Non-LTE Time-Dependent Radiative Transfer of Supernova Ejecta

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Synopsis

- Summary of supernova ejecta properties. Relevant quantities.
- Nature of the radiative transfer problem
- Step 1: non-LTE, D/Dt=0 Steady-State approach
- Step 2: non-LTE, De/Dt, Dn/Dt ≠ 0, but DJ/Dt = 0
- Step 3: non-LTE, De/Dt, Dn/Dt, & DJ/Dt ≠ 0
- Past, Ongoing and Future Projects
Summary of SN Ejecta Properties

Sequence of Events

- Dynamical phase:
  - Initial seconds: Sudden Release of Energy (shock and/or combustion) unbinds progenitor.
  - From seconds to days: acceleration to asymptotic velocity ($dP/dr \neq 0; P_{rad} \gg P_{gas}$) and homologous expansion

- Kinematic Phase/Homologous Expansion:
  - $Dv/Dt = 0$ - No Hydro (ignoring interaction).
  - $V(m,t) = \text{const.}$
  - $R(m,t) \gg R(m,t_0)$
  - $R(m,t) \sim V(m,t) \times t$
  - $\rho(m,t) \sim \rho(m,t_0) \left(\frac{t_0}{t}\right)^3$

Energy budget: **heating** (unstable isotopes + recombination)

**cooling** (expansion + radiation)
Summary of SN Ejecta Properties

Connection to Light Curves

- Photosphere at ~8000K and $10^{15}$cm => Large Luminosity $L_{bol}(t)$
- $\int dt \; L_{bol} \ll \int dm \; (e_{kin} + e_{th} + e_{rad})$: Radiation does work instead of escaping
- Radiative losses are key diagnostics of progenitor and ejecta properties

- Variations with $R_{star}$: ~$10^8$ (WD) to ~$10^{13}$cm (RSG). Impacts cooling losses due to expansion
- Variations with $M_{star}$: 1 to 10-20 Msun. Impacts $<v(m)>$ for given $E_{explosion}$
- Variations in Composition: impacts $\kappa_{es}$ (cm$^2$/g) through # of free electrons per baryon
  $\Rightarrow$ Impacts $\tau = \kappa \; M/R^2$ and $t_{diffusion} \sim \tau \; R/c$; Light curve conditioned by the time it takes to release stored ejecta energy
- Variations in Explosion properties: $E_{explosion}$ impacts $<v(m)>$ (modulo $M_{star}$); Radioactive isotope yields (half-life of weeks to months) affects long-term ejecta thermal evolution

One non-spectroscopic SN classification:

**Type I**: Small star with little mass

**Type II**: Big star with Large mass
Summary of SN Ejecta Properties
Illustration with a RSG Progenitor (Type II-P SN)

- $E_{\text{expl}} = 1.2 \times 10^{51}\text{erg}$; $\int dt \, L_{\text{bol}} \sim 10^{49}\text{erg}$
- At shock emergence, $E_{\text{int}} \sim E_{\text{kin}}$
- Homologous Expansion reached after $\sim 10\text{d}$
- Chemical stratification: H, He, O (main shells).
- Radioactive isotopes at depth ($v(m) < 1000\text{km/s}$)
- Fast Expansion $\Rightarrow$ Low density $\Rightarrow$ Dominance of scattering over collisional processes, non-LTE and time dependent effects.

Type II-Plateau SN ejecta at 4 days (RSG progenitor)

[Graphs and diagrams showing E$_{\text{int}}$ vs. E$_{\text{kin}}$ and mass fraction vs. interior mass]
Radiative Transfer Problem

**Aim:** Model evolution of Gas+Radiation from explosion (large $\tau$) until nebular phase (small $\tau$) with 1D non-LTE model atmosphere code CMFGEN

**Track:**
1. Changes in the radiation ($I, J, H$)
2. Changes in the gas ($e, N_i, N_e, \kappa$) + sources (decay)
3. Coupling between gas and radiation
4. Changes in composition ($X_i(t)$ through decay)

**Use** Comprehensive Atomic dataset for microphysics ($\kappa, \eta, \kappa_{th}$ vs. $\kappa_{scat}$)
Non-LTE Time-Dependent Radiative Transfer Problem

**Rate Equation:**
\[ \frac{\partial n_i}{\partial t} + \nabla \cdot (n_i \vec{V}) = \sum_{j \neq i} (n_j R_{ji} - n_i R_{ij}) \]

**Lagrangian Form:**
\[ \rho \frac{D(n_i / \rho)}{Dt} = \frac{1}{r^3} \frac{D(r^3 n_i)}{Dt} = \sum_{j \neq i} (n_j R_{ji} - n_i R_{ij}) \]

