1D and 3D radiative transfer in protoplanetary disks

Simon Hügelmeyer

Institut für Astrophysik Göttingen, Germany

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In collaboration with:
S. Dreizler (Göttingen), D. Homeier (Göttingen), P. Hauschildt (Hamburg)
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Motivation

Why modelling protoplanetary disks?

- we need to know disk structure to understand planet formation
- structure can be investigated by means of high-resolution IR spectroscopy
- look at inner disk region (where many exoplanets are observed) & use detailed model spectra
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Why a new radiative transfer code for protoplanetary disks?

- there are several structure and radiative transfer codes for protoplanetary disks (e.g. D’Alessio et al. 1998, Dullemond & Dominik 2004)
- use different approach: use stellar atmosphere code PHOENIX which can handle extensive lists of atomic and molecular lines as well as dust; adopt it to disks (geometry, heating sources)
- model detailed and self-consistent 1D disk structures
- expect that our line radiative transfer calculations can provide new insight about inner disk structure
Motivation 1D radiative transfer GQ Lup 3D radiative transfer

1D radiative transfer: Basics

- Assume standard accretion disk model for geometrically thin disks $H \ll R$ (Shakura & Syunyaev 1973, Lynden-Bell & Pringle 1974)
  - Parametrize viscosity
  - Decouple vertical and radial structure
- Separate disk in rings and calculate vertical structure and RT for each ring assuming physics does not change over ring width

**Figure:** Disk ring structure as adopted for our calculations. The radius of the rings increases exponentially.

**Input parameters**

- Central star properties: $M_*, R_*, T_{\text{eff}}$
- Radius of disk ring: $R$
- Mass accretion rate: $\dot{M}$
- Reynolds number: $Re$ (sets viscosity: $\nu = \sqrt{GM_*/R}/Re; Re \propto \alpha^{-1}$)
1D radiative transfer: Model basics

Hydrostatic equilibrium:

Unlike classical stellar atmosphere problem, gravity $g$ is function of height $z$

$$\frac{dP}{dm} = \frac{GM_\star}{R^3} z$$

(1)

Radiative transfer:

Solve the radiative transfer equation for a given number of quadrature points $\mu_i$

$$\mu_i \frac{dI_\nu}{d\tau_\nu} = I_\nu - S_\nu$$

(2)

With boundary conditions

$$I_\nu(-\mu, z_{\text{max}}) = I_{\nu}^{\text{ext}}(-\mu, z_{\text{max}}) \quad \text{and} \quad I(-\mu, 0) = I(\mu, 0)$$

Radiative equilibrium:

Radiative energy has to balance dissipated mechanical energy

$$E_{\text{mech}} = E_{\text{rad}} \iff \frac{9}{4} \frac{GM_{\star}}{R^3} \nu \rho = 4\pi \int_0^\infty (\eta_\nu - \chi_\nu J_\nu) d\nu$$

(3)
## Dust formation

- Condensate formation treated by assuming chemical and phase equilibrium for several hundred species (Dusty setup; Allard et al. 2001)
- Grain opacities calculated for 50 most important refractory condensates (for which optical data is available)
- Absorption and scattering using Mie formalism
1D radiative transfer: Dust treatment & irradiation

**Dust formation**
- Condensate formation treated by assuming chemical and phase equilibrium for several hundred species (Dusty setup; Allard et al. 2001)
- Grain opacities calculated for 50 most important refractory condensates (for which optical data is available)
- Absorption and scattering using Mie formalism

**Irradiation geometry**
- Blackbody or PHOENIX spectrum as input
- Determine corresponding star surface fraction for each quadrature point $\mu_i$
1D radiative transfer: Line profiles

- disk usually not resolved $\Rightarrow$ total flux $F_{\nu}(i)$ integral of specific intensities $I_{\nu}(i)$ over disk ring area

$$F_{\nu}(i) = \pi \cos(i) \sum_{j=1}^{NR} \left[ (R_{j}^{out})^2 - (R_{j}^{in})^2 \right] I_{\nu}(i, \nu)$$

- due to Keplerian rotation of matter disk rings show different line broadening
- ring symmetry causes double peak profile for $i \neq 0$

**frequency shift**

$$\nu_{i}' = \nu \sqrt{\frac{1 - v_i / c}{1 + v_i / c}}$$

$$v_i = v_r \sin(i)$$
1D and 3D radiative transfer in protoplanetary disks

**Motivation**

- **1D radiative transfer**
- **GQ Lup**
- **3D radiative transfer**

**1D radiative transfer: Line profiles**

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$$F_\nu(i) = \pi \cos(i) \sum_{j=1}^{NR} \left[ (R_{j}^{out})^2 - (R_{j}^{in})^2 \right] I_\nu(i, \nu)$$

![Graph showing line profiles for different inclination angles](image)

- $\nu' = \nu_s - v_i/c$  
  - $v_i = v_r \sin(i)$
Analysis of GQ Lup

- GQ Lup is a classical T Tauri star (CTTS) with a lately discovered sub-stellar companion GQ Lup B (Neuhäuser et al. 2005)
- very active: more than 2 mag variability ($V_{\text{max}} = 11.33$ mag and $V_{\text{min}} = 13.36$ mag)
- Broeg et al. (2007) and Seperuelo Duarte et al. (2008) derive different parameters from lightcurves (orbital period) and spectroscopy (rotational period $v \sin i$)

