SECOND VISCOSITY OF THE GAS IN THE OUTER SOLAR ENVELOPE

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ABSTRACT

We apply the theory of second viscosity to the gas in the outer solar envelope. We have assumed that this viscosity arises because of the finite relaxation time characterizing the exchange of translational energy of electrons and the internal, binding energy of the electrons in hydrogen atoms. Radiative and three-body recombinations with electrons, and charge exchange with H\textsuperscript{-} are assumed to be the dominant processes determining the relaxation times. The second viscosity so calculated is seven orders of magnitude larger than the ordinary viscosity in the upper photosphere (log $P_\odot = 3.2$). Some 2000 km below this level, it is still nearly 2 orders of magnitude larger, according to current estimates. We are unaware of experimental verification that the theory may be applied as we have done here. However, if the application is valid, the results may have a great impact on our understanding of the state of motion of the solar gas, and on heating of the chromosphere and corona. Subject headings: hydrodynamics — Sun: atmosphere — Sun: chromosphere — Sun: corona

I. INTRODUCTION

In a simple fluid, the dynamic viscosity ($\eta$) is essentially the product of the mass density, mean free path, and the thermal speed. However, in compressible fluids consisting of particles with internal energy states, an additional source of viscosity occurs. Landau and Lifshitz (1959, henceforth LL) call this quantity the second viscosity and designate it $\zeta$.

We shall be concerned here with the numerical value of $\zeta$ in the solar photosphere and convection zone (see § V below), which we shall call the outer solar envelope. Our results should also be relevant to solar-type stars. In Bray, Loughhead, and Durrant's (1984, p. 100 [henceforth BLD]) The Solar Granulation, $\zeta$ is briefly considered. These authors state that "atomic relaxation processes are sufficiently rapid to allow us to neglect $\zeta$." In the present paper, we show that this may not be the case.

Viscosity in the outer solar envelope plays a role in the theoretical interpretation of the fluid motions. It is well known that the Reynolds (Re) and Rayleigh (Ra) numbers in the solar convection zone are enormous. In Schwarzschild's (1959) classical description of the balloon photographs, he noted that in spite of the large Rayleigh number, the observed motions appeared to resemble an intermediate, "nonstationary" convection rather than the fully turbulent convection that large Ra might imply.

Recent discussions (see BLD) of the transition to convective turbulent fluid motion treat a variety of "intermediate" domains with the help of Ra and Prandtl numbers. The second viscosity should be included in the calculation of these numbers, and as we shall see, it can reduce Ra by several orders of magnitude.

Viscosity is also important in the Sun and stars because it governs the dissipation of sound. The characteristic length over which a sound wave is damped is inversely proportional to the kinematic viscosity $\nu = \eta/\rho$. For the solar atmosphere, $\nu$ is so small that damping lengths for sound waves are of the order of $10^7$ km for ordinary viscosity (see Osterbrock 1961, eq. [48]). This length is, of course, much larger than the thickness of the region in which energy is being deposited to heat the chromosphere and corona; this is the reason why the damping of ordinary sound waves has been considered unimportant for this process. We shall see that if $\eta/\rho$ is replaced by $\zeta/\rho$, the damping lengths for the upper solar atmosphere become startlingly short!!

II. THE SECOND VISCOSITY

The second viscosity is alternately known as the bulk or dilatation viscosity (see Hirshfelder, Curtiss, and Bird 1964). It is usually thought to apply to fluids consisting of molecules, with rotational and vibrational degrees of freedom. The viscous effects manifest themselves as a result of the interaction of these internal degrees of freedom with translation. Energy is exchanged between the internal and translational motions, on a "relaxation" time scale. In the present study, we consider the exchange of translational energy with the binding energy of hydrogen atoms.

Let us summarize LL's discussion (see § 78) of second viscosity within the context of a gas undergoing ionization. We then associate their quantity $\zeta$, with the degree of ionization of the gas, which we designate $x$. For a pure hydrogen gas (plus ions and electrons),

$$x = N_i/(N_i + N_\text{H}) = N_e/(N_e + N_\text{H}) \ .$$

(1)

The second viscosity arises in the present context because the finite relaxation time for recombination prevents the fluid from instantaneously attaining its equilibrium value of $x$. This relaxation time which LL call $\tau$, will be identified with the recombination time for the electrons. Where collisional ionization is of importance, an analogous ionization-time lag could be relevant, but it will not be considered here. The relaxation time must be considered within the context of other times that characterize the fluid motion. LL's analysis is made for the specific assumption of periodic fluctuations of the fluid, as would occur in a sound wave. The relaxation time should then be compared with the period of the sound waves. It is readily seen, however, that LL's development will retain dimensional or order-of-magnitude validity for less organized motions, for which it is convenient to consider an approximate, "characteristic" time.

