Coronal Solar Magnetism Observatory
Technical Note #1, Rev #1

Measurement Errors in Coronal Magnetic Field Parameters

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Summary: This note calculates the errors in the line-of-sight (LOS) magnetic field strength, the LOS velocity, and the plane-of-sky (POS) direction of the magnetic field from observations of the coronal Fe XIII emission line at 1074.7 nm in the presence of background light. The first part derives expressions for the errors in the parameters as a function of the number of detected photons for a coronal emission line, assuming photon noise. The second part presents a flux budget for the coronal observations. The note concludes with plots of the expected errors in the magnetic field parameters as a function of the coronal intensity, the background intensity, and the coronagraph aperture.

Errors in the Magnetic Field Parameters

The general problem is to determine the characteristics of the intensity profile of a spectral line given a set of observations at various wavelengths across the line. Assuming a model for the line profile which is a function of some number of parameters, we can fit the model to the observations to obtain the best fit values of the parameters. Here we are only interested in the uncertainty which will be achieved in the estimation of the spectral line parameters.

In principle, the value of each parameter can be evaluated at each measured point across the line profile, given the intensity and the partial derivative of the intensity with respect to each parameter. The error in the individual measurement of the parameter, a_i, is given from standard propagation of errors, assuming uncorrelated parameters as:

$$\sigma_{a_i}^2 = \frac{\sigma_{I_i}^2}{\left(\frac{\partial I}{\partial a_i}\right)^2}$$

Where I_i is the intensity profile observed at i points and $\sigma_{I_i}^2$ are the variances of the intensity measurements.

The best fit value of the parameter, $\langle a \rangle$, is given by the optimally weighted mean of the individual parameter measurements. The error in this best fit value is given by:

$$\sigma_{\langle a \rangle}^2 = \frac{1}{\sum W_i} \text{ , where } W_i = \frac{1}{\sigma_{a_i}^2}$$

Where W_i are the weights. (When the weighting is performed optically as by a filter, the filter bandpasses should approximate this weighting function).

For a Gaussian line profile appropriate for coronal emission lines we have:

$$I = I_0 e^{\frac{-(\lambda-\lambda_0)^2}{\Gamma^2}}$$

where I_0, λ_0, and Γ correspond to the central intensity, central wavelength, and e-folding half-width of the emission line, respectively. Requiring that the integral of the line profile over wavelength be equal to the total number of photons in the emission line, N, will assure proper normalization of the intensity into units of photons per unit wavelength:

$$\int_{-\infty}^{\infty} I_0 e^{\frac{-(\lambda-\lambda_0)^2}{\Gamma^2}} d\lambda = N$$

which results in the constraint that:

$$I_0 = \frac{N}{\Gamma\sqrt{\pi}}$$

We will now find the uncertainty in the value of λ_0. First, the partial derivative is evaluated:

$$\frac{\partial I}{\partial \lambda_0} = 2I \frac{\lambda - \lambda_0}{\Gamma}$$

Assuming photon noise, $\sigma_{I_i}^2 = I_i$, we obtain:

$$\sigma_{\lambda_0 i}^2 = \frac{I_i}{\left(2I_i \frac{\lambda_i - \lambda_0}{\Gamma^2}\right)^2} = \frac{1}{4I_i \frac{(\lambda_i - \lambda_0)^2}{\Gamma^4}}$$

and:

$$\sigma_{\langle \lambda_0 \rangle}^2 = \frac{1}{\sum_i 4I_i \frac{(\lambda_i - \lambda_0)^2}{\Gamma^4}}$$

Which, for a well sampled line profile and observations over the entire line profile, can be approximated by the integral:

$$\sigma^2_{\langle\lambda_0\rangle} = \frac{1}{\int_{-\infty}^{\infty} 4I \frac{(\lambda-\lambda_0)^2}{\Gamma^4} d\lambda}$$

Using the normalization of I_0 from above, the evaluation of the integral gives:

$$\sigma_{\langle\lambda_0\rangle} = \frac{\Gamma}{\sqrt{2N}}$$

We can estimate the value of the e-folding half width, Γ, for the FeXIII line at 1074.7 nm by assuming thermal Doppler broadening at a temperature of 1.6 MK, which results in Γ=0.0782 nm, then:

$$\sigma_{\langle\lambda_0\rangle} = \frac{55.3}{\sqrt{N}} \quad \text{(pm)}$$

Multiplying by c/λ gives the uncertainty in the Doppler shift for the 1074.7 nm line:

$$\sigma_{\langle v\rangle} = \frac{c\Gamma}{\lambda\sqrt{2N}} = \frac{15.4}{\sqrt{N}} \quad \text{(km/s)}$$

