Damping of sound waves propagating in optically thin plasmas

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The effects of the second viscosity on sound waves propagating in plasmas at thermochemical equilibrium are analyzed. It is found that this viscosity can be more important than the dynamical viscosity as well as than the thermometric conductivity, in particular, in photoionized plasmas with arbitrary metallicity Z. The strongest damping per unit wavelength always occurs for sound waves with a period of the order of the ionization relaxation time. Additionally, such a damping is more efficient for plasmas with solar abundances $(Z=1)$ than for primordial plasmas $(Z=0)$. In collisionally heated pure hydrogen plasmas the second viscosity becomes zero. © *2004 American Institute of Physics*. [DOI: 10.1063/1.1792633]

I. INTRODUCTION

In analyzing the linear wave propagation as well as in determining the local stability by linear theories in optically thin fluids where the heat gain/loss term in the energy equation is denoted by a local function of the thermodynamical variables, say density and temperature, usually it is assumed that the damping of the wave modes[1–3] or the stabilizing effect of local perturbations[4–6] is due to the thermal conduction. The above is justified by the fact that the *dynamical viscosity* (*shear viscosity*) term in the energy equation is a second-order term, and even though it is a first-order term in the momentum equation, it is argued that the viscosity coefficients are smaller than the thermometric conductivity. The above is a reasonable approximation when the chemical relaxation time τ of the gas is larger (chemistry frozen) or smaller (fluid in chemical equilibrium) than other relevant time scales, say the dynamical and thermal times. However, when these time scales are of the order of τ, the *second viscosity* (*bulk viscosity*) $\nu_2=\zeta/\rho$ becomes larger than the thermometric conductivity χ as well as than the dynamical viscosity $\nu=\eta/\rho(\lesssim\chi)$, and the corresponding dissipative effects become important, in particular, in determining the threshold value for wave amplification or the marginal values for stability. Additionally, as pointed out in Ref. 7 (p. 309) the second viscosity which is due to the irreversibility of chemical reactions, is quite different in nature from the dynamical viscosity ν because the coefficient ζ is dependent, in addition to the thermodynamical properties of the gas, on the frequency of the fluctuations (it is dispersive). The analysis of the above effects on the propagation of otherwise isentropic sound disturbances is the aim of the present paper.

II. BASIC EQUATIONS

Due to the fact that the interest will be focused on the bare second viscosity effects on the sound waves propagation, instead of handling the full linearized gas dynamics equations for reacting gases it is simpler to start from the second viscosity expression quoted from Ref. 7 (Chap. VIII, Sec. 81), i.e.,

$$\nu_2=\frac{(c_\infty^2-c_0^2)\tau}{1-i\omega\tau}, \tag{1}$$

which holds for disturbances $\sim\exp(-i\omega t)$ in a reacting gas with chemical relaxation time

$$\tau=[\partial X(\rho,T,\xi)/\partial\xi]^{-1}, \tag{2}$$

with $X(\rho,T,\xi)$ being the net chemical rate and ξ the chemical parameter, denoting the net advance of the reactions, and the sound velocities c_∞^2 and c_0^2 are defined as

$$c_\infty^2=\left(\frac{\partial p}{\partial\rho}\right)_\xi,\quad c_0^2=\left(\frac{\partial p}{\partial\rho}\right)_{eq}=\left(\frac{\partial p}{\partial\rho}\right)_\xi+\xi_0'\left(\frac{\partial p}{\partial\xi}\right)_\rho, \tag{3}$$

where ξ_0 is the value of the chemical parameter at chemical equilibrium and $\xi_0'=(\partial\xi_0/\partial\rho)$ also at equilibrium.

On the other hand

$$c^2=\left(\frac{\partial p}{\partial\rho}\right)=\frac{c_0^2-i\omega\tau c_\infty^2}{1-i\omega\tau}, \tag{4}$$

where $(\partial p/\partial\rho)=(\partial p/\partial\rho)_\xi+(\partial p/\partial\xi)_\rho(\partial\xi/\partial\rho)$. From Eq. (4) it follows that c no longer denotes the sound velocity, being complex. However, for sound waves the relation $k=\omega/c$ remains valid; therefore

$$k=\frac{\omega}{c_0}\sqrt{\frac{1-i\omega\tau}{1-i\beta^2\omega\tau}}, \tag{5}$$

where $\beta=c_\infty/c_0$ and the wave number also becomes complex, $k=k_1+ik_2$, with k_1 and k_2 being real. Therefore, fluctuations propagating in any direction (say x), $\sim\exp(ikx)=\exp(ik_1x)\exp(-k_2x)$, are damped provided $k_2>0$. As it is well known the positivity of k_2 is assured by the irreversible character of the dissipative processes.