For the solar convection zone, the characteristic times are of the order of $l/v$, where $l$ is a pressure scale height and $v$ is a "characteristic" velocity, which we take to be a little less than the sound speed. If we use $l \approx 300$ km and $v \approx 3$ km s\textsuperscript{-1}, the
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characteristic time \( t_e \) is 100 s, which, as we shall see, is longer by many orders of magnitude than most of the relaxation times that we shall calculate.

The fact that typically \( t_e \gg \tau \) is not in itself sufficient to make second viscosity unimportant. Rather, we must compare the second viscosity \( \zeta \) with the gas kinetic, dynamical, coefficient of viscosity, \( \eta \). In the present work, we have used Edmonds (1957) calculations for \( \eta \).

The reciprocal of \( t_e \) is essentially the frequency \( \omega \) of LL, so that for most of the solar envelope, we have the condition \( \omega t_e \ll 1 \). This means that \( \zeta \) is given simply by
\[
\zeta_0 = \frac{\omega \rho(c_o^2 - c_i^2)}{2},
\]
(2)

The quantities \( c_o^2 \) and \( c_i^2 \) are, respectively, \( (\partial P/\partial \rho)_{h,s} \) and \( (\partial P/\partial \rho)_{h} \). Here \( P = P_h \) is the total gas pressure, \( P_e + P_i + P_{ht} \), \( \rho \) is the density, and \( s \) is the entropy. Thus \( c_o \) is the sound speed with the degree of ionization frozen, while \( c_i \) is the sound speed that would apply if equilibrium could be reached instantaneously.

We note that LL's treatment also assumes that the perturbations of the fluid are adiabatic. We make the same assumption in what follows, but assert that our results concerning the second viscosity should also be relevant for nonadiabatic processes, for order-of-magnitude results.

III. EVALUATION OF THE ADIABATIC SOUND SPEEDS

We have evaluated LL's \( c_o^2 \) and \( c_i^2 \) numerically for a mixture of protons, electrons, and neutral hydrogen atoms, using the procedure outlined by Unsöld (1955; see eq. [56,11]). This composition was chosen for computational convenience. Some numerical inaccuracies are thus accepted insofar as the application of the results to real stars is concerned. However, we do not consider these inaccuracies to be acceptable, in view of other approximations as well as the exploratory nature of the present work.

Note that a factor \( (1/kt) \) is missing from Unsöld’s last term in curly brackets. Here, \( k \) is Boltzmann’s constant, and \( T \) is the temperature. The argument in Unsöld’s first logarithmic term is a number per unit volume, which we shall call \( N_j \) for the \( j \)th species. The quotient of the arguments of the two logarithmic terms is then dimensionless. We use a script \( \mathcal{N} \) to denote the (dimensionless) number of particles in the \( j \)th species, which contribute to the entropy \( S_j \) of the \( j \)th species. The correct equation is
\[
S_j = k \mathcal{N} \left\{ \frac{5}{2} \ln(N_j) + 5.80 - \frac{2\pi M_j kT}{h^2} \right\}^{3/2} + \frac{\langle E \rangle}{kT}.
\]

(3)

Here, \( \langle E \rangle \) is the average internal energy of the \( j \)th species.

Equation (3) follows equations (3–3.9) and (3–3.10) given by Cowley (1970). His \( Q \) is the product of a translational and an internal partition function; his \( N \) is the (script) \( \mathcal{N} \) of the current notation. For the electrons and protons, we take \( \langle E \rangle = 0 \). For hydrogen, \( \langle E \rangle \) is very nearly \( -13.6 \text{ eV} = -X_h \), with a small positive correction for occupied higher states. Thus all three species have a common zero-point energy. Note that it is also necessary to use the same zero point in the expression for \( u_h \), the hydrogen partition function. Thus, \( u_h \) is not \( \sim 2 \), but \( \approx 2 \exp \left( \frac{E_h}{kT} \right) \). With these zero points, the last two terms in equation (3) approximately cancel for a cool gas, and the two logarithmic terms dominate. This ensures that \( P \sim \rho^{5/2} \) in the classical, constant-\( S \), low-temperature limit, when the total entropy is dominated by neutral hydrogen.

