CONSTRAINING SOLAR CORE ROTATION WITH GENETIC FORWARD MODELLING

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ABSTRACT

We present a technique which uses forward modelling in conjunction with a genetic algorithm to find the internal rotation profile which best reproduces a set of input frequency splittings. The genetic algorithm provides an efficient and robust method for global optimization. We evaluate the ability of this technique to determine the rotation profile in the solar core by applying it to artificial datasets. In addition, we apply this technique to real data from the LOWL instrument.

Keywords: solar oscillations, internal rotation.

1. INTRODUCTION

Recently, the LOWL instrument (Tomczyk *et al.*, 1995a, 1995b) has begun providing high precision measurements of the frequency splittings of low- and intermediate-degree solar p-mode oscillations. The availability of this data has motivated the development of new techniques which allow the efficient extraction of information about internal rotation from the frequency splittings to complement existing inversion techniques.

This study represents the synthesis of three elements: the LOWL frequency splittings, a forward model, and a genetic algorithm. The LOWL frequency splittings (Tomczyk *et al.*, 1995b; Schou *et al.*, these proceedings) have been described elsewhere and will not be discussed in detail here. The forward model (Schou *et al.*, 1994) is described briefly in the next section, and a description of the genetic algorithm follows in section 3.

2. THE FORWARD PROBLEM

The variation of mode frequency ω_{nlm} with azimuthal order m is generally expanded in a set of a coefficients as:

$$\omega_{nlm}/2\pi = \nu_{nl} + \sum_{j=1}^{j_{\max}} a_j(n,l) P_j^{(l)}(m), \qquad (1)$$

where ν_{nl} is the average mode frequency, the a_j are the fitted coefficients and the $P_j^{(l)}$ are a set of orthogonal functions of degree j (cf. Schou *et al.* 1994). The odd a coefficients are related to the rotation rate Ω at radius r and colatitude θ by

$$2\pi\ a_{2s+1}(n,l)\ = \int_{r,\theta} K_{nls}^{(a)}(r,\theta)\,\Omega(r,\theta)\,r\,\mathrm{d}r\mathrm{d}\theta + \epsilon_{nls}\ , \qquad (2)$$

(Schou *et al.* 1994). That is, the frequency splittings are averages of the rotation rate weighted by the kernels $K_{nls}^{(a)}(r,\theta)$ which are assumed to be known functions of the solar structure. The ϵ_{nls} represent data errors.

Various linear inversion techniques have been used to infer the Sun's internal rotation. The solutions thus obtained are weighted averages of the true rotation rate, the weighting being given by averaging kernels. In the absence of further assumptions about the true rotation, the success of such an inversion can be judged by how well the averaging kernels are localized. According to this criterion, recent inversions of LOWL data (Tomczyk *et al.*, 1995b; Schou *et al.*, these proceedings) are unable to provide localized information below $0.2\,R_\odot$, because so few modes penetrate this deeply into the Sun.

Yet indubitably the low-degree frequency splittings do contain information about the core rotation. To illustrate this, we have taken two artificial rotation profiles – one flat and the other with a rotation below $0.2\,R_\odot$ at about twice the surface rate (Cases 1 and 2 of Figure 2) – and plotted in Figure 1 the difference in the a_1 coefficients that they would produce. We have added error bars to indicate the significance of the differences: these represent realization noise limits appropriate to observations of 6 months duration. Particularly for degrees $l = 2-4$, the differences in a_1 coefficients would easily be detectable in available measurements of low-degree frequency splittings.

Given the importance of measuring the Sun's core rotation, and the relative lack of success of present inversions in that region, we are motivated to look at alternative techniques which may extract information about solar internal rotation more successfully than conventional inversion techniques. Specifically, in this paper we consider the direct application of forward modelling techniques, utilizing genetic algorithms. We first test the approach on artificial data, and then apply it to LOWL observations.