The equivalent uncertainty in the measurement of the Zeeman shift due to magnetic field can be derived assuming a shift of $6.5\cdot10^{-6}$ nm/G for the FeXIII 1074.7 nm line and unit efficiency for V modulation:

$$\sigma_{\langle B\rangle} = \frac{8.5}{\sqrt{N}} \quad \text{(kG)}$$

Measurements of Stokes Q and U can be used to determine the plane-of-sky azimuth of the magnetic field, ϕ, using the equation:

$$\phi = \frac{1}{2}\tan^{-1}\frac{U}{Q}$$

Applying error propagation to this equation gives:

$$\sigma^2_{\phi} = \left(\frac{\partial\phi}{\partial U}\right)^2 \sigma^2_U + \left(\frac{\partial\phi}{\partial Q}\right)^2 \sigma^2_Q$$

Assuming that Stokes Q and U are integrated over the emission line, and that their uncertainty is equal to the uncertainty in Stokes I, as expected, it can be shown that:

$$\sigma_\phi = \frac{1}{2p\sqrt{N}}$$

where p is the degree of linear polarization:

$$p = \frac{\sqrt{Q^2 + U^2}}{I}$$

M. Penn and collaborators[1] have given a thorough discussion of the effect of background on coronal measurements. They show that if the model for the line profile includes a background term, and where many samples of the background are taken, as in a spectrograph, then all of the above uncertainties will be multiplied by the quantity:

$$\left(1 + \frac{B}{N}\right)^{\frac{1}{2}}$$

where B is the number of photons in the background and N is the number of photons in the corona. In the case where one observation of the background is taken, then the noise in the measurement of the background is not negligible and the multiplicative factor is:

$$\left(1 + 2\frac{B}{N}\right)^{\frac{1}{2}}$$

In either case, for background dominated measurements, the errors in the magnetic field parameters scale as the square root of the background level.

COSMO Flux Budget

The measured[2] flux in the continuum at the center of the solar disk at a wavelength of 1.1μ is:

$$0.99 \cdot 10^6 \text{ erg sr}^{-1} \text{ cm}^{-2} \text{ Å}^{-1} \text{ s}^{-1}$$

combined with $1.81 \cdot 10^{-12}$ erg/photon @ 1.1μ, $4.25 \cdot 10^{10}$ ϖ^2/sr, and a 1.4 Å FWHM for the emission line gives:

$$1.80 \cdot 10^7 \text{ photons } \varpi^{-2} \text{ cm}^{-2} \text{ s}^{-1}$$

at solar disk center.

Then, the number of detected photons will be:

$$N = 1.41 \cdot 10^5\, I\, \Delta x^2\, \Delta t\, \epsilon\, D^2 \text{ photons}$$

where I is the brightness of the corona in units of millionths of the solar disk center intensity ($\mu B_\odot$), Δx is the spatial size of a resolution element in arcseconds, Δt is the integration time in seconds, ϵ is the system efficiency, and D is the telescope aperture in meters.

We can now derive the errors in the magnetic field parameters by substituting the number of photons into the above equations for the errors including the multiplicative factors for the effect of background. We assume the worst case, where the noise in the measurement of the background is not negligible and the factor of 2 appears in the expression for the effect of the background.

The following are plots for the error in the LOS magnetic field, the LOS velocity, and the POS azimuth of the magnetic field as a function of the telescope aperture, for values of the coronal intensity of 2, 5, 10, 20 and 50 $\mu B_\odot$, and for cases of the sky background of 5 and 50 $\mu B_\odot$. The coronal intensity in the Fe XIII line is about 10-30 $\mu B_\odot$ in typical loops and can sometimes exceed a brightness of 50 $\mu B_\odot$. A value for the background of 5 $\mu B_\odot$ is achieved regularly at excellent coronal sites, while a value of the background of 50 $\mu B_\odot$ corresponds to poor coronal conditions. All plots assume a resolution element of 4 x 4″ (Δx=4), an integration time of 10 minutes (Δt=600) and a system efficiency of 2% (ϵ=0.02). The POS azimuth plots assume a degree of linear polarization of 5%.