We neglect both radiation pressure, and Coulomb (Debye) interaction energies. According to Unsöld (equation [56.14]), we have
\[
S_e/kN = 4P_e/(P - P_e),
\]
(4)

where \( S_e \) is the entropy of radiation per massive particle, and \( P_e \) is the radiation pressure. The right-hand side of equation (4) increases rapidly with temperature, but for \( T = 10,000 \text{ K} \) and \( P - P_e = 10^{5} \text{ atm} \), it is \( \approx 10^{-3} \), which should be compared with the corresponding quantity for the gas particles which is in the range 10–100 (see Unsöld's Table 39 or Fig. 93).

The energy per unit volume due to Coulomb interactions may be calculated by the formula given by Landau and Lifshitz (1958, eq. [74.11]). For the conditions of interest to us (\( P \approx 10^{5}, T \approx 10^{9} \)), this energy is less than a few percent of the thermal energy per unit volume of the particles, even if we assume complete ionization.

The relevant quantity to hold constant for a moving, convective element is the sum of the \( S \)'s for a fixed mass of material, or \( S \) per gram. In practice, we calculate and hold fixed the dimensionless ratio
\[
\frac{(S_e + S_i + S_{ht})}{(m_e \mathcal{N}_e + m_i \mathcal{N}_i + m_{ht} \mathcal{N}_{ht})} = \frac{P_e(S_e) + P_i(S_i) + P_{ht}(S_{ht})}{m_e P_e + m_i P_i + m_{ht} P_{ht}}.
\]

(5)

where the quantities in curly brackets on the right-hand side of equation (5) stand for the sum in equation (3), similarly enclosed, for electrons, ions, and hydrogen atoms, respectively. The pressures on the right are partial pressures of electrons, ions, and neutral hydrogens.

Our numerical evaluation begins with an assumed temperature and electron pressure. One obtains the corresponding gas pressure with the help of the Saha equation. Then, for an incremental temperature increase or decrease, the program iterates until that \( P_e \) and corresponding \( P_h \) are obtained for which the total entropy per gram is a constant. By numerical differentiation, we obtain the equilibrium sound speed, or \( c_o \).

The quantity \( c_o \) follows from similar numerical differentiation of the \( P - \rho \) relations provided we freeze the degree of ionization relative to that obtained after the initial entry of \( P_e \) and \( T \). Because the gas now behaves as a mixture of noninteracting particles with a constant number density, the result for \( c_o \) is very nearly \( \gamma P/\rho \), with \( \gamma = 5/3 \).

IV. RECOMBINATION TIME

Recombination can take place by a variety of channels: radiative, three-body, charge exchange, and so on. In general, one adds the rates for processes so that the effective recombination time \( \tau \) is given by
\[
1/\tau = \sum (1/\tau_i),
\]
(6)

where the summation is over individual processes "i." The most rapid processes of relevance in the solar convection zone that we know are charge exchange of protons with the \( H^- \) ion, three-body recombinations involving two electrons, and in the upper layers of the model, radiative recombinations.

Fussen and Kubach (1986) summarize the theoretical and experimental results for the charge exchange cross sections of \( \text{H}^+ - \text{H}^- \) collisions. They also give a useful interpolation formula (see their eq. [6]), which we have used to calculate the average rate coefficient \( \langle \sigma v \rangle \) by numerical integration over the Maxwell-Boltzmann distribution. The reciprocal of the recom-

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bination time by charge exchange with $H^- \text{ is } N(H^-)\langle \sigma v \rangle$. We have assumed that $N(H^-)$ could be obtained from an LTE calculation using the Saha Equation.

Bates, Kingston, and McWhiter (1962a, henceforth BKM) have calculated radiative and three-body recombination rates under a variety of "optically thick" conditions. We have chosen to work primarily with their results for a plasma optically thick in all lines. This condition should be met for the important early Lyman and Balmer lines in all but the very highest layers concerned here. We shall briefly consider possible changes if results for an optically thin plasma are used.

BKM write the recombination coefficient, usually designated by an $z$ (cm$^3$ s$^{-1}$) as $N_e k_i + a_i$, where $N_e$ is the number density of electrons. The first term represents the three-body recombinations, and the second, the radiative, under the assumed conditions. Between 4000 and 64,000 K, the BKM parameters $k_i$ and $a_i$ may be fitted by interpolation formulae, viz.

$$\log (a_i) = -11.735 - 1.264 \log (T) + 0.2295 \left[ \log (T) \right]^2,$$

and

$$\log (k_i) = -19.868 - 8.948 \log (T) + 2.429 \left[ \log (T) \right]^2.$$

The maximum error of these fits is 0.031 for log($a_i$) and 0.047 for log($k_i$) between 4000 and 64,000 K.

