# OBSERVATIONAL CONSTRAINTS ON THE DYNAMICAL PROPERTIES OF THE SHEAR LAYER AT THE BASE OF THE SOLAR CONVECTION ZONE

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## 1. INTRODUCTION

Although many helioseismic observations have shown that a shear layer exists at the base of the convection zone, it has been difficult to establish the precise physical properties of this region. This shear layer is important as the probable seat of the solar dynamo, therefore, observational determinations of the depth and thickness of this layer are critical in constraining dynamo theories. A recent determination of the properties of the shear layer by Kosovichev (1996) estimated that the layer has a thickness of .09 $\pm$.04 $R_\odot$ and that the bulk of the shear lies below the convection zone. We have employed frequency splitting data obtained with the LOWL instrument to tighten the constraints on the dynamical properties of this shear layer. We present here constraints on the depth and thickness of the shear layer resulting from analyses of the data by both inverse and forward methods.

## 2. OBSERVATIONS

The observations used for this study were obtained with the LOWL instrument (Tomczyk *et al.*, 1995a). The observations were taken over a two year period from 1994 Feb 26 until 1996 Feb 25. Mode parameters were derived for 1102 modes with degrees ranging from 1 to 99 and frequencies ranging from 1 to 3.5 mHz. The variation of mode frequency with azimuthal order was expanded in a set of $a$-coefficients following Schou *et al.*(1994). The odd $a$-coefficients are related to the rotation rate $\Omega$ at radius $r$ and colatitude $\theta$ by

$$2\pi\, a_{2s+1}(n,l) = \int_{r,\theta} K^{(a)}_{nls}(r,\theta) \qquad (1)$$

(Schou *et al.* 1994). Then, the frequency splittings are averages of the rotation rate weighted by the kernels $K^{(a)}_{nls}(r,\theta)$ which are assumed to be known functions of the solar structure. The $\epsilon_{nls}$ represent data errors. For this study we have available the odd $a$-coefficients ranging from $a_1$ to $a_5$.

## 3. FORWARD METHODS

Given observed frequency splittings, one can employ Eq. 1 along with the known kernels to compute the solar rotation profile. This is usually done using inversion methods, which are discussed in the next section. It is also possible to solve for the rotation profile using Eq. 1 and forward methods. These methods generally make a guess at the rotation profile, compute the frequency splittings via Eq. 1, evaluate the goodness of the solution against observed splittings, then iterate the solution while minimizing the difference with observations.

We have performed such forward calculations using minimization algorithms based on genetic (Charbonneau, 1995; Tomczyk *et al.*, 1995b) and simulated annealing (Press *et al.*, 1986) algorithms. We will discuss only the simulated annealing results here.

To simplify the forward modelling, we have constrained the possible rotation profiles by parameterizing the angular velocity as:

$$\Omega(r,\theta) = \Omega_c + 0.5[1. + erf(\frac{r-r_0}{0.5w})][\Omega_s - \Omega_c] \qquad (2)$$

where $\Omega_c$ is the core rotation rate which is assumed to be independent of depth and latitude, $erf$ is the error function which parameterizes the transition at $r_0$ from the constant core rotation to the surface-like rotation given by $\Omega_s$, $w$ is the full-width of the transition, $r$ is the distance from the solar center, and $\theta$ is the co-latitude. Also,

$$\Omega_s = \Omega_{eq} + Acos^2\theta + Bcos^4\theta \qquad (3)$$

where $\Omega_{eq}$ is the equatorial rotation rate and A and B are the constants of the surface differential rotation.

To limit the computation to the region of the Sun where this simple 6 parameter model is valid, we restricted the modeset to modes with lower turning point radii between 0.5 and .8 $R_\odot$. Also, we restricted the mode frequencies to be between 2.0 and 3.5 mHz. At each iteration we minimize $\chi^2$ defined as:

$$\chi^2 = \sum_{n=1}^{N} \left(\frac{A_n - a_n}{\sigma_n}\right)^2, \qquad (5)$$

where the $A_n$'s are the observed frequency splittings, the $a_n$ are the computed frequency splittings, and $\sigma_n$ is the error associated with the $n^{\rm th}$ frequency splitting.