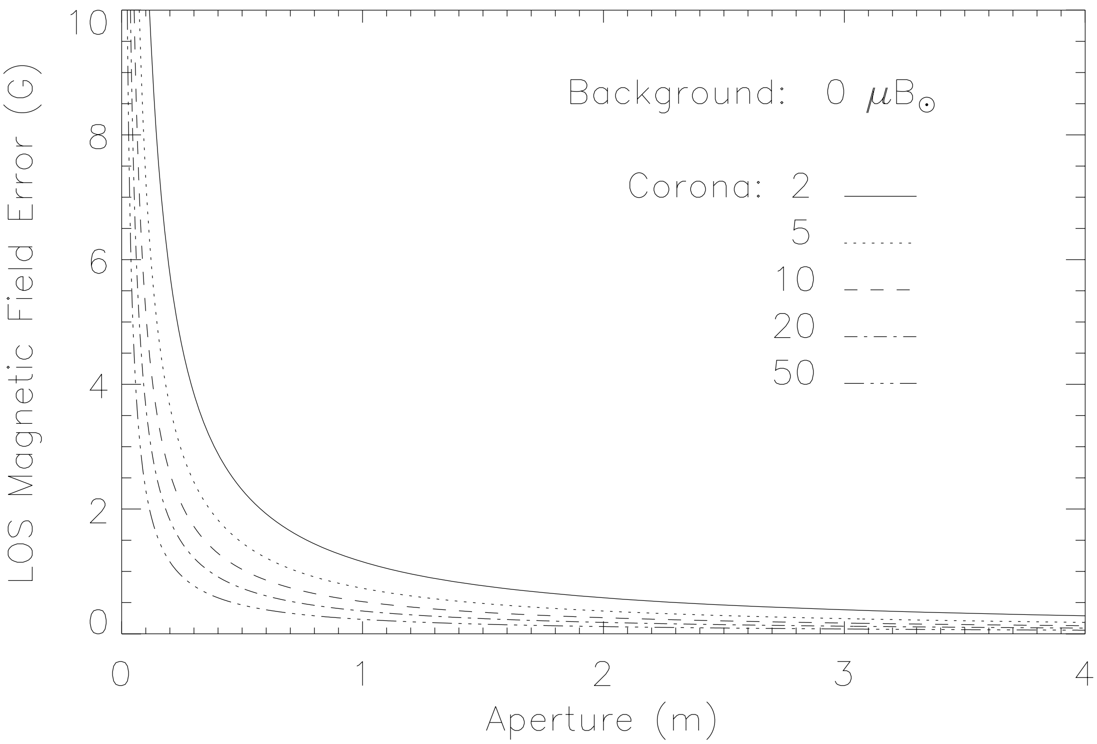
For reference, the first pair of plots show the expected error in the LOS magnetic field with no background. In that case, an error less than 1 G is achieved for coronal intensity greater than 2 $\mu B_\odot$ for apertures greater than about 1.2 m. The following plots illustrate the difficulty in measuring magnetic field parameters for faint coronal features in the presence of significant background levels.

Implications for COSMO: Given the COSMO requirement to reach 1 G error levels in the LOS magnetic field, it is clear from this analysis that it is critically important for the COSMO coronagraph to have very low levels of background light. This demands that the site be of exceptional coronal quality, and that the coronagraph be designed to insure a very low level of instrumental scattered light.

