For vectors $\mathbf{u}$ and $\mathbf{B}$, to $O(u/c)$:

Reynolds stress tensor

$$R_{ik} = \rho u_i u_k$$  \hspace{1cm} (1)

Maxwell stress tensor

$$M_{ik} = \frac{1}{\mu_0} \left( B_i B_k - \frac{1}{2} \delta_{ik} B^2 \right)$$  \hspace{1cm} (2)

Equation of motion, including pressure tensor $p$, external (gravity) force $\mathbf{g}$,

$$F_i = \frac{\partial}{\partial t} \rho u_i = \frac{\partial}{\partial x_k} \left( M_{ik} - R_{ik} - p_{ik} \right) + \rho g_i$$  \hspace{1cm} (3)

Reynolds and Maxwell stresses are quadratic in $\mathbf{u}$, $\mathbf{B}$. Hydrodynamics supports shocks, MHD also supports tangential discontinuities.

Plasma $\beta = \text{gas pressure to magnetic pressure} = \text{trace} p_{ij} / \text{trace} M_{ik}$.

Similarly, ratio of Reynolds to Maxwell stresses is $\text{trace} R_{ij} / \text{trace} M_{ik}$.
Explicitly:

Equation of motion

\[ F_i = \frac{\partial}{\partial t} \rho u_i = \frac{\partial}{\partial x_k} \left\{ \frac{1}{\mu_0} \left( B_i B_k - \frac{1}{2} \delta_{ik} B^2 \right) - \rho u_i u_k - p_{ik} \right\} + \rho g_i \]  

(4)

Force free equilibrium

\[ \frac{\partial}{\partial t} \rho u_i = 0 = \frac{\partial}{\partial x_k} \left\{ \frac{1}{\mu_0} \left( B_i B_k - \frac{1}{2} \delta_{ik} B^2 \right) \right\} \]  

(5)

\[ \nabla^2 \mathbf{B} + \alpha^2 \mathbf{B} = \mathbf{B} \times \text{grad} \alpha \]  

(6)

Complex characteristics. But

\[ \mathbf{B} \cdot \text{grad} \alpha = 0 \]  

(7)

\( \alpha = \) real constant along field lines: real characteristics. There is no requirement that \( \alpha(\mathbf{r}) \) (a measure of circulation or torque) be continuous from one field line to the next.
Dot product with $\mathbf{u}$ gives, with continuity equation, the mechanical energy equation

$$\frac{\partial}{\partial t} U = - \frac{\partial}{\partial x_k} (P_k + \rho u^2 u_k) + \rho g u_k$$ (8)

Energy density

$$U = \frac{1}{2} \rho v^2 + \frac{1}{2 \mu} B^2,$$ (9)

and Poynting vector

$$\mathbf{P} = \mathbf{E} \times \mathbf{H}$$ (10)
Switch on 2D motions (Parker 2007, p 122) between two plates with \( \sigma = \infty \): 

\[ \begin{align*}
  v_x &= +kz \frac{\partial \psi}{\partial y} \\
  v_y &= -kz \frac{\partial \psi}{\partial x} \\
  v_z &= 0
\end{align*} \]  \hspace{1cm} (11)

Frozen fields mean 

\[ 
B_x(t) = +Bkt \frac{\partial \psi}{\partial y} \quad B_y(t) = -Bkt \frac{\partial \psi}{\partial x} \quad B_z(t) = B, \]  \hspace{1cm} (12)

where \( \psi(x, y, zkt) \) is an arbitrary, bounded, \( n \)-times differentiable function of its arguments.