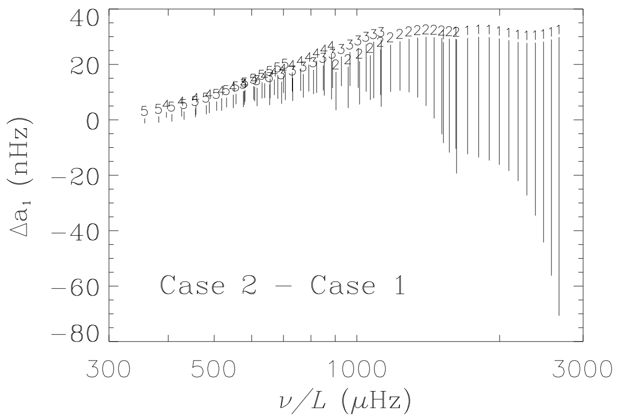

Figure 1: Differences between a_1 splitting coefficients computed using two artificial rotation profiles that differ below 0.2 $R_\odot$ (The profiles are Case 1 and Case 2 of Figure 2). The mode degree is indicated by number. Theoretical 1-σ error bars corresponding to 6 months of observations are shown.

3. GENETIC FORWARD MODELLING

Genetic Algorithms (hereafter GAs) are a class of heuristic search and optimization techniques inspired by the biological notion of evolution by means of natural selection (Holland, 1975). GA-based optimization involves producing a sequence of trial solutions to some optimization problem, evaluating how well each of these trial solutions satisfies the problem at hand, and generating a new sequence of improved trial solutions by breeding members of the sequence. This may seem like a simple variation on the Monte Carlo theme, but GA's differ from random heuristics in two fundamental ways: first, individual trial solutions are not selected completely randomly from the parent sequence; instead, the probability of being selected for breeding is proportional to a trial solution's fitness, i.e., how well it does at satisfying the problem at hand. In other words, only the original sequence is truly random. Second, breeding does not proceed by averaging or taking linear combinations of selected individuals, but rather by applying genetic-like operators (mutation and crossover in most basic implementations) to pairs of chromosomes, strings of digits that encode in some way the parameters defining each trial solution. New solutions are thus constructed and used to replace the original parent population, after which the process is repeated for however many generations are required to produce a trial solution that satisfies the original problem within some tolerance criterion.

Not only does this seemingly odd strategy work, but it often works extremely well. Unlike most conventional optimization methods (see e.g. Press et al. 1992, chap. 10), the technique works equally well on discrete or continuous problems, and does not require the computation of derivatives with respect to model parameters. Furthermore, the application of the crossover and mutation operators on the encoded trial solutions, coupled to selection based on fitness, leads to an automated and adaptive search of parameter space, an essential feature when multiple local extrema are present; the resulting algorithms are thus global and in general extremely robust, in that they do well on wide classes of problems. Goldberg (1989) and Davis (1991) are good textbooks with tutorial-like introductions to GAs. The application of GAs to a few typical and not-so-typical astronomical/astrophysical fitting and optimization problems is discussed in Charbonneau (1995).

In principle, the application of GAs to inverse problem is straightforward; one begins as with a Monte Carlo simulation, by constructing a series of trial solutions (here random rotation profiles). Using a forward model, frequency splittings are calculated for each trial rotation profile using eq. (2). Fitness is evaluated, being made proportional to some goodness-of-fit criterion, typically a χ^2 computed against observed splitting data. "New and Improved" rotation curves are then constructed using the selection and breeding principles outlined above. We will refer to this approach as *genetic forward modelling.* In practice things are complicated by the fact that the inverse problem is formally ill-posed. This is essentially due to the integral (or global) nature of the inverse problem; widely different rotation curves can produce similar sets of frequency splittings (see Craig & Brown 1986, §1.3 for an illustrative and instructive example). Consequently, helioseismic inversion (whether treated as a forward or inverse problem) must make use of regularization. More specifically, the GA-based optimizer used herein does not minimize directly the χ^2 associated with the set $[s_n]$ of N frequency splittings corresponding to a (discretized) rotation curve Ω_j $j = 1, ..., J$, but rather the quantity