Therefore, from Eq. (5) it follows that

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$$k_1 = \frac{\omega}{c_0}\left[\frac{1+\beta^2\omega^2\tau^2}{2(1+\beta^4\omega^2\tau^2)} + \frac{1}{2}\sqrt{\frac{1+\omega^2\tau^2}{1+\beta^4\omega^2\tau^2}}\right]^{1/2}, \tag{6}$$

$$k_2 = \frac{\omega^2\tau}{c_0}\frac{\beta^2-1}{\{2(1+\beta^4\omega^2\tau^2)[1+\omega^2\tau^2+\sqrt{(1+\beta^4\omega^2\tau^2)(1+\omega^2\tau^2)}]\}^{1/2}}, \tag{7}$$

from where the damping rate per unit wavelength k_1/k_2 as well as the phase velocity $v_{ph}=\omega/k_1$ can be straightforwardly calculated.

III. DAMPING OF SOUND WAVES IN OPTICALLY THIN PLASMAS

There are two plasma models of current importance in astrophysics as well as in the laboratory: (1) a collisionally ionized hydrogen plasma and (2) a photoionized hydrogen plasma with metallicity Z, for both of which the pressure is simply

$$p = N_o k_B(1+\xi)\rho T,$$

where N_o is the Avogadro number, k_B the Boltzmann constant, and ξ the degree of ionization.

On the other hand, for these plasmas the thermometric conductivity can be written as[4,8]

$$\chi = \frac{1}{\rho c_p}\left[2.5\times 10^3(1-\xi)T^{1/2} + 1.84\times 10^{-5}\frac{\xi T^{5/2}}{\ln\Lambda(\rho,T,\xi)}\right]. \tag{8}$$

A. Collisionally ionized hydrogen plasma

For the hydrogen plasma model studied in a previous work,[6] i.e., a collisionally ionized pure hydrogen plasma, the net rate function $X(\rho,T,\xi)$ and the net cooling rate per unit mass $L(\rho,T,\xi)$, respectively, are given by

$$X(\rho,T,\xi) = N_0\alpha_B(T)\rho\xi^2 - N_0 q(T)\rho\xi(1-\xi), \tag{9}$$

$$L(\rho,T,\xi) = N_0 R\rho\xi^2 T\beta_B(T) + N_0^2\chi_h\rho\xi(1-\xi)[q(T)+\Phi(T)] - L_0, \tag{10}$$

where $\chi_h=13.598$ eV, L_0 is a constant per unit mass heating, and the coefficients $\alpha(T)$, $\gamma_c(T)$, $\beta_B(T)$, and $\Phi(T)$ are given by Refs. 9–11. The galactic value $L_0=3.25\times 10^{-4}$ erg g^{-1}. s^{-1} (Ref. 12) has been taken as a reference value.

For this plasma the ionization at equilibrium $X(\rho,T,\xi)=0$ becomes $\xi_0=q(T)/[\alpha_B(T)+q(T)]$, i.e., only a function of temperature, and therefore the second viscosity is strictly zero. Therefore, for this particular plasma the relevant dissipative processes are the thermal conduction as well as the dynamical viscosity.

