A NEW THEORY OF CORONAL HEATING

RANDOLPH H. LEVINE

High Altitude Observatory, National Center for Atmospheric Research,* Boulder, Colorado

Received 1973 September 17

ABSTRACT

This paper examines an alternative to the heating of the corona by shocks or waves. It is argued that the corona is interspersed with magnetic neutral surfaces which collapse and accelerate particles to a few times their thermal velocity in the manner described in the preceding paper. The accelerated particles travel through the corona and lose their increased energy through Coulomb collisions. It is shown that this mechanism can provide the energy balance needed by the corona and that such acceleration can account for regions of enhanced or decreased heating in the corona.

Subject headings: corona, solar — hydromagnetics — magnetic fields, solar

1. INTRODUCTION: THE CORONAL HEATING PROBLEM

No one has convincingly demonstrated that a sufficient flux of energy can be carried into the corona by waves generated in lower layers of the solar atmosphere. The problem is so complex that simplifying assumptions must be made in all cases. The justifications for these are not always clear, nor is it clear that the solar atmosphere can be treated by ordinary hydrodynamics. In addition, the adequacy of generation mechanisms depends on solar parameters which are not well known, such as the velocity spectrum of the turbulence in the hydrogen convection zone (HCZ). Examples of such work are Kuperus (1969), Lighthill (1967), Osterbrock (1961), and Stein (1971). While further efforts may bear fruit, alternatives to heating by wave action should also be considered.

It is noteworthy that there is almost no observational evidence to support the idea that the corona is heated by shocks. Regular or pulsed motions (observed from Doppler broadening of spectral lines) are smaller in the corona than in the photosphere, a most disturbing fact in view of the theoretical requirement for the wave amplitude to increase before it develops into a shock. There are regular motions observed in the corona, and some have a periodicity near the value of 300 s characteristic of the oscillations of the HCZ. But their amplitude is small and their nature unknown (Durasova, Dobrin, and Yudin 1971; Glencross 1970). Recently, Canfield and Musman (1973) have studied the vertical propagation of longitudinal-mode waves with periods near 300 s. They can trace wave motion upward from the photosphere, but the waves are either totally dissipated or have changed to some other mode before they even leave the chromosphere.

The shortcomings of wave theories fall into the general category of not getting sufficient energy far enough out in the solar atmosphere—for various reasons not enough reaches the corona. But there are other conditions on a theory of the heating of the corona to which wave theories have, in general, not been addressed. Some of these are the distribution of the dissipated mechanical energy, its dependence on the solar cycle, and the nature of the heating mechanism in relation to other sources of solar energy.

The distribution requirement provides a simple and interesting test on theories of coronal heating. One can simply assume that enough energy reaches the corona and see how it distributes itself. Most theorists have only approached the problem of getting the proper total amount of energy to the base of the corona, an equally interesting and important test of heating theories, but not the only one.

The required distribution of mechanical energy in the corona is not precisely known. The estimates of Brandt, Mitchie, and Cassinelli (1965) are based on temperature distributions that are not steep enough at the transition zone. Hence they underestimate both the required deposition at the base of the corona and the total energy required. Other detailed work applicable to this aspect of the problem is by Kopp (1968). He calculated models of the corona that include self-consistent heating by shocks in the equations, whereas the Brandt models attempt to calculate the required deposition without reference to its source.

The problem of energy distribution is not a trivial one, for, as the work of Brandt et al. (1965) indicates, the deposition of mechanical energy should extend out to almost two solar radii to balance conductive and radiative losses completely. Even if heating waves can be shown to reach the base of the corona, some would have to continue several times farther than they had already gone in order to provide an energy balance in the 1.2–1.8 solar radii region, unless a new mechanism is suddenly invoked. It is hard to see how this is possible in the presence of the continuing steep density drop in the corona. The work of Kopp (1968) does not agree with this large a range of energy deposition. After his data are put in a form that can be compared with the work of Brandt et al., the deposition of shocks ceases to be significant around 1.3 solar radii. This supports the contention that shocks cannot reach very far into the corona to supply an energy balance. The need for mechanical heating out to around 2 solar radii

* The National Center for Atmospheric Research is sponsored by the National Science Foundation.
is argued for by Scarf and Noble (1965) and Parker (1965) and against by Kopp (1968) and Blackwell and Petford (1966).