**Energy Equation:**
\[ \rho \frac{De}{Dt} - \frac{P D\rho}{\rho Dt} = 4\pi \int_{0}^{\infty} \chi_v (J_v - S_v) \, dv + \text{De}_{\text{decay}}/Dt \]

where \( e = \text{internal energy/unit mass} \)
\[ = \frac{3}{2} kT (n + n_e) + \sum n_i E_i \]
\[ = \frac{3}{2} n \frac{kT}{\mu m_H n} + \sum n_i \frac{E_i}{\mu m_H n} \] (Excitation + Ionization)

**RTE 0th moment:**
\[ \frac{1}{cr^3} \frac{D(r^3 J_v)}{Dt} + \frac{1}{r^2} \frac{\partial (r^2 H_v)}{\partial r} - \frac{\nu V}{r c} \frac{\partial J_v}{\partial v} = \eta - \chi J_v \]

**RTE 1st moment:**
\[ \frac{1}{cr^3} \frac{D(r^3 H_v)}{Dt} + \frac{1}{r^2} \frac{\partial (r^2 K_v)}{\partial r} + \frac{K_v - J_v}{r} - \frac{\nu V}{r c} \frac{\partial H_v}{\partial v} = -\chi H_v \]
Step 1: Non-LTE Steady-State Approach

- Non-LTE treatment
- Ignore all D/Dt terms
- Focus on Decoupling region: $\tau \in [10^{-8}, 10^2]$ 
- Assume Diffusion Approximation at base (CMF flux)
- Analytical Description of SN Ejecta: $L_{0,\text{CMF}}, R_0, \rho(R), v(R)$ 
- Chemical Homogeneity: $X_i(R) = X_i$
- Approach: Guess and check parameters describing SN ejecta properties until sensible fit (stellar approach)

- Focus on Type II SNe because well-defined diffusing inner boundary, massive homogeneous hydrogen shell, little $^{56}\text{Ni}$
Step 1: Non-LTE Steady State Approach

Application: Modeling of Young Type II SNe

The case of the Type II-P SN 2005cs in NGC 5194 (Dessart et al. 2008)
Step 1: Non-LTE Steady State Approach
Application: Modeling of Young Type II SNe
The case of the Type II-P SN 2005cs in NGC 5194 (Dessart et al. 2008)

SN2005cs – 3 July 2005
E(B−V)=0.04
Data source: CfA

+ : Swift UVOT
Step 1: Non-LTE Steady-State Approach
Application: Modeling of Young Type II SNe

Properties of the Type II-P SNe 1999em, 2005cs, and 2006bp (Dessart et al. 2008)

- Inferences on evolution of $R_{\text{phot}}$, $T_{\text{phot}}$, $V_{\text{phot}}$, reddening, chemistry (He/CNO; non-LTE effects)
- Line identification (HI, HeI/HeII, CNO (I,II), FeII etc.)
- Peculiar Line formation (e.g.: peak blueshift)
- Correlation $v_{\text{phot}}/L_{\text{bol}}$
- Typically: $R_{\text{phot}} \sim 10^{15}$ cm, $T_{\text{phot}} \in [7,20]$kK, $L \sim 10^{8-9}L_{\text{sun}}$
Modeling of multi-epoch spectroscopic & photometric observations yields $T_{\text{phot}}(t)$

Modeling of brightness evolution for homologous expansion yields $R_{\text{phot}}(t)$ and distance

Methods: Expanding Photosphere Method (EPM; Kirshner & Kwan 1974) and Spectral-fitting Expanding Atmosphere Method (SEAM; Baron et al. 2000)

Application to cosmology: Independent determination of the Hubble Constant.

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### Step 1: Non-LTE Steady State Approach

**Application:** ~10% Accurate Distance Determination to Type II SNe

<table>
<thead>
<tr>
<th>SN</th>
<th>EPM</th>
<th>SEAM</th>
<th>Other</th>
</tr>
</thead>
<tbody>
<tr>
<td>1999em</td>
<td>11.5 ± 1.0 (D06)</td>
<td>12.2 ± 2.0 (D06)</td>
<td>11.7 ± 1.0 (Cepheid; L03)</td>
</tr>
<tr>
<td>2005cs</td>
<td>8.9 ± 0.5 (D08)</td>
<td>8.9 ± 0.7 (D08)</td>
<td>7.7 ± 1.0 (SBF; T01)</td>
</tr>
<tr>
<td>2006bp</td>
<td>17.5 ± 0.6 (D08)</td>
<td>17.1 ± 0.4 (D08)</td>
<td>17.0 ±? (TF; T88)</td>
</tr>
</tbody>
</table>
In Interacting SNe, the photosphere forms at large distances where the SN ejecta rams into a pre-SN ejected shell.

