsilon_z\) curves on the 180° side of their peaks is caused by the difference between particles that can reach the x-motion corresponding to \(\epsilon_z\) in the -x-direction and those that cannot and have to go through a much larger range of angles to escape in the +x-direction. The addition of a z-velocity will be dealt with below, under Coulomb losses.

For \(r = \infty\) and \(\cos \phi = 1\) the acceleration can be calculated analytically. The result is

\[ \tan (\theta_f/2) = \exp (\epsilon_z) \tan (\theta_0/2), \]  

(32)

\[ \tan \theta_f = e \tan \theta_0, \]  

(33)

the first representing the cutoff in \(\epsilon_z\), the second the cutoff in time. The final crossing angle is the \(\theta_f\) in equations (32) and (33) such that \(|\theta_f - \theta_0|\) is smallest, i.e., the first final crossing angle \(\theta_f\) reached in the above equations as the crossing angle nears 90°. The resulting energy gain is shown in figure 2. The \(r = \infty\) curves are symmetric about 90°, the critical angle (\(G = 0\)) and the angle of the maximum of \(F\) having both moved to 90°.

The values of these curves at initial crossing angles of exactly 90° are not determined by the above equations. The relation between change in the angle \(\theta\) and

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change in energy breaks down since the angle \( \theta \) does not change if it starts at 90°. But the particle still has a net \( x \)-motion and so a cutoff can be made in \( \epsilon_x \) and \( t_c \), as for other angles. With \( \phi = 0 \) and \( \theta \) understood to be 90°, and \( F \) and \( G \) thus constant, the result is

\[
\alpha = \exp \left( \frac{2r \epsilon_x}{r + 1} F_r/G_r \right) - 1, \quad \epsilon_x \geq G_r,
\]

\[
= \exp \left( \frac{2r}{r + 1} F_r \right) - 1, \quad \epsilon_x \geq G_r. \tag{34}
\]

In the preceding \( r = 1 \) case, \( F_1 = G_1 = 0.66 \); \( G_r = 0.45 \), so for \( \epsilon_x \geq 0.45 \), \( \alpha = 0.94 \). For \( \epsilon_x = 0.1 \), \( \alpha = 0.155 \); for \( \epsilon_x = 0.05 \), \( \alpha = 0.075 \). These points fall exactly on the calculated curves of figure 2. Indeed, one reason for calculating \( \alpha \) at an initial crossing angle of 90° is to provide just such a check on the earlier calculations of \( \alpha \) for general \( G \). The other major reason is to give an idea of the scale of the acceleration curve at \( \phi = 0 \) for any \( r \) without having to calculate all of the hypergeometric functions. To facilitate this calculation the formula (Abramowitz and Stegun 1965)

\[
F(a, 1 - a; b; \frac{1}{2}) = \frac{2^{1 - b} \pi^{1/2} \Gamma(b)}{\Gamma(\frac{1}{2} a + \frac{1}{2} b) \Gamma(\frac{1}{2} + b - \frac{1}{2} a)} \tag{35}
\]

is useful since \( \sin^2 45° = \frac{1}{2} \).

For \( r = \infty \), \( G_r(90°) = 0 \). This means that the time cutoff is appropriate for all \( \epsilon_x \). The reason is that for \( r = \infty \), particles with initial crossing angles of 90° have no net \( x \)-motion at all: the critical angle is also 90°. Thus for all \( \epsilon_x \), \( \alpha = 6.39 \) at \( \theta = 90° \) (see fig. 2).

With \( \theta = 90° \) we can obtain an analytic solution for \( \phi \neq 0 \). Using the relation between \( d\phi t \) and \( \phi \) we find that the \( \epsilon_x \) cutoff, \( \epsilon_x \), cutoff, and time cutoff solutions become

\[
\tan \frac{\phi_t}{2} = \tan \frac{\phi_0}{2} \exp \left[ -\frac{r}{r + 1} F_r \epsilon_x \right], \tag{36}
\]

\[
\tan \left( \frac{\phi_t + \pi}{4} \right) = \tan \left( \frac{\phi_0 + \pi}{4} \right) \exp \left[ -\frac{r}{r + 1} F_r \epsilon_x \right], \tag{37}
\]

\[
\tan \phi_t = \tan \phi_0 \exp \left[ -\frac{r}{r + 1} F_r \right]. \tag{38}
\]