B. Photoionized hydrogen plasma with metallicity Z

For an optically thin hydrogen plasma with metallicity Z heated and ionized by a back-ground radiation field of mean photon energy E and ionization rate ς, the net rate function $X(\rho,T,\xi)$ and the net cooling rate per unit mass $L(\rho,T,\xi)$ are, respectively, given by[13]

$$X(\rho,T,\xi) = N_0\rho[\xi^2\alpha-(1-\xi)\xi\gamma_c]-(1-\xi)(1+\phi)\varsigma, \tag{11}$$

$$L(\rho,T,\xi) = N_0^2\rho[(1-\xi)Z\Lambda_{HZ}+\xi Z\Lambda_{eZ}+(1-\xi)\xi\Lambda_{eH} + \xi^2\Lambda_{eH^+}] - N_0(1-\xi)\varsigma[E_h+(1+\phi)\chi_h], \tag{12}$$

where ϕ is the number of secondary electrons, E_h the heat released per photoionization,[14] Λ_{HZ}, Λ_{eZ}, Λ_{eH}, and Λ_{eH^+}, respectively, are the cooling efficiencies by collisions of neutral hydrogen ions and metal atoms,[15,16] electron ions and metal atoms,[16] Lyα emission by neutral hydrogen[17] and hydrogen recombination, on the spot approximation.[9] The metallicity Z is defined such that $n_Y/n_H=YZ$ where n_Y and n_H are the number density of the heavy element with cosmic abundance Y and of the hydrogen atoms, respectively. Therefore, $Z=0$ for a pure hydrogen plasma and $Z=1$ for a plasma with solar abundances. The heavy elements considered are O, C, N, Si, Fe, and S.[16,17]

From Eq. (11) it follows that

$$\tau = (No\ \rho)^{-1}\left[2\ \alpha\xi + (\xi-1)\ \gamma_c + \frac{(1+\phi)}{No\ \rho}\varsigma\right]^{-1}, \tag{13}$$

$$\xi_0' = -\frac{1}{2}\frac{(1+\phi)\varsigma[\gamma_c+2\ \alpha-\sqrt{B}+(1+\phi)\varsigma/No\rho]}{(\alpha+\gamma_c)No\rho^2\sqrt{B}}, \tag{14}$$

where

$$B = \gamma_c^2 + 2(2\alpha+\gamma_c)\frac{(1+\phi)\varsigma}{\rho No} + \left[\frac{(1+\phi)\varsigma}{No\rho}\right]^2.$$

Figure 1 is a plot of the chemical relaxation time [Eq. (2)] as a function of temperature for a photoionized plasma at thermochemical equilibrium $X=0$ and $L=0$ for two representative values of the metallicity, $Z=0$ and 1. For $Z=0$ and the photon energy $E=15$ eV (dash-dot-dot line) and $E=10^2$ eV (dot line). For $Z=1$ the dashed line corresponds to $E=15$ eV and the continuous to $E=10^2$ eV. Note that the range of temperature at which the above equilibria exist strongly depends on the values of the parameters Z and E. The corresponding ionization values as functions of temperature are shown in Fig. 2. For $Z=1$ and low temperatures ($T<10^4$ K) the relaxation time is maintained at a quasicon-

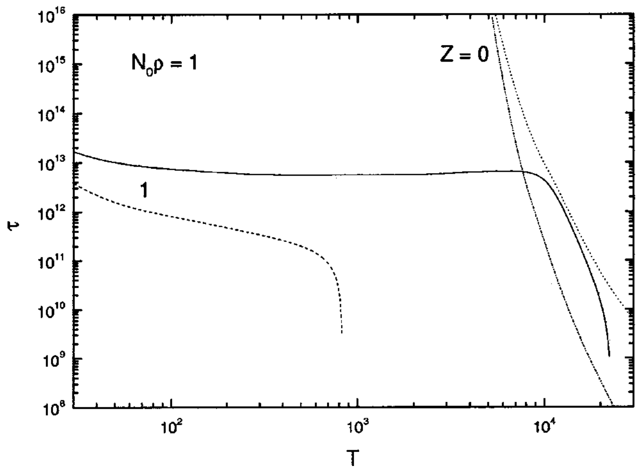

FIG. 1. The chemical relaxation time as a function of temperature for a gas with metallicity $Z=0$, photon energy $E=15$ eV (dash-dot-dot line) and $E=10^2$ eV (dot line). For solar abundances $Z=1$, the dashed line corresponds to $E=15$ eV and the continuous to $E=10^2$ eV.

stant value due to the slow decreasing of the ionization and it sharply decreases when the hydrogen recombination becomes very effective ($T>10^4$ K), whereas for a pure hydrogen plasma ($Z=0$) the relaxation time sharply increases for $T<10^4$ K due to the strong decreasing of the ionization (see Fig. 2). The above effects are due to the contribution to the density of electrons of the heavy metals for plasmas with solar abundances and the lack of these in a pure hydrogen plasma.