Though most phenomena of the solar magnetic cycle are well known, the dependence of the heating of the corona on the cycle is not. The usual view that there is more energy deposition at coronal maximum is not well supported. The fact that the corona is brighter at solar maximum does not necessarily mean that it is receiving more mechanical energy. The radiation from the corona is such a small part of its energy balance (Brandt et al. 1965; Kopp 1968) that many things besides enhanced mechanical energy deposition could result in its increase. In fact, Brandt et al. find the energy needs of the minimum corona to be equal to or slightly greater than those of the maximum corona. While this is probably not an accurate calculation, due to inadequacies in the treatment of the extreme lower corona, work they consider more accurate on the range of the deposition does require the extent of coronal heating at solar minimum to be greater than at solar maximum. Theirs seems to be the only work comparing the energy balance of the corona at different parts of the solar cycle.

The numbers agreed upon as representing the energy balance of the corona cannot tell us the nature of the heating mechanism. But they do suggest a framework. It is now thought that around $5 \times 10^9$ ergs cm$^{-2}$ s$^{-1}$ in mechanical energy is necessary at the base of the corona to heat it, though recent work of Kopp (1972) on the channeling of conductive flux through the chromospheric network may lower this greatly. This is about $4 \times 10^{20}$ ergs s$^{-1}$, apportioned evenly over the solar surface. While this is an appreciable amount of energy, it is not large by solar standards. The radiative output of the Sun is five orders of magnitude higher and the energy released by just one large flare per day represents nearly the same flux as is needed by the corona. This kind of consideration leads to the notion that the heating of the corona is a "spillover" of a more energetic process elsewhere in the Sun. Or it may come from a process that operates elsewhere but at little or no more intensity than in the corona, making it unimportant anywhere but there. In this sense the heating of the corona is merely an afterthought of solar activity.

For example, the HCZ generates about 100 times the energy needed by the corona. But, as noted above, dissipation in the photosphere and chromosphere may be large enough to leave nothing for the corona; the entire mechanical energy output of the convection zone may be dissipated before reaching the top of the chromosphere. And the waves which heat the corona have little connection with other coronal activity, though there are ties to chromospheric phenomena.

This leads us in the direction of an alternative energy source that is tied to coronal activity and capable of explaining both the heating of the corona and the variations in the heating (e.g., over active regions; at solar minimum), in a straightforward way. The solar magnetic field is the obvious choice. In the following sections we present a theory of the corona based on considerations of the nature of the coronal magnetic field. Section II describes the proposed heating mechanism, which is based on the presence of neutral sheets in the corona, and § III presents the results of some simple calculations that show the possibilities of the mechanism.

II. A CORONAL HEATING MECHANISM

We know that the magnetic field of the Sun, whatever its nature, is turbulent on small scales. It is probable that this turbulence produces a twisting and compressing of the field that would release under appropriate circumstances. Given the known magnetic activity on large scales in the Sun, it is hard to believe that these circumstances do not occur. Thus we can expect a continual process of untwisting of the small-scale turbulence of the solar field. It is also likely that these untwisting elements expand into the solar atmosphere, or are actually carried or formed there by large-scale magnetic activity such as prominences or the magnetic loops of the Wilcox sector field, an ever-expanding phenomenon, with new sectors forming and emerging into the atmosphere, often carrying magnetic loops into the corona to distances of several solar radii (Wilcox 1971). And Piddington (1972) points out the probability that the general solar field is expanding as an integral part of its nature.

In fields that have been so tangled, and in the untwisting process itself, neutral sheet regions will occur. These are areas where oppositely directed magnetic fields meet at a surface of zero field. Such a field configuration will collapse toward the neutral surface, except that the amount of matter, and hence the gas pressure, will be so great initially that the collapse will be slowed. The still-twisted nature of the field will tend to perpetuate this condition, holding off significant collapse until the region is farther out in the atmosphere. As this occurs the excess matter will begin to flow back along the now fairly untwisted field lines toward the solar surface. This will lessen the gas pressure in the neutral region, but the flow along field lines will stabilize the configuration even more. When the amount of material left in the region is no longer sufficient to hold off the force of rapid collapse by gas pressure, a rapid collapse will occur and the field will eventually rearrange itself through a series of instabilities. The density is the controlling factor because of the high conductivity and resulting slow annihilation of fields in the solar atmosphere.