The value of \( \phi_t \) for which \( |\phi_t - \phi_0| \) is the smallest is then the appropriate final value of \( \phi \). The energy gain, determined by an analysis similar to equations (24) and (25), is

\[
E/E_0 = \sin^2 \phi_0/\sin^2 \phi_t. \tag{39}
\]

III. COULOMB LOSSES

To deal properly with particles near thermal velocity we must take their Coulomb losses into account. The energy gains from acceleration by the magnetic field, which depend largely on \( r_p \), can be offset by Coulomb losses, i.e., energy transfer in collisions with other particles. These losses depend mainly on the density of particles in the accelerating region. The modification of the acceleration coefficient in figures 2–4, along with the elaboration of the conditions for acceleration to overcome losses, are the goals of this section.

The Coulomb energy loss of a test particle of mass \( M \), charge \( Ze \), and velocity \( v \) in a medium consisting of particles of mass \( m \), charge \( Ze \), and number density \( n \) is given by Butler and Buckingham (1962) as

\[
dE/dt = -8\pi^2(Ze^2)^2 \ln \Lambda(n/mv)H(v/v_t), \tag{40}
\]

where \( \ln \Lambda \) is the Coulomb logarithm (approximately equal to 20 in the corona), and \( v_t \) is the most probable velocity in the thermal distribution of the particles in the medium:

\[
v_t = (2k_B T/m)^{1/2}, \tag{41}
\]

where \( T \) is the temperature and \( k_B \) is the Boltzmann constant. The function \( H \) is given by

\[
H(x) = \int_0^x \exp (-x^2)dx - (1 + m/M)x \exp (-x^2). \tag{42}
\]

This function goes through zero at \( x^2 = 3m/2M \), which corresponds to \( Mv^2/2 = 3k_B T/2 \), or the rms velocity of the thermal distribution. Particles with velocities below this have a net acceleration \( (H < 0) \), and those above a net deceleration \( (H > 0) \).

For losses in a two-component plasma, as found in the corona, equation (40) is used twice, producing two terms, each with parameters of density, temperature, etc., corresponding to one of the plasma components. If, as in the corona, the components are protons and electrons, and their densities and temperatures are approximately equal we have

\[
dE/dt = -8\pi^2 e^4 \ln \Lambda(n/e)(H_pe/v_pe)m_p + H_e(v/v_pe)m_e, \tag{43}
\]

where \( m_p \) is the proton mass, \( m_e \) the electron mass, and \( H_p \) refers to equation (42) with \( m = m_p \), \( M = \) mass of test particle, and \( H_e \) is equation (42) with \( m = m_e \), \( M = \) mass of test particle. The electron and proton thermal velocities \( v_e \) and \( v_p \) are similarly gotten from equation (41) and are related by \( v_e = v_p (m_p/m_e)^{1/2} \).

Equation (43) is the form of the loss equation we will use for the corona and, unless otherwise stated, we will consider the test particles as protons \((M = m_p) \) since, as shown below, they are more likely to be accelerated.

It will be useful to have the loss equation as a velocity equation:

\[
dv/dt = -0.9 \times 10^{-11}(n/e^3)(H_pe(n/v_p^2)\sqrt{w}), \tag{44}
\]

\[
= -4.24 \times 10^{-24}(n/T^{3/2})(H_pe(n/v_p^2)\sqrt{w}),
\]

where \( w = v/v_p \). Clearly the functions \( H_pe, H_p, H_pe, \) and...
$H_{ep}/w$ are of prime importance. Since we are mostly concerned with protons near thermal velocity, we will introduce the arbitrary normalization $h_{ep}(w) = H_{ep}(w)/0.152 \times 10^{24}$, where $0.152 \times 10^{24}$ is the approximate value of $H_{ep}(w)/w^2$ at its peak near $w = 1.5$. Thus $h_{ep}/w^2$ will have values on the order of 1 for the values of $w$ that will interest us. The velocity equation then becomes

$$\frac{dw}{dt} = -0.644(\eta/T^{3/2})[h_{ep}(w)/w^2], \quad (45)$$

where

$$t_d = 1.553T^{3/2}/n. \quad (46)$$

Without doing any detailed calculations we can now discuss the criteria for acceleration. The gains of the particle through magnetic acceleration are given by equation (19) (or eq. [7]). In terms of velocity this is

$$\frac{dw}{dt} = w/t_a, \quad (47)$$

where

$$t_a = \frac{r + 1}{r} \frac{t_B}{F_i(\theta) \cos^2 \phi} \geq t_B. \quad (48)$$