$$\tilde{\chi}^2 = \sum_{n=1}^{N} \left(\frac{S_n - s_n}{\sigma_n} \right)^2 + \frac{\lambda}{\langle \Omega \rangle} \sum_{j=1}^{J} |\nabla \Omega|_j^2, \qquad (3)$$

where the S_n's are the observed frequency splittings, λ is a Lagrange multiplier, σ_n the error associated with the $n^{\rm th}$ frequency splitting, $\langle \Omega \rangle$ some suitably defined average, and the rotation curve gradient is computed with a standard two-point finite difference formula. Fitness is then defined as the inverse of $\tilde{\chi}^2$. The proper choice of a numerical value for λ is nontrivial and a bit of an art; up to now we seem to obtain optimal results when λ has a value such that the second term on the RHS of (3) is larger than the first by about a factor of 10 early in the evolution, with both terms becoming of the same order near the end of the run. There are many other ways to formulate a regularization criterion (i.e., similar to eq. [3] but using second derivative, maximum entropy criteria, etc.). We make no claim that eq. (3) represents the optimal choice; quite simply, eq. (3) seems to work well, so we use it.

4. RESULTS

To test the genetic forward modelling technique, we generated sets of artificial data based on the internal

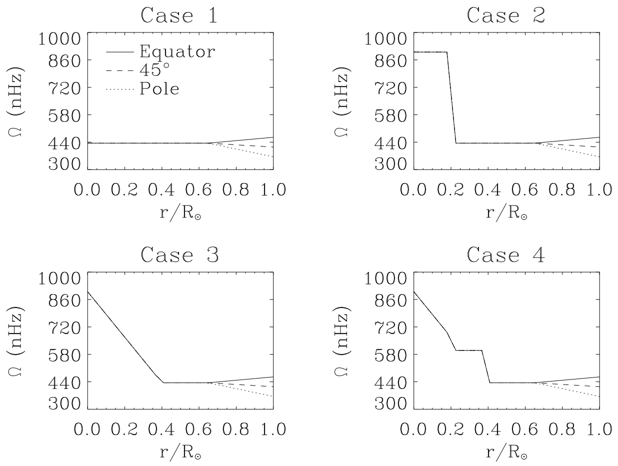

Figure 2: The four rotation profiles used to evaluate the genetic forward algorithm. The profiles above 0.5 $R_\odot$ are identical in all cases.

rotation curves of Figure 2. These curves differ only in the rotation rates below 0.4 $R_\odot$. We generated both noiseless frequency splittings, as well as splittings perturbed by gaussian errors. The errors were computed assuming realization noise limits with an observation length of 6 months and mode linewidths obtained from a fit to the observations of Libbrecht (1988).

To simplify our initial attempts at genetic forward modelling, we have reduced the problem in two important ways. First, we attempt to reconstruct the solar rotation profiles only below $\simeq 0.5\ R_\odot$, and additionally we assume that in this region the solar rotation profile has no latitude dependence. This reflects our primary aim to investigate whether genetic forward modelling can provide meaningful rotation information to greater depths than conventional inversion techniques. We have therefore restricted our modeset to only a_1 coefficients with degrees ranging from 1 to 20. Above 0.5 $R_\odot$ we constrain the trial rotation curves to be identical to the target rotation curves.