Figure 3 is a plot of the viscosity coefficient ν_2 (the set of four upper lines) and the thermometric conductivity χ (the set of four lower lines), corresponding to the representative values for Z and E of Figs. 1 and 2. As it has been shown above, the second viscosity is a complex quantity; therefore, in this figure ν_2 represents the asymptotic case $\omega\tau \ll 1$. The second viscosity is larger than the thermometric conductivity in the range of temperature under consideration, i.e., $30<T<3\times10^4$ K. The above holds for the plasmas with

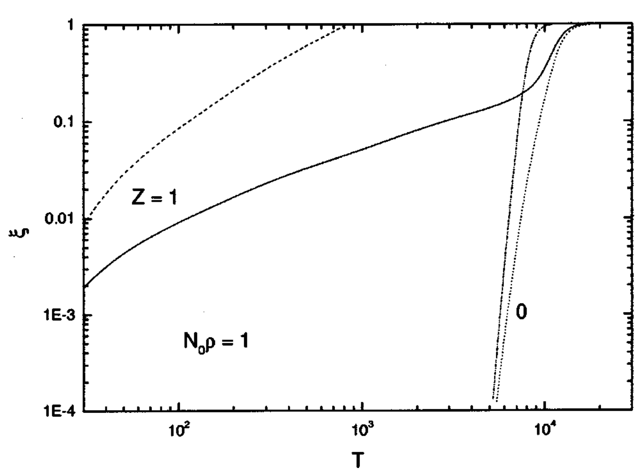

FIG. 2. The equilibrium ionization as a function of temperature for the plasmas of Fig. 1.

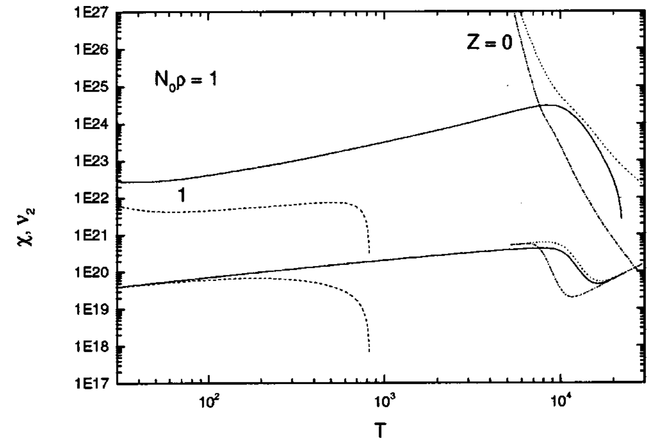

FIG. 3. The thermometric conductivity χ and the second viscosity ν_2 as functions of temperature for the plasmas of Fig. 1.

arbitrary metallicity, in particular, for plasmas with solar abundances ($Z=1$) as well as for a pure hydrogen plasma ($Z=0$).

Figures 4 and 5 show the wave numbers k_1 (straight lines) and k_2 (bent lines) defined by Eqs. (6) and (7) for $Z=0$ and $Z=1$, respectively ($N_0\rho=1$ for both figures) as functions of the dimensionless frequency $\omega\tau$. Figure 4 corresponds to an equilibrium temperature $T=10^4$ K and photon energy $E=15$ eV (dash-dot lines) and $E=10^2$ eV (dotted lines). In Fig. 5, $T=10^2$ K, $E=15$ eV (dotted lines), and $E=10^2$ eV (dashed lines). The continuous lines correspond to $T=10^4$ K and $E=10^2$ eV.