Thus the full velocity equation can be written in the form

$$t_d \frac{dw}{dt} = \left(t_d/t_a\right)w - h_{ep}(w)/w^2. \quad (49)$$

As long as the first term is greater than the second, acceleration will result. If not, the particle will decelerate. The two terms of equation (49) are shown in figure 3 for various ratios $t_d/t_a$. The loss term is the curve and the gain terms are simply straight lines with different slopes. For a given such line, all velocities for which the loss curve is above the straight line are decelerated while those velocities for which the loss curve is below the line are accelerated, at least initially. For example, for $t_d/t_a = 0.5$ velocities below $w = 1.2$ are accelerated up to $w = 1.2$; velocities between this value and $w = 1.85$ are all decelerated to $w = 1.2$; velocities above $w = 1.85$ are accelerated. This cannot be taken as implying any change in the velocity distribution in such a case because $t_a$ depends on $\theta$ and $\phi$, and those angles are continually changing. The real usefulness of this is to see that net acceleration will not occur unless $t_d/t_a \geq 1$. Thus to accelerate near-thermal protons in a collapsing magnetic region as we have described we must have

$$1.553T^{3/2}/n \geq \frac{r + 1}{r} \frac{t_B}{F_i(\theta) \cos^2 \phi}, \quad (50)$$

or, since $t_a \geq t_B$, the acceleration condition is more generally stated as

$$t_B \leq 1.553T^{3/2}/n. \quad (51)$$

This second condition (eq. [51]) allows us to make general statements about the possibility of any acceleration at all, while the first (eq. [50]) shows that there will be initial crossing angles $\theta$ that are not accelerated for any values of $t_d$, $T$, $\phi$, and $n$ since $F_i(\theta) \rightarrow 0$ as $\theta \rightarrow 0^\circ$ or $180^\circ$.

If we consider the acceleration of electrons the gain equation (47) is the same while the loss equation (44) has $M = m_e$ instead of $M = m_p$ as above for protons. The variable is still $w = v/v_p$, so electrons near thermal have $w \approx 40$. This means that instead of a characteristic acceleration time given by equation (46) we have a characteristic deceleration time for electrons given by

$$t_d' = t_d(m_e/m_p) = 0.545 \times 10^{-5}t_d$$

$$= 8.46 \times 10^{-4}T^{-4/3}T^{3/2}/n. \quad (52)$$

And in place of equation (49) we have a net velocity equation given by

$$t_d' \frac{dw}{dt} = (t_d'/t_a)w - [h_{ep}(w)/w^2], \quad (53)$$

where, again, the values of $w$ that are appropriate for thermal electrons are near $w = 40$.

Plots of the two terms of this equation are shown in figure 4. The same considerations apply here as for figure 3 to decide which velocities are initially accelerated. We can see from the plot that to accelerate most of the electrons a ratio $t_d'/t_a \geq 0.05$ is needed. So to accelerate near-thermal electrons in our collapsing magnetic configuration requires

$$0.0169T^{3/2}/n \geq \frac{r + 1}{r} \frac{t_B}{F_i(\theta) \cos^2 \phi}, \quad (54)$$
or, in general,

\[ t_B \leq 0.0169T^{-3/2}/n. \]  \hspace{1cm} (55)

Notice that if \( t_d/t_a \approx 1 \) so that the proton acceleration condition is just fulfilled, \( t_d'/t_a \approx 0.005 \) so no electrons are being accelerated at all; but if the electron acceleration condition is just fulfilled, \( t_d'/t_a \approx 0.05 \), then \( t_d/t_a \approx 100 \) and protons will be accelerated with essentially no losses. Thus the acceleration of electrons in this mechanism is much harder to achieve than the acceleration of protons.

Since the magnetic neutral surface can be thought of as a sheet current, it may be wondered whether the nonplanar orbits of particles undergoing acceleration significantly disrupt the sheet current and destroy the magnetic region. If all the particles in the region had looplike orbits, for instance, the magnetic field generated by them would severely alter the magnetic configuration, producing a magnetic field across the once-neutral sheet.

But there is a definite distinction between those particles that are accelerated and those that are not. As we saw above, there are \( \theta \) angles near \( 0^\circ \) and \( 180^\circ \) which are never accelerated if Coulomb losses are considered, and under coronal conditions at least, electrons are not accelerated at all. It is precisely these particles that maintain the sheet current. In addition, solar gravity may produce a “combing out” process whereby infalling matter helps maintain the neutral surface configurations in the solar atmosphere. The result is two distinct groups of particles—one that is accelerated and one that maintains the sheet current.