Figure 3 shows the results of the genetic reconstruction of the four synthetic rotation curves shown in Figure 2. Parts (A)—(D) are genetic fits to the set of error-free synthetic frequency splittings, while parts (E)—(H) show the corresponding solutions with noise introduced in the frequency splittings, as described above. The dotted lines are the original rotation curves corresponding to the four cases considered, and the dots and triangles are genetic solutions for two values of the regularization parameter λ ($\lambda = 10^{-3}$, solid dots, is currently our preferred value). All solutions were obtained by evolving 60 individuals over 500 generations under full generational replacement with elitism and variable mutation rate. Rotation curves were encoded to 4 digits accuracy in the form of decimal integer strings, on which standard one-point crossover and mutation operators were applied at breeding. Constant selection pressure was enforced through ranking, and parent selection was carried out using the roulette wheel algorithm. See Charbonneau (1995) for an introduction to these and other genetic algorithm concepts (and jargon).

Figure 3 illustrates several aspects of these solutions which we consider noteworthy: 1) With or without errors in the frequency splittings, genetic forward modelling succeeds in reproducing the rotation curves to within ~ 20 % down to $\simeq 0.1\ R_\odot$. The clear detection of a mere twofold increase in rotation rate below $0.2\,R_\odot$ (case 2) or of a change in slope (case 4), are particularly encouraging. 2) The technique fails below $0.1\,R_\odot$; the flat shape of the computed rotation curves indicate that below this depth the solution is completely dominated by the regularization constraint. Over-regularization produces rotation curves that “look good”, yet systematically underestimate rotation rates in the deep core ($< 0.25\ R_\odot$); this is readily seen upon comparing the two sets of curves in Fig. 3. This problem is of course not specific to genetic forward modelling, but in one form or another plagues essentially all inversion methods.

We also applied the genetic forward modelling to real data obtained with the LOWL instrument. The determination of these frequency splittings is discussed elsewhere (Tomczyk *et al.*, 1995b; Schou *et al.*, these proceedings). Our approach is similar to that used to reconstruct the synthetic rotation curves. We use genetic forward modelling to reconstruct the rotation curve below $0.5\,R_\odot$ while above 0.5 $R_\odot$ we assume a rotation profile as given by a standard inversion (Schou *et al.*, these proceedings). Above 0.85 $R_\odot$ we interpolate the inversion to surface values. This is admittedly rather simplistic, and our results, shown in Figure 4, should be considered as preliminary and viewed with corresponding caution. We do note that the rotation profile of Figure 4 gives no indication of a rapidly rotating solar core, even for a solution that is likely under-regularized (triangles).

5. CONCLUSIONS

Our investigation is still at a preliminary stage, and we are not yet in a position to say that the approach presented here is better or worse suited to probing the solar core rotation than conventional inversion methods. However, the low-ℓ frequency splittings do contain information about the rotation of the solar core beneath 0.2 $R_\odot$, and genetic forward modelling appears capable of extracting this information. The application of this technique to LOWL data strengthens the case against the existence of a rapidly rotating solar core, but any firmer statement must await a fully consistent application of genetic forward modelling to this problem.

We are continuing work in this area and have identified future improvements to the technique including: optimization of the regularization scheme, the generation of error limits on the inferred rotation curves,