To evaluate the forward and inverse methods, we have generated sets of artificial frequency splittings based on the above parameterization and Eq. 1. Noise was added by perturbing the computed splitting values with gaussian errors having standard deviations given by the observational errors for that mode. Numerical experiments showed that the forward method converged to the correct solution for a variety of positions and widths of the shear layer. Also, the forward method recovered very narrow transitions with widths much below the resolution limit of the linear inversion methods.

We applied the forward model to the LOWL 2-year dataset using only the $a_3$ coefficients in the computation of the $\chi^2$ (following Kosovichev, 1996). In this case the method converged to the solution:

$r_c$=0.704 ± .003 $R_\odot$ $w$=.0498 ± .012 $R_\odot$

The quoted errors on the rotation parameters were derived from a Monte Carlo computation.

## 4. INVERSE METHODS

The data have been inverted using two 'traditional' inversion techniques, a 2D Subtractive Optimally Localized Averages (SOLA) method and a 2D Regularized Least Squares (RLS) approach. Some details of the SOLA inversion method are given in another poster (W.J. Chaplin et al., *Rotation of the solar core*): combinations of the data were taken so as to construct averaging kernels resembling 2D Gaussians of the form

$$\exp\left[-(r-r_0)^2/\Delta_r^2\right]\exp\left[-(\theta-\theta_0)^2 r_0^2/\Delta_\theta^2\right] , \quad (6)$$

normalized to have unit area. Here $(r_0, \theta_0)$ is the chosen target location, and $\Delta_r$,$\Delta_\theta$ are parameters which control the spread of the averaging kernel. We took $\Delta_r = 0.04R_\odot$ and $\Delta_\theta = 0.2R_\odot$, where $R_\odot$ is the solar radius. The results of the inversion are shown in Fig. 1.

The 2D RLS inversion is described by Schou et al. 1994 (cf the poster by W.J. Chaplin et al.): we chose trade-off parameter values $\mu_r = \mu_\theta = 10^{-6}$. The results of the inversion are shown in Fig. 3.

In addition to the above inversions for the rotation rate $\Omega(r,\theta)$, we also performed a SOLA inversion for the radial gradient of the rotation, $\partial\Omega/\partial r$. The results of the inversion for the rotation gradient can be seen in Fig. 4.

## 5. INTERPRETATION OF THE INVERSION RESULTS

The result of the inversion at any point is a convolution of the true rotation profile with the averaging kernel (plus propagated data noise). This even if the base of the convection zone were a sharp discontinuity, the finite extent of the averaging kernels would produce a smoothed-out transition in the inversion solution. This can be quantified as follows. For simplicity, suppose that the transition has the form

$$\int_{-\infty}^{r} \exp[-4(r'-r_b)/(w_b)^2]\,\mathrm{d}r' \quad (7)$$

(cf. Kosovichev 1996): this is a step from zero to unity, and $w_b$ is the width over which the step goes from 0.08 to 0.92 If the averaging kernel is Gaussian with radial width $\Delta_r$, then the inversion solution has a step of the same form as the true transition but with a width $(w_b^2+4\Delta_r^2)^{1/2}$ instead of $w_b$. Clearly, in the limit of very fine resolution (small $\Delta_r$) the width of the transition would be correctly recovered, while in the limit $\Delta_r \gg w_b$ the width of the transition in the solution would be $2\Delta_r$, independent of the width of the true transition.