It is at this initial point that particles are significantly accelerated by the collapsing magnetic regions, a topic discussed in the preceding paper (Levine 1974). The collapse stops with pressure forces building up again (see Parker 1963 for a discussion of the steady-state version of this process, Sweet's mechanism) or with the onset of a hydromagnetic instability (see Syrovatskii 1966 for a discussion of plasma instabilities near a neutral line). But the continual action of the untwisting, the plasma expansion, and gravity will continue to create new regions of collapsing field configuration.
Such regions will not, of course, be confined to the corona. In fact, those accelerating regions that reach the corona may be only the final remnant of stronger accelerating regions whose major effects occur lower in the atmosphere. It is possible, for example, that several such regions could lie along neutral sheets starting low in the atmosphere and expanding far into the corona. It is well known that solar flares can occur at any observable intensity and there is no reason to suppose that flarelike activity does not extend downward in intensity toward the magnetic accelerating regions we have discussed. Recently, Martres, Pick, and Soru-Escant (1972) have found enhanced features of the chromosphere that are the source of fast particles, are not associated with flares, and lie along a magnetic neutral line. These are excellent candidates for intense magnetic accelerating regions.

The possible connection of these accelerating regions with the chromospheric network is intriguing. The network is a mottled pattern in emission lines (and in magnetic field) of characteristic cell separation \( \sim 30,000 \text{ km} \) in the chromosphere. It certainly extends into the transition zone (Noyes 1971), and may extend into the low corona (Simon and Noyes 1971) in cells of characteristic size \( \geq 17,000 \text{ km} \) and separation perhaps twice that. This is very suggestive of the lattice-like array and size of accelerating regions in the models that follow. It would be interesting to find a stronger connection of the network with possible magnetic accelerating regions.

Such a connection may exist in the observations of Krieger, Vaiana, and Van Speybroeck (1971). They find bright spots of X-ray emission embedded in the network, and always associated with a magnetic neutral line. The observation of these features in the X-ray region indicates that they are either hot or extend towards the corona or both. From the figures shown by Krieger et al. the characteristic size of these areas is \( \sim 20,000 \text{ km} \).

The observational evidence on the small-scale structure of the corona, where we might find many collapsing field configurations, is slight. Most reliable information is obtained by averaging techniques that obscure small-scale data in favor of better statistics. The facts that are known (Newkirk 1967) indicate that there are small-scale elements of density enhancement in the corona, often with a preferred alignment, but always with a randomly oriented component as well. It is difficult to associate a magnetic field structure with these areas, or with any of the corona. All that is known about the coronal fields is deduced from the gross shape of large-scale structures where the field is strong enough to measure or from extrapolation of photospheric fields:

Though there is no direct evidence, Newkirk interprets the data he reviews as possibly implying "a corona completely interlaced with tangled filaments which originate over the chromospheric network forming the boundaries of the supergranulation," and having a constraining magnetic field associated with it. The "tangled filaments," which we would identify with collapsing magnetic null surface regions, may have this origin, and may be responsible for providing the mechanical energy balance known as the heating of the corona. Models based on these considerations are presented in the next section.

III. MODEL CALCULATIONS

We want to show a variety of conditions and assumptions under which the heating requirements of the corona can be met by the acceleration of near-thermal particles and their consequent loss of energy through Coulomb interaction. The emphasis will be on showing a range of possibilities that solve the problem and which cover the probable state of conditions in the corona and in the accelerating regions. In this way we will see the great flexibility of this heating mechanism without having to be overly precise about coronal conditions or about the behavior of the magnetic accelerating regions (MARs). The question of where the actual conditions of the corona and of the MARs fall in this range is still open but not necessary to the point at hand.

We envision a corona interspersed with magnetic null surfaces (not necessarily isolated from one another) which collapse and accelerate particles in the manner described in the preceding paper (Levine 1974). Their size and spacing are matters of investigation, and their distribution could vary with the solar cycle to produce the necessary changes in coronal heating.

A collapsing magnetic null surface accelerates particles by continual reflection off the approaching magnetic "walls." The acceleration of each particle is characterized by a parameter \( \epsilon = \Delta E/E_0 \) whose average value for a spectrum of conditions is in the range of 1.0-5.0 and does not depend on the strength of the magnetic field, given the fact that it is collapsing. Because of Coulomb losses within the accelerating region, the accelerated particles will consist almost entirely of protons. With this picture in mind we can calculate the effect of the acceleration of near-thermal particles on the energy deposition in the corona.

\[ P = \sum_{\text{MARs}} \frac{\int dv v^2 A(v) \exp \left( -v^2/k_B T \right)}{\int dv v^3 \exp \left( -v^2/k_B T \right)} , \]

where \( N \) is the number of particles accelerated per second and \( A(v) \) is the average over angles of the excess energy of a particle which has velocity \( v \) before acceleration.