The corresponding damping per unit wavelength k_1/k_2 has been plotted in Fig. 6 as dash-dot-dot lines ($Z=0$, $T=10^4$ K, $E=15$ eV), dash-dot lines ($Z=0$, $T=10^4$ K, $E=10^2$ eV), dot lines ($Z=1$, $T=10^2$ K, $E=15$ eV), dash lines ($Z=1$, $T=10^2$ K, $E=10^2$ eV), and continuous lines ($Z=1$, $T=10^4$ K, $E=10^2$ eV). The strongest damping always occurs at dimensionless frequencies $\omega\tau\approx1$, the exact value depending on the values of the plasma parameters, as expected from physical considerations because the maximum efficiency of

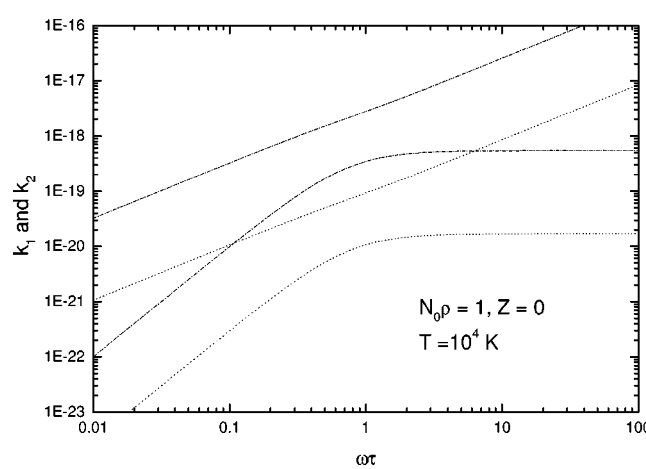

FIG. 4. The wave numbers k_1 (straight lines) and k_2 (bent lines) defined by Eqs. (6) and (7) for $N_0\rho=1$ and $Z=0$ as functions of the dimensionless frequency $\omega\tau$ for an equilibrium temperature $T=10^4$ K and photon energy $E=15$ eV (dash-dot lines) and $E=10^2$ eV (dotted lines).

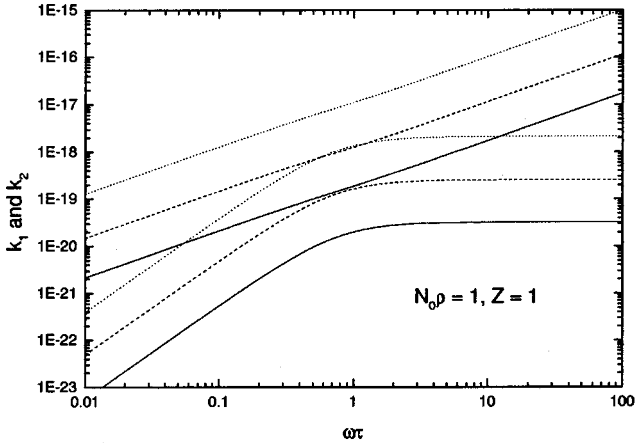

FIG. 5. As Fig. 4 for $Z=1$, $T=10^2$ K, $E=15$ eV (dotted lines), and $E=10^2$ eV (dashed lines). The continuous lines corresponds to $T=10^4$ K and $E=10^2$ eV.

energy transfer from the macroscopic kinetic energy to the random one just occurs when the period of the macroscopic oscillation is close to the recombination time τ. So, sound waves with frequency close to the above value become strongly damped over about 8–10 wavelengths (for the values of the parameters under consideration). The damping strongly decreases for $\omega\tau \ll 1$ and $\omega\tau \gg 1$, which corresponds to the asymptotic cases of a gas in equilibrium ionization at any instant and with a fix ionization, respectively. Additionally, the damping is more effective at high than at low temperatures because such an energy transfer becomes much more effective at high temperatures.

The damping scale length decreases, i.e., the damping becomes more effective when density increases. However, the damping length per wavelength becomes independent of density. Therefore, k_1/k_2 depends only on the plasma temperature for any set of values of the free parameters Z and E which determine the equilibrium values of the ionization and