<table>
<thead>
<tr>
<th>authors</th>
<th>$d$ [pc]</th>
<th>$P$ [d]</th>
<th>$v \sin i$ [km s$^{-1}$]</th>
<th>$R_\star$ [$R_\odot$]</th>
<th>incl. [$^\circ$]</th>
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</thead>
<tbody>
<tr>
<td>Broeg et al.</td>
<td>140</td>
<td>8.45</td>
<td>6.8</td>
<td>2.55</td>
<td>27</td>
</tr>
<tr>
<td>Seperuelo D. et al.</td>
<td>150</td>
<td>10.7</td>
<td>6.5</td>
<td>1.80</td>
<td>51</td>
</tr>
</tbody>
</table>

calculated sets of disk ring structures/spectra

$R = 0.031$ AU $- 0.422$ AU

$T_{\text{eff}} = 4060$ K

$M_\star = 0.8$ $M_\odot$

$\dot{M} = 2 \cdot 10^{-8}$ $M_\odot$/yr $- 7 \cdot 10^{-10}$ $M_\odot$/yr

$R_e = 1/5 \cdot 10^4$ ($\alpha \sim 0.05$)
Model fit
Model fit

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1D radiative transfer

GQ Lup

3D radiative transfer

Disk ring contribution

Fig. 4. Optical depth structure for a disk ring model with $R = 0.065$ AU, $M = 0.8 M_\odot$, $R_\star = 2.55 R_\odot$, $\dot{M} = 3 \cdot 10^{-9} M_\odot \text{yr}^{-1}$, and $Re = 5 \cdot 10^4$. The left plot shows the run of $\tau_{\text{ext}}$ with height ($z/R = 0$ is the midplane of the disk). The asterisks denote the optical depth at the center of the line, the crosses are for a continuum point (see right panel). The dotted line at $\tau = 1$ marks the region where the line and the continuum become optically thick, i.e., where the radiation that we see comes from. The plot on the right is a temperature-log $\text{cm}$-diagram showing at which temperatures and column masses the optical depth for the line (dashed line) and the continuum (dotted line) become unity.

In Eq. 26 we assumed that the disk is axis-symmetric and that the intensity is constant for all radii between inner and outer radii for each ring. In addition to the integration over all disk rings, the influence of the disk's rotation on the line profile is taken into account by applying the Doppler shift $\nu' = \sqrt{1 - \frac{u(i, \phi)}{c} + \frac{u(i, \phi)}{c}}$ to the line. Here the velocity $u(i, \phi)$ is determined for a given inclination $i$ and for a set of disk ring segments with azimuthal angle $\phi$. We use $N_A = 100$ disk ring segments, i.e., 100 steps in $\phi$, to determine the rotationally broadened line profile. This method is a simple way to determine line profiles for rotating accretion disks if the lines originate in the very upper layers of the disk. However, Horne & Marsh (1986) noted that an anisotropy in the local emission pattern changes the global line shape: line photons are trapped in optically thick emission layers and can more easily escape in directions where there are larger Doppler gradients. The true consequence of this on the line shape will be discussed in a future paper (Hügelmeier et al. in preparation) where we will present full 3D radiative transfer calculations in rotating accretion disks.

Fig. 5. The left panel shows normalised disk ring spectra. Intensities and wavelength are offset for clarity. The right panel depicts bars corresponding in height to the contribution of each disk ring spectrum to the total spectrum. The bar for the stellar contribution has to be multiplied by a factor of 25. The spectra and the weights are grey-scale coded corresponding to their ring radius $R$. 

3. Synthetic spectra for GQ Lup

We retrieved spectra of T Tauri stars taken with the high-resolution infrared spectrograph CRIRES at the VLT from the ESO Science Archive Facility (see Pontoppidan et al. 2008, for a description of the observations). The observations were reduced using a combination of the CRIRES pipeline and our own IDL routines. The telluric absorption lines in the spectrum were recalculated and the wavelength range was shifted to the vacuum wavelength. The telluric absorption lines in the spectra were recalculated and the wavelength range was shifted to the vacuum wavelength.
use 3D radiative transfer framework of Hauschildt & Baron (2006)
1D models (temperature, opacity) are interpolated on 3D grid (Cartesian or cylindrical grid)
simple 2-level model atom line transfer in moving media implemented
accelerated lambda iteration can be used to include scattering in RT

\[ nx = ny = 33 \quad nz = 65 \]
use 3D radiative transfer framework of Hauschildt & Baron (2006)

- 1D models (temperature, opacity) are interpolated on 3D grid (Cartesian or cylindrical grid)

- simple 2-level model atom line transfer in moving media implemented

- accelerated lambda iteration can be used to include scattering in RT

\[ nr = nz = 65 \quad nt = 17 \]
Coupling between stellar irradiation and disk structure

- In 1D case only disk surface is irradiated by central star
- In reality star light irradiates inner disk wall \( \Rightarrow \) puffed-up inner rim?
- 1D opacity sampling of \( \approx 10^5 \) frequencies \( \Rightarrow \) use Planck mean opacities for 3D RT with \( \approx 50 \) frequencies

![Graph showing opacity vs. wavelength](image)
3D RT: Cartesian grid
3D RT: Cylindrical grid
3D RT: Cylindrical grid - opacity

opacity [1/cm]

R [AU]

layer 1
layer 2
layer 3
layer 4
layer 5
layer 6
layer 7
layer 8
layer 9
layer 10
3D RT: Cylindrical grid - opacity
Acknowledgements

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References


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1D radiative transfer

GQ Lup

3D radiative transfer

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Fig. 6. Temperature structures for the eight... For our best model fit, we use a model which fits the
strength of the higher order CO v = 1 − 0 lines because these are...
Line origin

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Line origin

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Line origin

$\tau = 1$ continuum
$\tau = 1$ line

$\log(m/\text{[g cm}^{-2}\text{]})$ vs $\tau$ (K)

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