V. MODEL-NUMERICAL RESULTS

We have adopted physical depths $z$, temperatures $T$, and densities $\rho$ from Table 2 on p. 440 of the Landolt-Börnstein Tables (see Böhm-Vitense 1965). We then calculated appropriate gas and electron pressures for a pure hydrogen mixture. The $H^-$ number densities were also calculated for a pure hydrogen mixture. Since our primary focus is on the viscosity of the gas, the resulting changes in the pressure-structure of the gas from that of the Sun will be of secondary importance.

Table 1 gives the physical depth $z$, the temperature, density, logarithms of the gas and electron pressures, as well as number densities of electrons or ions and the number densities of $H^-$ ions in LTE for the pure hydrogen mixture.

The uppermost layers ($z = 50$ and 250 in Table 1) correspond to the upper solar photosphere: optical depths at 5000 Å of $\approx 10^{-3.5}$ and $10^{-2}$, according to Gingerich et al. (1971). In their model, when log($P_\rho$) = 3.24, log($P_e$) = -0.922, while the temperature is 4250 K. Our value for log($P_\rho$) = -1.15. The two values of log($P_\rho$) result from the different temperatures and compositions. Our neglect of the electron-donating metals has been somewhat compensated by the higher temperature.

Neglect of the electron-donating metals should be most serious in the higher layers of our model, where the degree of ionization is lowest. At these depths, radiative and $H^-$ charge-exchange recombinations are more important than those due to three-body collisions. The net recombination rate will therefore scale directly as the electron density, which in our model is only about $-0.92 - (-1.15) \approx 0.23$ dex too small. In the deeper layers, where three-body processes are more important, the metals are no longer the principal electron donors, and a pure hydrogen approximation should improve. Naturally, more detailed calculations should be made including both realistic abundances and the ionization log, when three-body interactions dominate the rates.

The radiative gradient exceeds the adiabatic gradient when log($P_\rho$) = 5.1, which falls between the entries for $z = 750$ and 850 km in Table 1. This is the point at which convection would begin, according to the classical Schwarzschild criterion.

In Table 2, we give the rate coefficients for $H^-$, reciprocal recombination times for charge exchange with $H^-$, and combined radiative and three-body rates [1/$\tau(BKM)$]. The three body rates dominate throughout the deeper layers of the model. The rates that are dominated by radiative recombination are marked in the table with an "r." At 450 km, the two rates are within 10% of one another. We also give the reciprocal of the combined rate, $\tau$, the quantities $c_\infty^2$ and $c_\infty^2$, and the second (dynamic) viscosity $\zeta = \zeta_\infty$ from equation (2). The final column of Table 2 gives Edmonds’s kinematic viscosities, for the temperatures and gas pressures of Table 1 obtained by quadratic interpolation in his Table 1.

It can be seen that for all depths in the model, the second viscosity is much larger than the value calculated by Edmonds. For the two upper layers, $\zeta/\eta \approx 10^7$. Even if we allow for an extra order of magnitude in the $H^-$ number density due to contributions from metals in the Sun, the ratio $\zeta/\eta$ would be six orders of magnitude!!