The orbits of accelerated particles can then be considered as perturbations on the sheet current. These may or may not lead to instabilities. The collapse times necessary for acceleration in the corona are fairly short, suggesting that other instabilities may be more important than this potential one. We note that if \( x_c \gg 10,000 \text{ km} \) then \( \epsilon_x \approx 10 \) for a thermal proton, indicating the likelihood of instability before the accelerated particles have had a chance to travel far. Hence the breakup of the region is probably due to some other sources, though we cannot completely rule this one out.

The equations for the energy gain of a proton, including Coulomb losses, depend only on the ratios \( t_d/t_a \) and \( v_d/v_a \) for given \( \epsilon_x, \epsilon_e \) and initial angles \( \theta_0 \) and \( \phi_0 \). Dependence on these ratios means that \( \alpha \) depends on the initial velocity of the proton explicitly instead of implicitly through \( \epsilon_x \) and \( \epsilon_e \). And do we expect the losses to be larger at some velocities than at others. We also expect the acceleration/loss balance to have a strong angle dependence. To give a comprehensive picture of the acceleration as a function of angle, an average over velocities is clearly needed. The quantity we use is the thermal average of \( \alpha \) over all velocities, i.e.,

\[
\bar{\alpha}(\theta_0, \phi_0) = \frac{\int_0^\infty \alpha(\theta_0, \phi_0, \nu) \exp \left( -\nu^2/v_p^2 \right) \nu^2 d\nu}{\int_0^\infty \exp \left( -\nu^2/v_p^2 \right) \nu^2 d\nu},
\]  \hspace{1cm} (56)

where only values of \( \alpha(\theta_0, \phi_0, \nu) \) which represent \( v_r > v_p \) are used in the integral since we want \( \alpha \) to represent energy gains that can be transferred to the medium outside the accelerating region and only acceleration above thermal velocity does this, i.e., acceleration to thermal velocity is a property of the thermal distribution rather than the magnetic configuration.

Examples of thermal averages taken this way for protons \( (v_r = v_p) \) are shown in figures 5a, b and 6a, b. These plots show \( \bar{\alpha}(\theta_0) \) for \( r = 1, r = \infty, \epsilon_x = \epsilon_e = 1.0, \) and \( t_d/t_a = 0.0 \) and 0.5. These averages are, in general, somewhat lower than we might expect because the contribution from many slow protons is nil, hence lowering the average. The case of \( t_d/t_a = 0.0 \) (no losses) for electrons is the same as the no loss case for protons, figures 5a and 6a.

Finally, we can also average over initial angles \( \theta_0 \) and \( \phi_0 \) to find the average \( \alpha \) for a given kind of particle, a given ratio \( t_d/t_a \), and given \( \epsilon_x \) and \( \epsilon_e \). This average is

\[
\langle \bar{\alpha} \rangle = \int_0^{90^\circ} \int_0^{180^\circ} \bar{\alpha}(\theta_0, \phi_0) \sin \theta \frac{d\theta_0}{180^\circ} \frac{d\phi_0}{90^\circ},
\]  \hspace{1cm} (57)

\footnote{We have defined \( \bar{\alpha} \) this way as the average over all particle velocities so that we can multiply it by the density of particles to get the total energy gain. We could have defined \( \bar{\alpha} \) in equation (58) by integrating from \( v_p \), or some such velocity, to \( \infty \), but then we would have to use a corresponding fraction of the number density to get the total energy gain.}
and will often be referred to as $\langle \alpha \rangle$ or $\langle a \rangle$ or even $\alpha$ when the context is clear. The values of this average for $r = 1$ as a function of $t_b/t_d$ for the $\epsilon_x$ and $\epsilon_y$ combinations noted are shown for protons and for electrons in figure 7. Figure 8 shows the same average, $\langle \alpha \rangle$, for $r = \infty$ as a function of $t_b/t_d$ for electrons and protons.

Even for the limiting case of no losses the largest values of $\alpha$ that can be expected are from 0.4 for $r = 1$ to 2.5 for $r = \infty$. For protons this value has dropped significantly for $t_b/t_d \approx 0.5$, while for electrons it has dropped by $t_b/t_d \approx 0.003$, though for $r = \infty$ the drop is not as much. This illustrates a major factor of the process, that the acceleration of electrons requires much smaller values of the ratio $t_b/t_d$ than acceleration of protons. In fact, significant acceleration of protons can occur with essentially no electron acceleration, and, conversely, if electrons are accelerated significantly, protons are being accelerated with essentially no Coulomb losses.