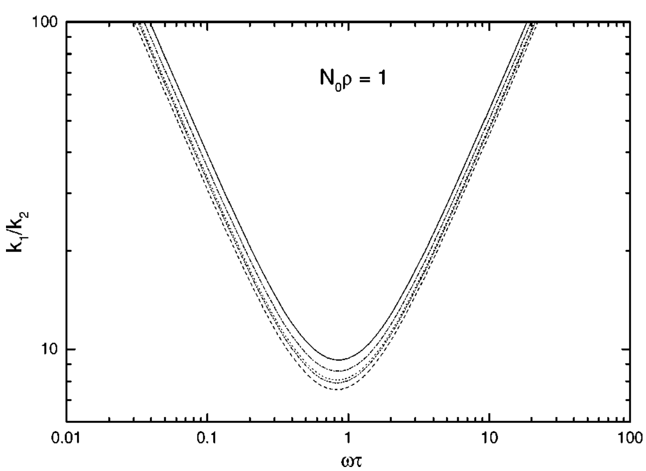

FIG. 6. The damping per unit wavelength k_1/k_2 corresponding to the wave numbers of Figs. 4 and 5 as functions of the dimensionless frequency $\omega\tau$, dash-dot-dot lines ($Z=0$, $T=10^4$ K, $E=15$ eV), dash-dot lines ($Z=0$, $T=10^4$ K, $E=10^2$ eV), dot lines ($Z=1$, $T=10^2$ K, $E=15$ eV), dash lines ($Z=1$, $T=10^2$ K, $E=10^2$ eV), and continuous lines ($Z=1$, $T=10^4$ K, $E=10^2$ eV).

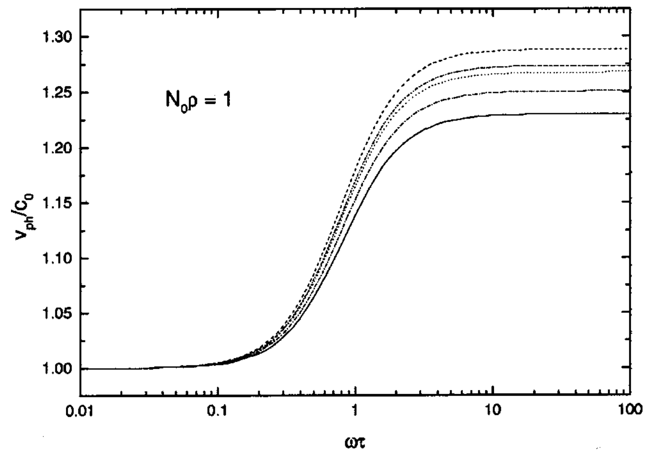

FIG. 7. The dimensionless phase velocities v_{ph}/c_0 as functions of the dimensionless frequency $\omega\tau$ for the same values of parameters used in Fig. 6, i.e., dash-dot-dot lines ($Z=0$, $T=10^4$ K, $E=15$ eV), dash-dot lines ($Z=0$, $T=10^4$ K, $E=10^2$ eV), dot lines ($Z=1$, $T=10^2$ K, $E=15$ eV), dash lines ($Z=1$, $T=10^2$ K, $E=10^2$ eV), and continuous lines ($Z=1$, $T=10^4$ K, $E=10^2$ eV).

the range of temperature where such equilibrium is possible. In particular, low values of E reduce such a temperature interval.

Figure 7 is a plot of the dimensionless phase velocities v_{ph}/c_0 as functions of the dimensionless frequency $\omega\tau$ corresponding to the same values of parameters used in Fig. 6. As it is expected, for $\omega\tau \ll 1$ the dimensionless velocity $v_{ph}/c_0 \to 1$ and for $\omega\tau \gg 1$, $v_{ph}/c_0 \to \beta$, i.e., $v_{ph} \to c_\infty$. Additionally, from Figs. 6 and 7 it follows that the strongest damping occurs at $\omega\tau$ values at which the transition from c_0 to c_∞ occurs.

The cases analyzed above typify the main effects of the second viscosity on the sound wave propagating through the plasma under consideration.

Finally, one could advance that the qualitative effect of the second viscosity on linear thermally unstable fluctuations is, as any irreversible process, to introduce stability. The same should be held in nonlinear stages. However, the calculations of their quantitative effects remain to be analyzed.

IV. SUMMARY

In summary, the second viscosity can be more important than the dynamical viscosity and the thermal conductivity in reacting plasmas. In particular, in photoionized plasmas with metallicity Z the above holds, except at very high temperatures when the plasma becomes completely ionized. The strongest acoustic damping always occurs for frequencies of the order of the inverse of the recombining relaxation time and it increases with temperature. Additionally, such a damping is more efficient for plasmas with solar abundances ($Z=1$) than for a primordial plasma ($Z=0$). In collisionally heated pure hydrogen plasmas the second viscosity becomes zero.

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