### Table 1

<table>
<thead>
<tr>
<th>$z$(km)</th>
<th>$T$(K)</th>
<th>$\rho$(gm/cm$^3$)</th>
<th>log($P_\rho$(cgs))</th>
<th>log($P_e$(cgs))</th>
<th>$N_e = N_i$ cm$^3$</th>
<th>$N(H^-)$ cm$^3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>4700</td>
<td>4.9(–9)</td>
<td>–1.15</td>
<td>3.28</td>
<td>1.1(11)</td>
<td>6.6(5)</td>
</tr>
<tr>
<td>250</td>
<td>4700</td>
<td>4.0(–8)</td>
<td>–0.70</td>
<td>4.19</td>
<td>3.1(11)</td>
<td>1.6(7)</td>
</tr>
<tr>
<td>450</td>
<td>5660</td>
<td>2.7(–7)</td>
<td>1.09</td>
<td>5.09</td>
<td>1.6(13)</td>
<td>2.9(9)</td>
</tr>
<tr>
<td>550</td>
<td>8190</td>
<td>3.2(–7)</td>
<td>3.28</td>
<td>5.34</td>
<td>1.7(15)</td>
<td>1.3(11)</td>
</tr>
<tr>
<td>650</td>
<td>11000</td>
<td>4.0(–7)</td>
<td>4.60</td>
<td>5.61</td>
<td>2.6(16)</td>
<td>1.1(12)</td>
</tr>
<tr>
<td>750</td>
<td>12100</td>
<td>4.7(–7)</td>
<td>4.97</td>
<td>5.75</td>
<td>5.6(16)</td>
<td>2.0(12)</td>
</tr>
<tr>
<td>850</td>
<td>12900</td>
<td>5.6(–7)</td>
<td>5.21</td>
<td>5.88</td>
<td>9.1(16)</td>
<td>3.0(12)</td>
</tr>
<tr>
<td>950</td>
<td>13800</td>
<td>6.6(–7)</td>
<td>5.42</td>
<td>6.01</td>
<td>1.5(17)</td>
<td>4.4(12)</td>
</tr>
<tr>
<td>1200</td>
<td>15300</td>
<td>1.1(–6)</td>
<td>5.82</td>
<td>6.30</td>
<td>3.1(17)</td>
<td>9.9(12)</td>
</tr>
<tr>
<td>1500</td>
<td>17000</td>
<td>1.7(–6)</td>
<td>6.17</td>
<td>6.59</td>
<td>6.2(17)</td>
<td>1.9(13)</td>
</tr>
<tr>
<td>2000</td>
<td>19400</td>
<td>3.3(–6)</td>
<td>6.58</td>
<td>6.95</td>
<td>1.4(18)</td>
<td>4.5(13)</td>
</tr>
</tbody>
</table>

* The parenthesized entries give the power of 10 to be applied.
### Table 2

**Kinetic Coefficients in the Convection Zone**

<table>
<thead>
<tr>
<th>$x$ (km)</th>
<th>$\langle \sigma v \rangle$</th>
<th>$1/\tau$ (H$^-$)</th>
<th>$1/\tau$ BKM</th>
<th>$1/\tau$ s$^{-1}$</th>
<th>$c_{\infty}^2$ cm$^2$ s$^{-2}$</th>
<th>$c_0^2$ cm$^2$ s$^{-2}$</th>
<th>$\zeta$ (gm cm$^{-1}$ s$^{-1}$)</th>
<th>$\eta$ (gm cm$^{-1}$ s$^{-1}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>4.5(-8)</td>
<td>3.0(-2)</td>
<td>3.7(-2)</td>
<td>6.6(-2)</td>
<td>6.4(11)</td>
<td>5.6(11)</td>
<td>5.7(3)</td>
<td>4.1(-4)</td>
</tr>
<tr>
<td>250</td>
<td>4.6(-8)</td>
<td>6.9(-1)</td>
<td>1.1(-1)</td>
<td>8.0(-1)</td>
<td>6.4(11)</td>
<td>5.6(11)</td>
<td>4.1(3)</td>
<td>4.1(-4)</td>
</tr>
<tr>
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<td>1.2(2)</td>
<td>8.3(0)</td>
<td>3.2(4)</td>
<td>7.8(11)</td>
<td>6.8(11)</td>
<td>2.0(3)</td>
<td>4.7(-4)</td>
</tr>
<tr>
<td>550</td>
<td>3.8(-8)</td>
<td>5.0(3)</td>
<td>6.8(3)</td>
<td>1.2(4)</td>
<td>1.1(12)</td>
<td>9.1(11)</td>
<td>5.7(0)</td>
<td>6.2(-4)</td>
</tr>
<tr>
<td>650</td>
<td>3.5(-8)</td>
<td>4.0(4)</td>
<td>3.5(5)</td>
<td>3.9(5)</td>
<td>1.6(12)</td>
<td>1.2(12)</td>
<td>4.4(-1)</td>
<td>6.2(-4)</td>
</tr>
<tr>
<td>750</td>
<td>3.5(-8)</td>
<td>7.0(4)</td>
<td>9.9(5)</td>
<td>1.1(6)</td>
<td>1.9(12)</td>
<td>1.4(12)</td>
<td>2.4(-1)</td>
<td>6.2(-4)</td>
</tr>
<tr>
<td>850</td>
<td>3.4(-8)</td>
<td>1.0(5)</td>
<td>2.0(6)</td>
<td>2.1(6)</td>
<td>2.2(12)</td>
<td>1.6(12)</td>
<td>1.6(-1)</td>
<td>6.2(-4)</td>
</tr>
<tr>
<td>950</td>
<td>3.4(-8)</td>
<td>1.5(6)</td>
<td>3.8(6)</td>
<td>3.9(6)</td>
<td>2.5(12)</td>
<td>1.8(12)</td>
<td>1.2(-1)</td>
<td>5.7(-4)</td>
</tr>
<tr>
<td>1200</td>
<td>3.6(-8)</td>
<td>3.5(6)</td>
<td>1.1(7)</td>
<td>1.2(7)</td>
<td>3.0(12)</td>
<td>2.2(12)</td>
<td>7.7(-2)</td>
<td>5.1(-4)</td>
</tr>
<tr>
<td>1500</td>
<td>3.3(-8)</td>
<td>6.4(5)</td>
<td>2.0(7)</td>
<td>3.0(7)</td>
<td>3.7(12)</td>
<td>2.8(12)</td>
<td>5.3(-2)</td>
<td>4.6(-4)</td>
</tr>
<tr>
<td>2000</td>
<td>3.3(-8)</td>
<td>1.5(6)</td>
<td>9.2(7)</td>
<td>9.3(7)</td>
<td>4.5(12)</td>
<td>3.5(12)</td>
<td>3.6(-2)</td>
<td>4.8(-4)</td>
</tr>
</tbody>
</table>