IV. SHORT COLLAPSE TIMES

We have extended the simple case to include Coulomb losses. In both of these cases we have seen that the acceleration of many particles is not limited by the size of the magnetic region but by its lifetime. The acceleration of some particles, an increasingly large proportion as $\epsilon_x$ and $\epsilon_y$ increase, requires encounter times on the order of $t_b$, the collapse time of the field. In this section we will attempt to account for the significant change that the field undergoes in these times. There are two levels of approach. First, we can assume that the field does not change significantly over a single cycle in the magnetic configuration but only relatively slowly over the whole encounter. But the calculations in the previous two sections only assumed that the behavior of a particle over a cycle was described by the functions $F_\phi$, $G_\phi$, etc. There is no explicit dependence on the values of field parameters. If in a subsequent cycle the field has changed (owing to $t$.
Fig. 7.—Average over angles of the acceleration parameter $a$ as a function of $t_a/t_a$. For protons and electrons for $r = 1$.

Fig. 8.—Average over angles of the acceleration parameter $a$ as a function of $t_a/t_a$ for protons and electrons for $r = \infty$.
single cycle. Thus as long as the change is uniform and as long as the change in the field is negligible over a nearing $t_B$ these same functions describe the motion, slow compared to a cycle, $t$ can reach $t_B$ and the situation will still be covered by the preceding analysis. The second approach is to consider collapse times so short that the field changes appreciably over one cycle. In considering this second case below we will see a confirmation of our expectations in the first. We will go no further than the assumption that

$$K(t) = K_0 \exp \left( t/t_B \right) = K_0 \exp \left( \tau/\tau_B \right),$$

(cf. eq. [8]), which is equivalent to saying that $t_B$ is a constant and is not large compared to orbit times.

In the above analysis the equations of motion are reduced to the differential equation (12) whose solution for $K = 0$ leads to the identification of the hypergeometric functions $I_n, J_n$, and $K_n$. In the present case this equation has no analytic solution. This rather unsatisfying result means that the question of short collapse times must be dealt with in an entirely numerical manner. This involves integrating the orbit equations until either $\tau = \tau_B$ or the $x$-motion or $z$-motion has exceeded the critical value corresponding roughly to the size of the region in the $x$- or $z$-direction. But in dealing with very short collapse times $t_B$ may be less than the orbital period. Thus the distance cutoff is not likely to be necessary.

With this in mind we cut off the integrations when $\tau = \tau_B$. The time cutoff corresponds to the $\epsilon_x = 1$ curve in the analytic calculations and it is with this curve that we will make comparisons to see the effect of short collapse times.

Since cycles considered here take times on the order of $t_B$, the time cutoff will catch some particles in mid-cycle—it is no longer a small perturbation to consider all quantities measured at the end of a full cycle. Thus there might be some fluctuation of the calculated values from a smooth curve or a change in the shape of the curve, depending on the initial angle $\theta$. There is also no guarantee that the acceleration will not be velocity-dependent.

Figure 9 shows the dependence of the acceleration on $\tau_B$ for $r = 1$ and a standard velocity near thermal with $\phi = 0$. Shown dotted is the curve from figure 2 for large $\tau_B$ and $\epsilon_x = 1$. This is the acceleration curve with a cutoff for cases where $\tau_B$ is large. The smooth curve is for $\tau_B = 10^4$, and lies generally below the $\epsilon_x = 1$ curve, and the jagged curve is for $\tau_B = 10^6$.

The erratic character of this curve is due to the arbitrariness of the time cutoff at this velocity. In a time $\tau_B$ particles are making a few cycles and are being cutoff in different particles of a cycle. The significant thing, though, is that the $\epsilon_x = 1$ curve is a rather good smooth fit through the points of the $\tau_B = 10^4$ curve.

So for $\tau_B \geq 10^4$ the effect of short collapse times is adequately represented by the long collapse time limit. And even for $\tau_B$ below this value the magnitude of the acceleration parameter $\alpha$ is comparable to the long $\tau_B$ case. The significant value $\tau_B = 10^4$ corresponds to $\tau_B = 1/B_0$ for protons and to $t_B = 5.7 \times 10^{-4}/B_0$ for electrons.

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REFERENCES