For the number of particles accelerated per second in a region we will take

\[ N = fnL^2/t_b , \]

where \( n \) is the number density of particles that are accelerated, \( L \) is a characteristic dimension of the...
region, and $t_B$ is the characteristic collapse time of the region and hence close to the lifetime of the region. Since there is some uncertainty in these numbers we have included a factor $\eta$ that can be adjusted to see the effect of changes in these parameters. Since the collapse is likely to begin when the gas density in the region reaches the neighborhood of the surrounding density, we will calculate using $n_e$, the coronal density, for $n$. As long as $n \propto n_e$, it is a valid assumption throughout the corona, this will be accounted for by the proportionality constant $\eta$.

The calculations of the integral in the sum for $P$ can be simplified greatly. The quotient of integrals represents the average energy gain per particle, which we can represent to sufficient accuracy by using $\alpha = \Delta E/E_0$ and taking $E_0$ to be the mean thermal energy. Thus,

$$P = \sum_{\text{max}} n_e L^3 \frac{\kappa}{t_B} \frac{3k_B T}{2}. \quad (3)$$

Now if we assume that the only parameters that change from region to region are $T$ and $n_e$,

$$P = \frac{3fL^3c}{2t_B} \sum_{\text{max}} n_e T. \quad (4)$$

This is certainly a very accurate assumption, for we can expect $t_B$ and $L$ to vary, but it is sufficient to get a good idea of the magnitude of the process and we will see below the effect of varying $t_B$ from one region to another.

In order to sum over all the accelerating regions, some assumption must be made about their distribution. We will use a quasi-cubic array of accelerating regions. This consists of a network of evenly spaced regions at $\eta = r/R_0 = 1.1$, another layer with the same total number of regions (and hence larger separation) a distance $D$ above that, and so on in layers separated by a distance $D$ out to the neighborhood of $\eta = 1.7$. The array is quasicubic because the (lateral) separation in the first layer is taken to be $D$, the separation distance between layers. Alternatively, one can imagine radial rays all over the Sun, separated from one another by a distance $D$ at a distance 1.1 $R_0$ from the center and with an accelerating region there and every distance $D$ along the ray from there to around 1.7 $R_0$. Equivalently for this total energy calculation, each level or shell of accelerating regions can be rotated in any way relative to the others as long as its radius is not changed. In this picture there are imaginary concentric spheres in the corona separated by a distance $D$, each of which has the same number of accelerating regions attached to it, but distributed in an arbitrary way. This conceptualization will allow us to claim validity for the total energy calculation in the case of the minimum coronal, where the distribution of accelerating regions is least likely to be uniform over the solar surface.

The number of regions in each layer is then $4\pi R_0^2(2/1.21)D^2$ and the number of layers is approximately $0.6/R_0/D$. Thus the total energy deposited is a

<table>
<thead>
<tr>
<th>TABLE 1</th>
<th>Calculated Properties of Accelerating Regions: Inhomogeneous Maximum Corona</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L/D$ for $f_e = 1^*$</td>
<td>$L/D$ for $f_e = 1\dagger$</td>
</tr>
<tr>
<td>$D$ (km)</td>
<td>$t_B = 1$</td>
</tr>
<tr>
<td>100,000</td>
<td>0.050</td>
</tr>
<tr>
<td>50,000</td>
<td>0.055</td>
</tr>
<tr>
<td>25,000</td>
<td>0.057</td>
</tr>
<tr>
<td>10,000</td>
<td>0.058</td>
</tr>
<tr>
<td>1,000</td>
<td>0.059</td>
</tr>
</tbody>
</table>

* Arbitrary $t_B$ (see eq. [6]).
† Maximum allowable $t_B$ for acceleration (see eq. [8]).

sum over layers,

$$P = \frac{1.54 \times 10^5fL^3c}{t_B D^2} \sum_{\text{layers}} n_e T. \quad (5)$$

We can calculate the sum from experimental values of $n_e$ and $T$. Then, knowing that $P = 4 \times 10^{28}$ ergs s$^{-1}$, we can find the ratio

$$fL^3c/t_B D^2 = 2.6 \times 10^{21} \sum_{\text{layers}} n_e T. \quad (6)$$

The collapse time $t_B$ may vary from region to region. It was shown in the preceding paper (Levine 1974) that the maximum collapse time necessary to accelerate protons is

$$t_B \leq 1.553 T^{3/2}/n_e. \quad (7)$$

If we use this for $t_B$ in equation (5) and put it back inside the sum we have instead of equation (6)

$$fL^3c/D^2 = 4.1 \times 10^{21} \sum_{\text{layers}} n_e^2 T^{-1/2}. \quad (8)$$

This is an upper limit to $t_B$ and hence an upper limit to $L$.

