OBSERVATIONAL CONSTRAINTS ON THE DYNAMICAL PROPERTIES OF THE SHEAR LAYER AT THE BASE OF THE SOLAR CONVECTION ZONE

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1. INTRODUCTION

Although many helioseismic observations have shown that a shear layer exists at the base of the convection zone, it has been difficult to establish the precise physical properties of this region. This shear layer is important as the probable seat of the solar dynamo, and hence observational determinations of its depth and thickness are critical in constraining dynamo theories. A recent determination of the properties of the shear layer by Kosovichev (1996) estimated a thickness of .09 $\pm$.04 $R_\odot$ and that the bulk of the shear lies below the convection zone. We have employed frequency splitting data obtained with the LOWL instrument to tighten the constraints on the dynamical properties of this region. We present here constraints on the depth and thickness of the shear layer resulting from analyses of the data by both inverse and forward methods.

2. OBSERVATIONS

The observations used for this study were obtained with the LOWL instrument (Tomczyk *et al.*, 1995a). The observations were taken over a two-year period from 1994 Feb 26 until 1996 Feb 25. Mode parameters were derived for 1102 modes with degrees ranging from 1 to 99 and frequencies ranging from 1 to 3.5 mHz. The variation of mode frequency with azimuthal order was expanded in a set of a-coefficients following Schou *et al.* (1994). The odd a-coefficients are related to the rotation rate Ω at radius r and colatitude θ by:

$$2\pi\, a_{2s+1}(n,l) = \int_{r,\theta} K_{nls}^{(a)}(r,\theta)\, \Omega(r,\theta) r \, \mathrm{d}r \, \mathrm{d}\theta + \epsilon_{nls} \; . \quad (1)$$

Thus the frequency splittings are averages of the rotation rate weighted by the kernels $K_{nls}^{(a)}(r,\theta)$ which are assumed to be known functions of the solar structure. The ϵ_{nls} represent data errors.

For this study we have available the odd a-coefficients ranging from a_1 to a_5. Given such data, one can employ Eq. (1) along with the known kernels to infer the solar rotation profile, either by means of forward methods or by inversion.

3. FORWARD METHODS

The forward methods generally make a guess at the rotation profile, compute the frequency splittings via Eq. (1), evaluate the goodness of the solution against observed splittings, then iterate the solution to minimize the difference with observations.

We have performed such forward calculations using minimization algorithms based on genetic (Charbonneau, 1995; Tomczyk *et al.*, 1995b) and simulated annealing (Press *et al.*, 1992) algorithms. We will discuss only the simulated annealing results here.

To simplify the forward modelling, we have constrained the possible rotation profiles by parameterizing the angular velocity as:

$$\Omega(r,\theta) = \Omega_c + 0.5\left[1. + \mathrm{erf}\left(\frac{r-r_0}{0.5w}\right)\right](\Omega_s - \Omega_c) \; , \quad (2)$$

where Ω_c is the core rotation rate which is assumed to be independent of depth and latitude, erf is the error function which parameterizes the transition at r_0 from the constant core rotation to the surface-like rotation given by Ω_s, w is the full-width of the transition, r is the distance from the solar center, and θ is the co-latitude. Also,

$$\Omega_s = \Omega_{eq} + A\cos^2\theta + B\cos^4\theta \; , \quad (3)$$

where Ω_{eq} is the equatorial rotation rate and A and B are the constants of the surface differential rotation.

To limit the computation to the region of the Sun where this simple six-parameter model is valid, we restricted the modeset to modes with lower turning point radii between 0.5 and 0.8 $R_\odot$ and frequencies between 2.0 and 3.5 mHz. At each iteration we minimize χ^2 defined as:

$$\chi^2 \;=\; \sum\left[\frac{a_{2s+1}(n,l) - a^*_{2s+1}(n,l)}{\sigma_{nls}}\right]^2 \; . \quad (4)$$

Here a_{2s+1} and a^*_{2s+1} are respectively the observed and computed frequency splittings, and σ_{nls} is the standard deviation of the data error ϵ_{nls} (assumed independent and Gaussian-distributed).

To evaluate the forward and inverse methods, we have generated sets of artificial frequency splittings based on the above parameterization and Eq. (1). Noise was added by perturbing the computed splitting values with gaussian errors having standard deviations given by the observational errors for that mode. Numerical experiments showed that the forward method converged to the correct solution for a variety of positions and widths of the shear layer. Also, the forward method recovered very narrow transitions with widths much below the resolution limit of the linear inversion methods.