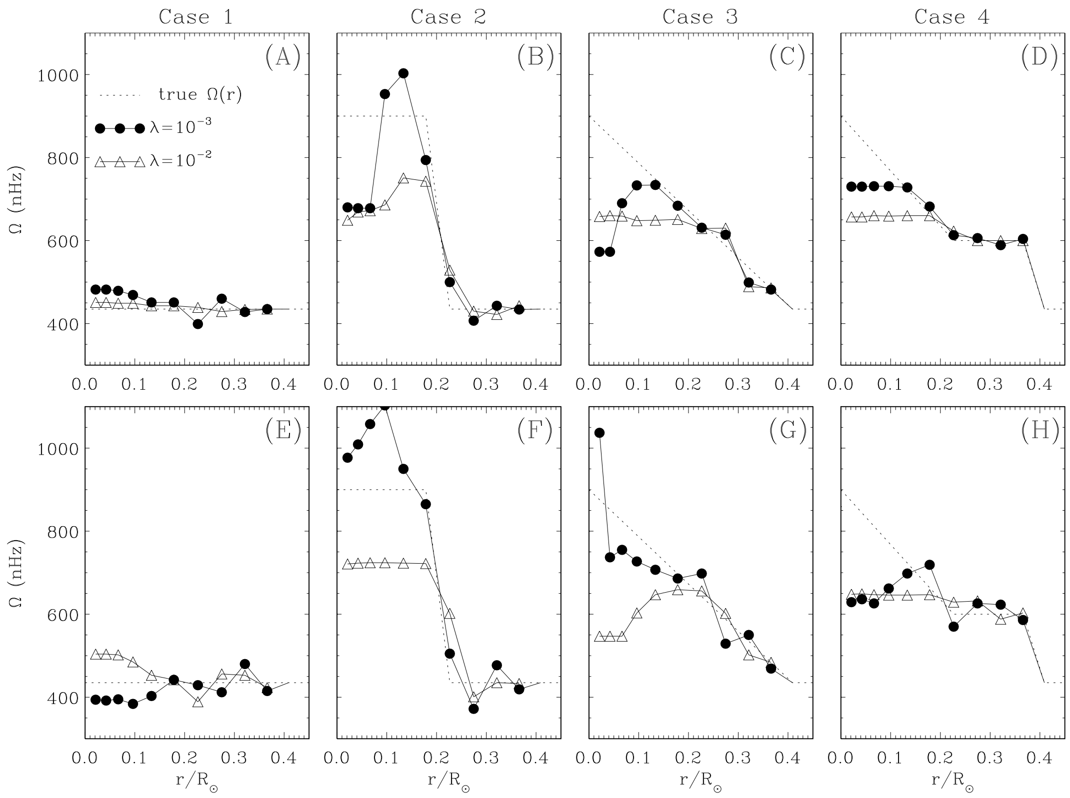

Figure 3: Inversion of the synthetic rotation curves of Fig. 2 in the interval $[0 < r/R_\odot < 0.5]$, using genetic forward modelling. Dotted lines are the original rotation curves, and the dots and triangles are inversions obtained for two different values of the regularization parameter λ (see text). Parts (A)—(D) are inversions of noiseless synthetic data, while for parts (E)—(H) noise was artificially introduced in the synthetic frequency splittings.

and the full 2-dimensional fitting of the complete rotation curve. Ongoing work indicates that the use of 2-D chromosomes, with suitably modified crossover and mutation operators, can lead to a viable 2-D generalization of the technique used in this paper to carry out 1-D inversion. We anticipate that, in the near future, genetic forward modelling will become a useful tool for helioseismic inference.

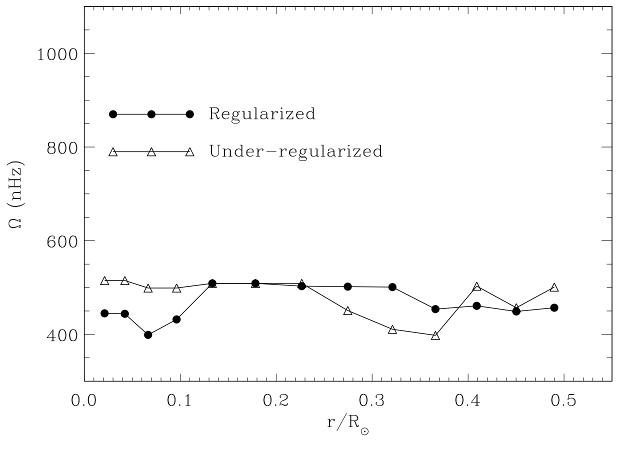

Figure 4: Inversion of the LOWL data using genetic forward modelling. The dots are the rotation curves produced by tuning the regularization parameter to correspond to the $\lambda = 10^{-3}$ curves on Fig. 3. The triangles are a solution with λ decreased by a factor of 10 with respect to the basic solution.

6. REFERENCES

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