Results of exact calculations of equations (6) and (8), using the coronal temperature and density from the calculations of Brandt et al. (1965), are shown in tables 1 and 2. Table 1 is for the inhomogeneous maximum coronal data (Allen 1955) and table 2 is for the inhomogeneous minimum corona (Pottasch 1960).

<table>
<thead>
<tr>
<th>TABLE 2</th>
<th>Calculated Properties of Accelerating Regions: Inhomogeneous Minimum Corona</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L/D$ for $f_e = 1^*$</td>
<td>$L/D$ for $f_e = 1\dagger$</td>
</tr>
<tr>
<td>$D$ (km)</td>
<td>$t_B = 1$</td>
</tr>
<tr>
<td>100,000</td>
<td>0.061</td>
</tr>
<tr>
<td>50,000</td>
<td>0.067</td>
</tr>
<tr>
<td>25,000</td>
<td>0.069</td>
</tr>
<tr>
<td>10,000</td>
<td>0.072</td>
</tr>
<tr>
<td>1,000</td>
<td>0.072</td>
</tr>
</tbody>
</table>

* Arbitrary $t_B$ (see eq. [6]).
† Maximum allowable $t_B$ for acceleration (see eq. [8]).
For collapse times near 10 s, $L/D \simeq 0.12$, while for $t_p$ near 1 s, $L/D \simeq 0.06$. The upper limit on $L/D$, imposed by the maximum permissible $t_p$ to accelerate protons, is $0.15$ for maximum corona and $0.02$ for minimum corona. If acceleration of electrons is important, $t_p$ will be approximately 100 times smaller and $L/D$ will be lowered by about a factor of 5. These calculations are for $\alpha = 1$. For $\alpha \neq 1$, $L/D$ varies as $(\alpha^3)^{-1/2}$.

\( b \) Distribution of Energy Dissipation

While this calculation is encouraging in pointing out the possibilities of magnetic accelerating regions for heating the corona, a convincing case is also made by calculating the distribution of the energy dissipation due to these regions. This distribution is simply the distribution of the Coulomb energy losses of the accelerated particles as they travel through the corona. Each accelerating region will thus be surrounded by a “cloud” of energy dissipation of rather irregular shape, but most intense near the region. The shape of the cloud will depend on the position of the region in the corona, its orientation, and the directions in which particles are emitted by the region. For example, particles emitted toward the Sun will be entering areas of higher density and hence will lose energy much more rapidly than particles moving away from the Sun into less dense zones. The shape of the total energy dissipation distribution is simply the sum of the energy dissipation in the clouds along any given radial ray. This can of course vary greatly, depending on how many clouds of dissipation are near the ray in question, their shape, and so on. The likely nonuniform character of the corona and its heating can be accounted for in this way.

But we are interested in the average heating along a representative ray. To this end, we will calculate the dissipation cloud about each region as a function of the radial position of the region in the corona and then calculate the distribution of the dissipation for arrangements of the regions corresponding to the quasi-cubic array pictured above.

The Coulomb energy loss of a proton as a function of distance is found from the equations of the preceding paper (Levine 1974) to be

\[
dE/dx = -1.38 \times 10^{-20}n_p(x)/T_p[\eta_\text{cf}(w)/w^3],
\]

where $w = v/v_p$ and the function $\eta_\text{cf}(w)$ is defined there. We have emphasized the fact that while the $n_p$ in this equation is the local density as the particle travels in the corona, the temperature $T_p$ appears only as the thermal energy of the accelerated particles and hence depends only on the region and its position. It will be useful to have this energy loss in terms of the ratio of the particle’s energy to the average thermal energy,

\[
dU/dx = \frac{2}{3k_bT_p} \frac{dE}{dx} = -6.66 \times 10^{-5} \frac{n_p(x)}{T_p^2} \left[\ln_\text{cf}(w)/w^3\right],
\]

because when the total energy lost, $\int dE/dx$, is equal to the total excess energy, $\Delta E$, with which it was ejected from the accelerating region, the particle has given up all its excess energy. This condition is equivalent to

\[
U = \int dU/dx dx = \alpha w_p^3/v^2 = w_p^3/\alpha.
\]