Knowing the radial width $\Delta_r$ of the averaging kernel, one can use the above formula to attempt to estimate the true width $w_b$ from the width $(w_b^2+4\Delta_r^2)^{1/2}$ of the transition exhibited by the inversion solution. To test how successful this procedure is, we have used the artificial splitting data which was generated as described above. These data were inverted in 'blind' fashion, i.e. the inverters were not told what the artificial rotation profiles looked like. From the SOLA inversions of the artificial data, the width of the transition was measured and, using the above formula with $\Delta_r = 0.04R_\odot$, the width $w_b$ of the transition in the artificial profile was estimated. The results of this blind experiment were:

## 6. RESULTS OF BLIND INVERSIONS OF ARTIFICIAL DATA

| Case | Measured transition | Inferred $w_b$ | Actual $w_b$ |
|---|---|---|---|
| 0 | 0.14 | <0.06 | $\simeq$0 |
| 1 | 0.14 | <0.06 | $\simeq$0 |
| 2 | 0.12 | <0.04 | $\simeq$0 |
| 3 | 0.20 | 0.09 | 0.10 |
| 4 | 0.16 | 0.07 | 0.10 |
| 5 | 0.16 | 0.07 | 0.10 |
| 6 | 0.23 | 0.11 | 0.20 |
| 7 | 0.31 | 0.15 | 0.20 |
| 8 | 0.19 | 0.09 | 0.20 |

In the cases of rotation profiles 0-2, the inverters inferred from their inversions for $\Omega$ and $\partial\Omega/\partial r$ together that the transition was not resolved. (If the transition is very narrow, the gradient at the transition will resemble a Dirac delta function, and the inferred gradient will be given simply in terms of the magnitude of the step and the amplitude of the averaging kernel. If the inferred gradient is indistinguishable from this value, then the inverters judged the transition to be too thin to be resolved.) This conclusion was shown to be correct when the 'truth' was revealed! In the other cases, the inverters' tended to underestimate the width of the transition.

The results of exactly the same procedure, applied to the real LOWL data, give a measured transition of $0.10R_\odot$ at the equator and $0.14R_\odot$ at a latitude of 60°, which imply values of $w_b = 0.03R_\odot$ and $0.06R_\odot$ respectively. However, by the same criteria as before, the inverters judge that the transition is unresolved, as in cases 0-2 above.

If the width of the transition were as great as $0.10R_\odot$, the blind experiment indicates that we would judge the transition to be resolved by the SOLA inversion. In fact, the transition appears to be unresolved and we conclude that the transition is less than $0.10R_\odot$. Indeed, if the inferred values are to be believed, then $0.06R_\odot$ is an upper bound on the width of the transition.

We note that the inversions also indicate that the transition appears to be slightly deeper at the equator than at higher latitudes, *i.e.* the shape of the transition is prolate (Figs 2,3). (Such a prolate figure was also unambiguously detected in a further blind experiment.) The prolate transition region appears not only in the inversions of the full 2-year dataset but also in inversions of the first and second years of LOWL data separately.

## 7. SUMMARY

We have employed forward modelling and inverse methods to derive estimates for the thickness and position of the shear layer at the base of the convection zone. Forward modelling yields values for these parameters of

$0.050 \pm .012\ R_\odot$ and $0.704 \pm .003\ R_\odot$, respectively. The value of the width is lower than the estimate of Kosovichev (1996) by one of his standard deviations. The value of the depth is consistent with his estimate and confirms that the bulk of the shear lies below the base of the convection zone. The value of $0.050\ R_\odot$ is consistent with upper limits set by linear inversions, which also indicate a transition thickness which is less than $0.10 R_\odot$ and possibly less than $0.06 R_\odot$. However, we note that the forward method employed here assumes spherical symmetry in the depth of the transition, as did the method of Kosovichev (1996). If, as our inversions indicate, this assumption is not correct, then forward modelling based on such simple models will overestimate the thickness of the transition. We conclude that forward modelling is a valuable complement to inverse methods. However, the constrained nature of the forward method requires care in the interpretation of the results.

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