The Physical Problem

Let us consider an incompressible viscous magnetofluid under the condition of perfect electrical conductivity initially at an equilibrium. For simplicity, let us further assume that the fluid lines are to be understood so that they can be represented by two independent family of global flux surfaces.

- Infinite conductivity guarantees invariance of magnetic topology as the fluid evolves with time in response to the non-linear Lorentz force. Magnetic energy gets converted to kinetic energy of flow and is lost irrecoverably through viscous dissipation.
- Magnetic field being frozen into the plasma (infinite conductivity) can not vanish entirely and the system will reach a minimum magnetic energy equilibrium state with time.
- The pre-specified condition for equilibrium is that a set of flux surfaces are also isotropic surfaces. But from early writings of Grad [1] to the Parker theory of spontaneous current sheet formation [2], it is well known that the organization of these particular global flux surfaces into isotropic surfaces is generally not possible for three-dimensional magnetic field. Yet the viscous flow must tend to the field evolution as the field runs out of free energy under the frozen-in condition.
- The minimum energy equilibrium state, in general, is not continuous in space. As the field approaches its terminal state extreme gradients in fluid displacement and tangential discontinuity in magnetic field or current sheets form discontinuously.

This work presents the first and simplified study in a series of investigations to understand the dynamical evolution of the plasma as itprogresses toward the formation of current sheets under frozen-in condition.

Governing Equations

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\begin{align*}
\frac{\partial \mathbf{v}}{\partial t} - \nabla \times (\mathbf{v} \times \mathbf{B}) & = \frac{\mathbf{J}}{\rho} + \mathbf{f} \\
\nabla \cdot \mathbf{v} & = 0
\end{align*}
\]

- Incompressible flow
- To avoid unnecessary complications due to internal fluid energy

As two fluid elements pertaining to two different portions of the same flux tube or two different flux tubes with their individual embedded magnetic field press into each other under frozen-in conditions, the preservation of topology prohibits them from interacting. Tangential discontinuities thus develop on the surface separating the interacting flux surfaces.

Initial and boundary conditions

- Trivial periodic boundary condition to avoid complications of boundary walls
- Curvilinear geometry employed

Computational Model

- A variant of the multi-scale computational fluid model EULAG [4] customized for the given problem
- The governing equations are solved numerically using a non-oscillatory forward-in-time (NFT) approach [5]
- The template algorithm represents a system implicit with respect to pressure and all velocity components.

Refined numerical dissipation steps in locally and breaks the flux surface) points to the formation of current sheets in the process (Figures 6 and 7).

Results and Discussions (Contd.)

- The initial Euler surfaces are periodic cylinders with non-circular cross-sections (Figure 1).
- Magnetic field lines on a given \( \xi \) constant surface is defined by its intersection with constant \( \zeta \) surfaces of different radii and vice versa.
- All the field lines are closed with O-points lying along the cylindrical axis. Also for a given cylinder, there are three X-points one at each cap and one at the middle (Figure 2).

References