Given a (radial) position in the corona, we can use equations (10) and (11) to calculate the energy loss of a particle as a function of distance in any direction for various initial velocities. These losses, as a function of distance from the region, can then be averaged over a thermal distribution to find the average energy deposited in that direction from a region at that radial position. In taking the thermal average we should not use $v = 0$ as a lower limit in the integral because even though particles below thermal velocity are accelerated in the magnetic configuration, unless they are accelerated to a velocity above thermal they remain at a velocity that does not deposit energy directly into the corona. (See footnote 1.) Thus the lower limit should be at $v = v_p/((1 + \alpha)^{1/2}$, the lower bound of the velocities which are accelerated to greater than thermal velocity. For $\alpha > 1$ the difference between this and $v = 0$ is significant because the Boltzmann factors of $v_p$ and $v_p/(1 + \alpha)^{1/2}$ are in the ratio

\[
1 : \exp[\alpha/(1 + \alpha)]/(1 + \alpha),
\]

which is not small. Also, the direction of emission of the particle can be specified by one angle, the angle of its initial velocity with respect to the radial direction, as all particles emitted in a cone of this angle about the radial direction are assumed to encounter the same temperature-density distribution as a function of their distance from the accelerating region. Finally, we assume that the particles travel in straight lines, an assumption that may not be correct in the outer layers of the Sun we are dealing with (but which will not matter because of the diffuse character and low intensity of the dissipation there).

A tremendous amount of information can be generated in this way. For each of several representative radial positions one can calculate the energy deposition as a function of distance in each of several representative directions and for each of several velocities, which are then averaged over a thermal distribution for each direction from each region. And all this depends on the coronal data used and on the acceleration coefficient $\alpha$. Until changed, we will henceforth discuss data for the inhomogeneous maximum corona and for $\alpha = 1$. The dependence on $\alpha$ is not a trivial scaling.
matter here as it was in the total energy calculation. A different $a$ will change the shape of the cloud about an accelerating region.

Calculations along these lines have been made for three cases of the quasi-cubic array. Recall that in the quasi-cubic array the accelerating regions lie in shells separated by a distance $D$ and the regions within a shell at radius $r$ are separated by a lateral distance $Dr^2/r^2$. Cases 1 and 2 are for a normal quasi-cubic array. This has the accelerating regions of one layer directly above those of the one below, i.e., all the MARs lie on radial rays whose separation is $D$ at $\eta = 1.1$ and the regions are separated along the rays by a distance $D$. Case 1 is the energy $-\nabla \cdot b(\eta)$ where $b$ is the energy flux as a function of distance along one of these rays. Case 2 is $-\nabla \cdot b(\eta)$ as a function of distance along a ray which is equidistant from each of the nearest rays of case 1, i.e., it is along a ray between the accelerating regions of the normal quasi-cubic array. Case 3 is for a shifted quasi-cubic array: every other layer of accelerating regions is moved over so that the regions of that layer lie along the ray described for case 2. In case 3, the function $-\nabla \cdot b$ is normalized, as in equation (18), to equal $P$, while in cases 1 and 2 it is assumed that each of these cases contributes equally to the integral of the total energy for the normal quasi-cubic array so the sum of cases 1 and 2 is normalized to $P$, giving each the proper magnitude relative to the other.

The results are shown in figures 1, 2, and 3 for $D = 50,000$, 25,000, and 10,000 km, respectively. Each plot shows $-\nabla \cdot b(\eta)$ for cases 1, 2, and 3 as well as a range of energy deposition requirements of the corona. These theoretical comparison curves are based on the work of Brandt et al. (1965) and are expected to be least accurate near the transition zone. The lower comparison curve represents the maximum corona with thermal conductivity reduced by half, the upper has radiative losses increased by a factor of 5. The curve representing the energy needs of the maximum corona with no such adjustments falls between these two. The upper curve is normalized to the same total energy deposited as the curves of the present models so only their shape is significant.