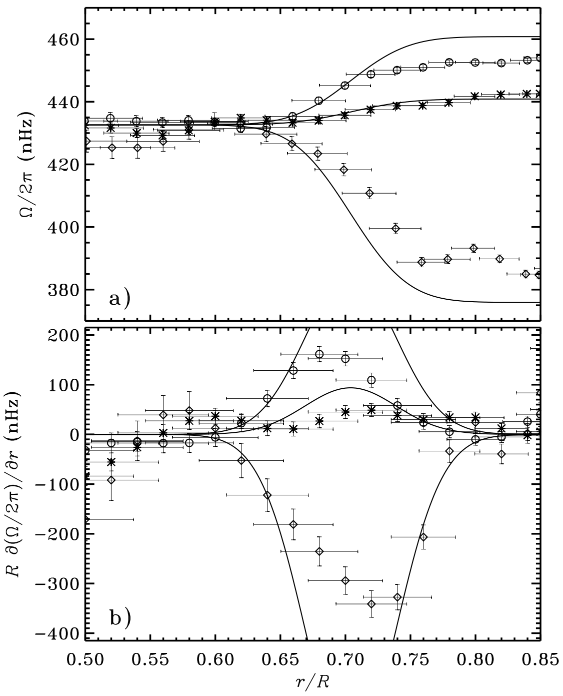

Figure 1: The angular velocity (panel a) and its radial gradient (panel b), as inferred by means of SOLA inversion; circles, stars and diamonds show results at the equator and latitudes of 30° and 60°. The curves show the solution obtained with the forward method, based on the parametrization in Eq. (2).

We applied the forward model to the LOWL 2-year dataset using only the a_3 coefficients in the computation of the χ^2 (following Kosovichev, 1996). In this case the method converged to the solution:

$$r_0 = 0.704 \pm .003\ R_\odot \quad w = .0498 \pm .012\ R_\odot$$

The quoted errors on the rotation parameters were derived from a Monte Carlo computation.

4. INVERSE METHODS

The data have been inverted using a 2D Subtractive Optimally Localized Averages (SOLA) method, in which combinations of the data were taken so as to construct averaging kernels resembling 2D Gaussians of the form

$$\exp\left[-(r-r_0)^2/\Delta_r^2\right]\exp\left[-(\theta-\theta_0)^2 r_0^2/\Delta_\theta^2\right] , \quad (5)$$

normalized to have unit area. Here (r_0, θ_0) is the chosen target location, and Δ_r,Δ_θ are parameters which control the spread of the averaging kernel. We took $\Delta_r = 0.04R_\odot$ and $\Delta_\theta = 0.2R_\odot$, where $R_\odot$ is the solar radius. The results of the inversion are shown in Fig. 1a. Similar results were obtained with a 2D RLS inversion. We also performed a SOLA inversion for the radial gradient of the rotation, $\partial\Omega/\partial r$. The results of the inversion for the rotation gradient can be seen in Fig. 1b.

The result of the inversion at any point is a convolution of the true rotation profile with the averaging kernel (plus propagated data noise). Thus, even if the base of the convection zone were a sharp discontinuity, the finite extent of the averaging kernels would produce a smoothed-out transition in the inversion solution. If the averaging kernel is Gaussian with radial width Δ_r, and the true rotation profile has a step $\mathrm{erf}[(r-r_0)/0.5w]$, then the inversion solution has a step of the same form as the true transition but with a width $(w^2+4\Delta_r^2)^{1/2}$ instead of w. Knowing the radial width Δ_r of the averaging kernel, one can use the above formula to attempt to estimate the true width w from the width of the transition exhibited by the inversion for Ω. The inversion for $\partial\Omega/\partial r$ is useful for judging whether the transition is resolved. For if the transition is very narrow, the gradient at the transition will resemble a Dirac delta function, and the inferred gradient will be given simply in terms of the magnitude of the step and the amplitude of the averaging kernel. If the inferred gradient is indistinguishable from this value, the transition may be judged to be unresolved.

To test this procedure we conducted blind experiments on the artificial data. Here the inverters successfully identified as unresolved transitions which were narrower than the averaging kernels, and made reasonable estimates of the widths of more gradual transitions. The results of exactly the same procedure, applied to the real LOWL data, give a measured transition of $0.10R_\odot$ at the equator and $0.14R_\odot$ at a latitude of 60°, which imply values of $w = 0.03R_\odot$ and $0.06R_\odot$ respectively. However, by the same criteria as before, the inverters judged that the transition is unresolved. If the width of the transition were as great as $0.10R_\odot$, the blind experiments indicate that we would find the transition to be resolved by the SOLA inversion. Thus we conclude that the transition is less than $0.10R_\odot$. Indeed, if the inferred values are to be believed, then $0.06R_\odot$ is an upper bound on the width of the transition.

We note that the inversions also indicate that the transition appears to be slightly deeper at the equator than at higher latitudes, *i.e.*, the shape of the transition is prolate (cf. Fig 1). (A similarly prolate figure was unambiguously detected in a further blind experiment.) The prolate transition region appears not only in the inversions of the full 2-year dataset but also in inversions of the first and second years of LOWL data separately.

In summary, the forward and inverse methods produce mutually consistent estimates of the thickness of the transition. These are lower than the central estimate of Kosovichev (1996). The inferred location is consistent with his estimate and indicates that the centre of the shear layer is slightly below the base of the convection zone. The forward method assumes spherical symmetry in the location of the transition, as did the method of Kosovichev (1996). If, as our inversions suggest, this assumption is not correct, then forward modelling based on such simple models will overestimate the thickness of the transition.

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