- Interaction leads to conversion of kinetic into internal energy.
- Large mass of resulting ionized material cools on diffusion time scale
- For large $\tau$, thermalization ensues and allows the modeling of the optically-thick shell.
- Good fits for SN 1994W
- Key finding: Line broadening dominated by non-coherent scattering with thermal electrons rather than expansion

Variation in expansion rate.
- P-Cygni profile for high $V_0$
- Symmetric profile for small $V_0$
Step 1: Non-LTE Steady State Approach
Shortcomings: Problem at recombination epoch in Type II SNe

- Synthetic Balmer line profile shapes systematically too narrow and weak
- Problem encountered in all Type II SNe (low/high $L_{bol}$, $E_{expl}$, or $^{56}$Ni)
- Origin: Recombination and expansion time scale comparable: $R/V \sim 1/\alpha N_e$
- Solution: Include time-dependent terms in statistical and gas energy equation
Step 2: Non-LTE, Dn/Dt and De/Dt≠0, but DJ/Dt=0

- Non-LTE treatment
- Include Dn/Dt and De/Dt terms in statistical and energy equations.
- Evolve outer ejecta layers: \( \tau \in [10^{-8}, 10^4] \)
- Assume Diffusion Approximation at base (CMF flux)
- Analytical Description of SN Ejecta: \( L_{0,CMF}(t), R_0, \rho(R), v(R) \)
- Chemical Homogeneity: \( X_i(R) = X_i \)
- Approach: Adopt initial SN ejecta properties and evolve, adjust \( L_{0,CMF}(t) \) to follow the typical SN \( L_{bo}(t) \)
Step 2: Non-LTE, $Dn/Dt$ and $De/Dt \neq 0$, but $DJ/Dt = 0$

Results for Type II SNe

- Balmer line strength and width reproduced
- All lines affected
Step 2: Non-LTE, Dn/Dt and De/Dt≠0, but DJ/Dt=0
Results for Type II SNe

- Impacts ejecta ionization ($N_e$, $N_i$, ∀ ion), with freeze-out in low density (high velocity) regions.
- Impacts $\tau_{\text{line}}$: broader line formation region, broader/stronger lines; impacts inferred chemistry/stratification
- Impacts $\tau_{\text{cont, ejecta}}$: ejecta optically-thick longer
D/Dt effects present even at early times

- HeI lines strongly affected 1-2 weeks after explosion
- Effect present at early time so not due to $^{56}$Ni
- Effect present under fully-ionized conditions so not fundamentally linked to recombination energy
  
  $\Rightarrow$ Dn/Dt effects likely important in all SN ejecta
Step 2: Non-LTE, $\frac{Dn}{Dt}$ and $\frac{De}{Dt} \neq 0$, but $\frac{DJ}{Dt}=0$

Shortcomings

- Inner boundary flux must be set: Tricky tuning
- Diffusion approximation potentially faulty: low $\tau$, weak thermalization
- Analytical description of ejecta not globally consistent; no chemical stratification
Step 3: Non-LTE, Dn/Dt, De/Dt, and DJ/Dt ≠ 0
Combination of Non-LTE and full time dependence

- Non-LTE treatment
- Include Dn/Dt and De/Dt terms in statistical and energy equations.
- Include DJ/Dt and DH/Dt terms in RTE
- Evolve ENTIRE ejecta: $\tau \in [10^{-8}, \sim 10^5]$
- Zero Flux adopted at inner boundary
- Initial SN ejecta conditions taken from a physical model of the progenitor star exploded with a “piston” (KEPLER, Woosley & Weaver)
- Chemical Stratification; Profile taken from evolved progenitor star
- Disadvantage/Advantage: No adjustable parameter to “fit” observations once a time sequence is started. Global consistency rather than fine tuning
Step 3: Non-LTE, Dn/Dt, De/Dt, and DJ/Dt ≠ 0
Application: Time Evolution of SN ejecta + emergent radiation

- Initial model: Im18a7Ad (Woosley) at 0.1 day
- Evolved with steps Δt = 1.1t
- Chemical Stratification
- Energy deposition from isotopes

Results: non-LTE light curve, sensible fits to spectra (although some discrepancies), Ionization freeze-out (as in step 2), recombination fronts (HI, HeI, HeII)
Summary and Ongoing/Future Projects

✓ Various approaches possible for quantitative spectroscopy of SNe.
✓ Non-LTE and steady state adequate at early times in Type II SN. Needs further checking. Allows “good fits” through tuning parameters.
✓ Modeling of recombination phase (changes in ionization at low-density high velocity) requires time-dependent terms (Dn/Dt, De/Dt)
✓ Better physical consistency with “Full-Ejecta” simulations based on hydrodynamical inputs of pre-SN evolution and explosion and NO artificial boundary condition.

Future Projects

✓ Parameter study of core-collapse SN ejecta (II, Ib/c), width-luminosity relation of Type Ia SN light curve
✓ Application to cosmology: systematic errors with Type Ia SNe, use of Type II for distance determinations and Hubble constant (EPM, SEAM, SCM)
✓ Application to stellar evolution and structure: Quantitative spectroscopy of SN ejecta to infer properties of progenitor (M, R, Ω, shell structure) and explosion (yields, morphology)