Many interesting features are brought out by these calculations. Consider figure 1. The large separation of 50,000 km means that, at least near the lower values of $\eta$, the energy-deposition clouds of separate regions will not overlap at all and this is indeed the case. There is great variability in the energy deposition below about $\eta = 1.4$, indicating that this great a separation would create definite hot spots in this region of the corona. The between-MARs curve of case 2 does not even show up until $\eta \approx 1.6$ and then only negligibly. The shifted array of case 3 predictably skips the peaks of case 1 when it is between regions and contains only those peaks where the radial ray considered passes through a region, i.e., every other peak, out to about $\eta = 1.5$.

In figure 2 the accelerating regions are closer together by a factor of 2 from figure 1. There are still large peaks and valleys, now out to about $\eta = 1.25$, by which time the variability of the energy deposition is much milder. The case 2 curve becomes important only after about $\eta = 1.4$, when the energy-deposition clouds start to penetrate the interregion area. The case 3 curve again picks up every other peak out to about where the lateral overlap begins. It is the important contribution from laterally neighboring regions that keeps the deposition approximately constant in the $\eta = 1.5-1.6$ area. If it were not for this the curve would continue to be wavy. For example, if the particles are channeled into the two radial directions with a $\cos^a$
weight, the case 1 curve would have the same general magnitude, but peaks extending all the way to the cutoff of accelerating regions. The drop at the end of the curve is caused solely by the lack of MARs past $\eta = 1.7$; if there were more areas beyond that they would contribute enough to keep the curve approximately flat. A corollary of this is that if the MARs were cut off sooner than $\eta = 1.7$, but still in the flat part of the curve, a similar drop would occur. In this way deposition requirements could be more nearly matched with fewer regions occupying less of the corona.

Figure 3 shows remarkable features. The separation of 10,000 km is so small that, except for the very lowest 0.1 $R_\odot$ of the corona, no structure attributable to individual accelerating regions can be detected. This is the reason the curve is so flat. Even though the contribution of regions further out is less in magnitude than that of those closer in, the increase in the size of the energy-dissipation clouds at larger $\eta$ means that more accelerating regions are contributing to the deposition at a given point as we move outward in the corona. The distinction between the three cases also becomes quite blurred. The drop in the tail of the curve appears here too. As for $D = 25,000$ km we can cut the range of MARs off anywhere in the flat part of the curve and get a similar drop and so match theoretical curves better. But here a new feature is encountered. Even if we reduce the number of layers of accelerating regions and match the tail of the curve, the calculated deposition will still be flat where the required
deposition is peaked. The reason again is that $D$ is relatively small and a solution which matches the particular theoretical curves shown would require a variable $D$: more regions producing more deposition where it is needed. It is important to notice that this is not necessary for $D = 25,000$ km, but only somewhere below that separation.

We can sum up the situation in the elementary cases considered here by saying that for $D = 50,000$ km the corona is heated very irregularly, with many separated regions of intense heating. While this is probably not appropriate to the steady-state heating of the corona, regions of this size and intensity may provide an interesting link with the continuum of flarelike activity that extends down to the limit of current observations.

The magnetic accelerating regions considered here may be only a weaker version of more intense activity elsewhere in the solar atmosphere. It is not unlikely that they should exist in a range of intensities. For a separation of 25,000 km the energy deposition requirements of the corona can be met, with some irregularity in the extreme lower corona, by MARs extending from $\eta = 1.1$ to about $\eta = 1.5$. This is only two-thirds as many regions as we assumed in table 1 so the size of the regions, $L$, calculated in table 1 will be larger by approximately a factor of $(3/2)^{1/3} = 1.14$. (The exact value will also depend on how $\sum n_i T_i$ changes, but this change will be slight when regions with low $n_i$ are deleted.) For separations much smaller than this, and certainly for $D = 10,000$ km, the accelerating regions must not only extend no further than $\eta = 1.5$ but their separation must be a function of $\eta$ so that more energy can be deposited in the region of the deposition peak than would be the case for even separations, which give an essentially flat curve. More MARs in the lowest corona means that $D$ must either be an increasing function of $\eta$ or have a minimum near $\eta = 1.25$.

We may add a directional weight proportional to $\cos^2 \theta$, where $\theta$ is the angle with the radial direction, because the general magnetic field in the corona may confine particle motion in more or less radial rays, or because a raylike inhomogeneous structure was assumed in the calculations of Brandt et al. (1965). This does not change the general shape of the energy-deposition distributions or the above conclusions. Only the smoothness of the curves is affected because of the decreased contribution from laterally placed accelerating regions.