VI. Alternate Rates

Bates, Kingston, and McWhirter (1962b) give recombination coefficients for *optically thin* plasmas. Their Table 2A gives combined collisional-radiative recombination coefficients for singly charged ions as a function of electron temperature and density under the condition that all radiation can escape. For low electron densities, where three-body recombinations are negligible, this condition is known to astronomers as (Baker and Menzel's [1938]) case A, and the entries in Bates, Kingston, and McWhirter approach, for example, the familiar values listed by Osterbrock (1974, Table 2.1) for case A, in the limit as $N_e$ goes to zero.

It seems clear that the optically thin case is *not* relevant to the solar model we have considered. Nevertheless, these values give insight into the possible changes if our assumption of optically thick lines were to be relaxed. The rates for an optically thin gas are higher, than those for the gas that is optically thick in all of the lines, but the largest difference is only a factor of $\sim 6$, at the top of the model, where $\zeta$ is many orders of magnitude larger than $\eta$. In the deepest layers, the two coefficients approach one another, although, possibly, somewhat slower than one might have expected, granted that three-body recombinations dominate throughout most of the optically thick model. At 2000 km, the two rates are within 10% of one another.

We have assumed here that three-body recombinations involving a proton, electron, and hydrogen atom are much slower than those of two electrons and a proton, which were explicitly included in the rates of BKM.

We found no experimental or theoretical rates for the process H$^+$ + H$^+$ $\rightarrow$ H$^+$. The inverse process, the self ionization of hydrogen has been studied experimentally by Gealy and van Zyl (1987a, b), although for energies far exceeding those of interest to us. We made the assumption that the relevant cross section could be extrapolated from the last experimental point to threshold as the energy squared. It was then possible to use the principle of detailed balancing to obtain the desired rate (see Osterbrock 1989, eqs. [4.38] and [4.39]). We found the rates for H$^+$ + H$^+$ $\rightarrow$ 2H to be many orders of magnitude below those given in Table 2.

We are unaware of experimental verification of the application of the LL formalism to ionization. While we can think of no reason why the present analysis should not apply to the viscosity of the outer solar envelope, it is well to withhold final judgement until laboratory work has investigated the phenomena.

If we use $\zeta/\rho$ instead of $\eta/\rho = v$ in the formulae for the Reynolds and Rayleigh numbers, both will decrease directly by the ratios $\zeta/\eta$ of in the last columns of Table 2. These decreases, although substantial, do not affect the general conclusion that the solar convection zone is highly unstable. However, they do influence conclusions drawn about the degree of instability and may account for the resemblance of the solar motions to an intermediate rather than a fully turbulent convection. We recognize that any realistic characterization of the motions in the solar atmosphere must also include the magnetic field.

We note, finally, that the damping lengths for sound waves will decrease by the ratios $\zeta/\eta$. If we naively apply this ratio to the figure $10^7$ km quoted by Osterbrock (1961), we obtain a damping length of 1 km!! It is beyond the scope of this paper to apply the present results to the old problem of heating of the chromosphere and corona. We merely remark that if the LL theory is relevant, it could clarify many aspects of this complicated and puzzling phenomena.

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