To calculate the energy deposition at coronal minimum, the regions were allowed to extend from $\eta = 1.25$ to $\eta = 1.7$. This is not to say that we do not expect any magnetic null regions below $\eta = 1.25$, but we want to compare the heating estimates with the calculation of Brandt et al., and the deposition there includes roughly this region. This means that we can meaningfully normalize the resulting deposition curves of our model to the total energy deposition of one of the curves of Brandt et al., and talk about the deposition in this region independently of whether there is significant activity below this level or not. In addition, the minimum coronal data are valid only for equatorial regions, but both the theoretical curves and the following energy deposition curves of our models are normalized to a total $P$ in the same way, i.e., involving the same integral multiplied by the same surface area. So even though the $P$ used refers to a corona with uniform activity over the solar surface, the resulting curves and those of Brandt et al. are comparable on an equal basis without regard for how much of the corona contains heating activity at solar minimum. (The same is true for the maximum coronal calculations above, where this fact is not as important.)

Figures 4 and 5 show the results for the inhomogeneous minimum corona for $D = 25,000$ and $D = 10,000$ km, respectively. The energy deposition for cases 1, 2, and 3 is plotted, as before, and the curve of Brandt et al. corresponding to this case is shown also. For $D = 25,000$ km the curves are too flat and too large in magnitude beyond about $\eta = 1.45$. A much better fit would be found by having regions of this

![Figure 4](https://example.com/fig4.png)

**Fig. 4.—Same as fig. 1 for coronal minimum and $D = 25,000$ km**
Fig. 5.—Same as fig. 4 for \( D = 10,000 \) km

separation only out to about that level. There would then probably not be any need to have \( D \) vary with \( \eta \) because the peak would increase to meet the normalization requirement. This would increase \( L \) in tables 1 and 2 by a factor of about \( 3^{1/3} \). With a separation of \( D = 10,000 \) km the agreement is so good that it seems unnecessary to contemplate any adjustments.

At coronal minimum, then, the quasi-cubic array (shifted or normal) provides good agreement with sample calculations of energy-deposition requirements of the corona. Only the extent of the penetration of the MARs into the corona is in question, and then only the upper limit because we have used an artificial lower limit here to enable comparison with the calculations of Brandt et al. (1965). For separations of 25,000 km the regions need extend no farther than \( \eta \approx 1.45 \) while for separations of 10,000 km they can extend all the way to \( \eta = 1.7 \).

It is appropriate to note here that if the acceleration parameter \( \alpha \) is larger than the value 1.0 we have used, each accelerating region will have a larger deposition cloud and the curves will be smoother and flatter. We should also keep in mind the fact that \( \alpha \) is more generally a function of the angle at which a particle begins its acceleration in a magnetic null surface region. There is also a tendency for MARs to emit accelerated particles in a direction perpendicular to the null surface. Also, collisions may soon change the direction of motion of the particles and the null planes of various accelerating regions are not likely to be correlated in direction. Thus our use of energy-deposition clouds with the characteristics we have assumed is an idealization of an average and cannot be taken too literally as corresponding closely to features of the corona. The properties of such clouds can be compared with the similarly idealized requirements of the corona, as we have done, and regions like these clouds probably exist in the corona. But the actual situation contains many complicating features that we are not yet justified in dealing with and have not dealt with in this analysis. The rationale for examining this mechanism of coronal heating more closely will hopefully come from encouraging observations of the corona and its magnetic field.

The help of Professor David Layzer was instrumental in the completion of this work, some of which was done while the author was a Ph.D. candidate at Harvard University. I thank Dr. Roger Kopp for reviewing the manuscript.

REFERENCES

Note added in proof.—Several recent observational results lend further support to the above ideas. It is now known that surface magnetic fields of the Sun are concentrated into small bundles of intense field strength. The details of this small-scale structure are not presently known, however. Two other recently seen and probably connected phenomena are ephemeral active regions and bright emission points seen in X-ray and extreme-ultraviolet observations, particularly those from Skylab-ATM. These small and apparently evenly distributed enhancements may correspond to neutral sheet acceleration regions similar to those discussed here, but much stronger in intensity. Their distribution, and the small, bipolar character of the ephemeral active regions suggest that the kind of processes postulated in this paper do occur in the solar atmosphere.