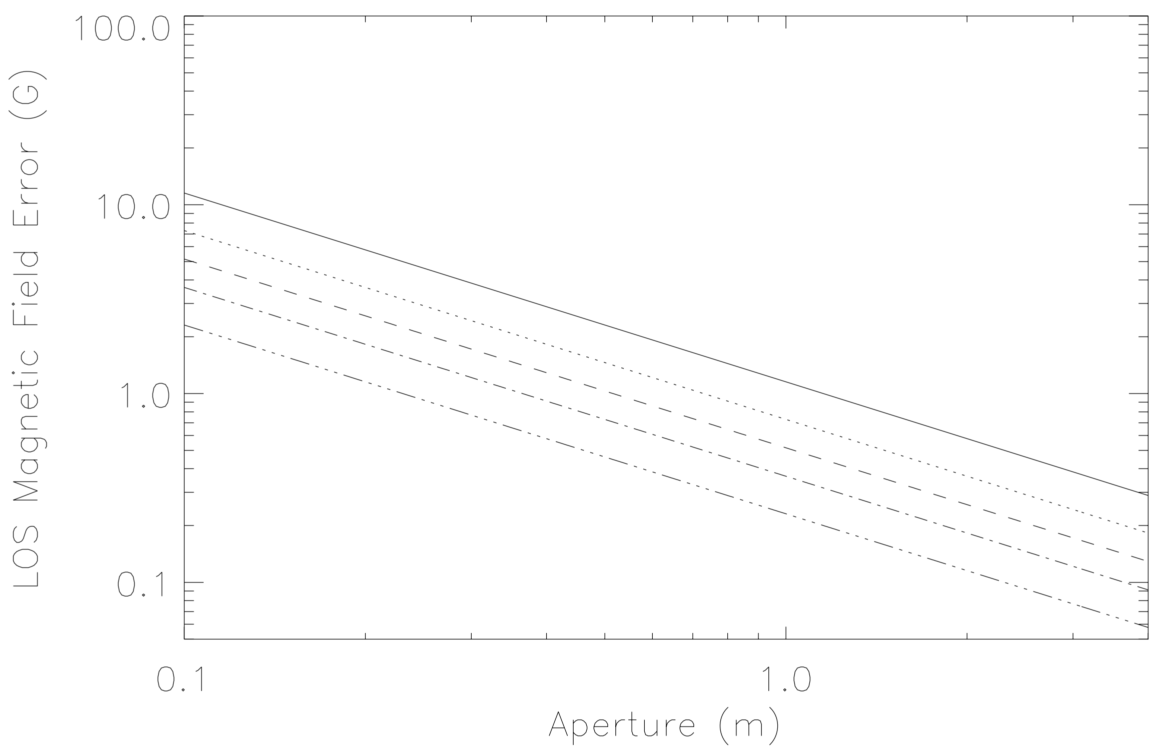
References

1. Penn, M., Lin, H., Tomczyk, S., Elmore, D., and Judge, P.G. "Background-Induced Measurement Errors of the Coronal Intensity, Density, Velocity and Magnetic Field", 2004, *Sol Phys*, **222**, 61.

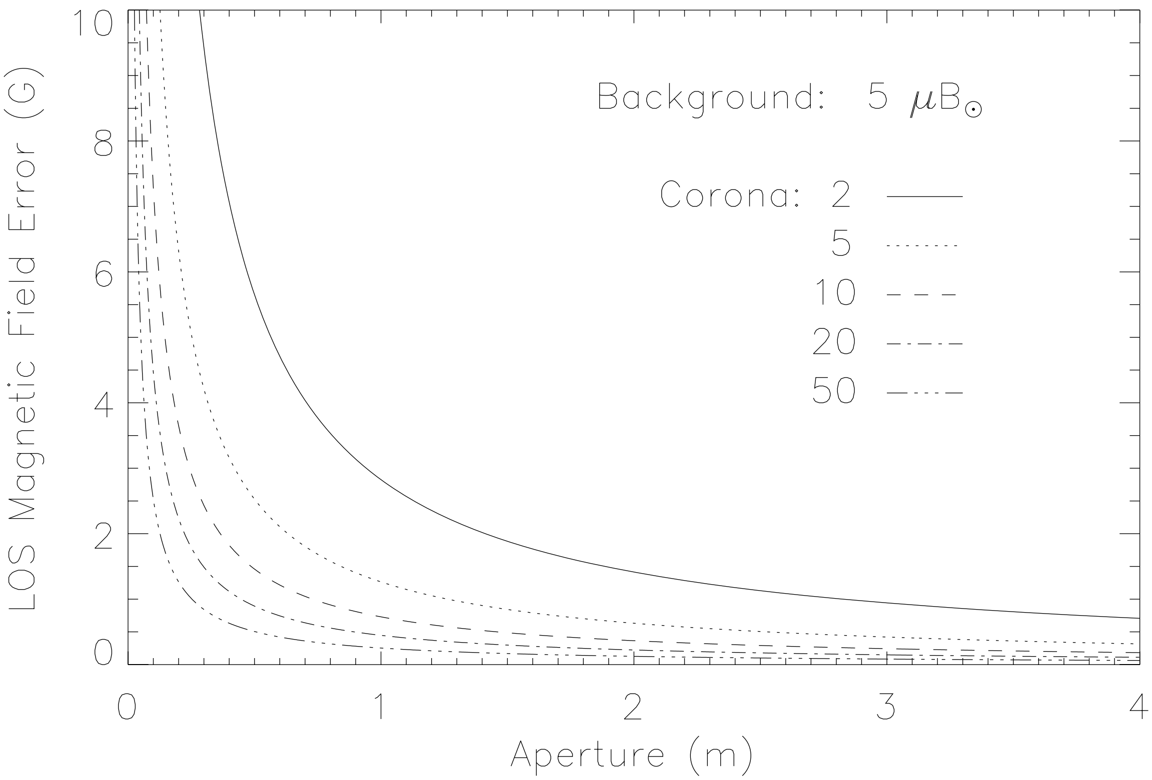
2. Allen, C.W., *Astrophysical Quantities*, 1983, p. 172.



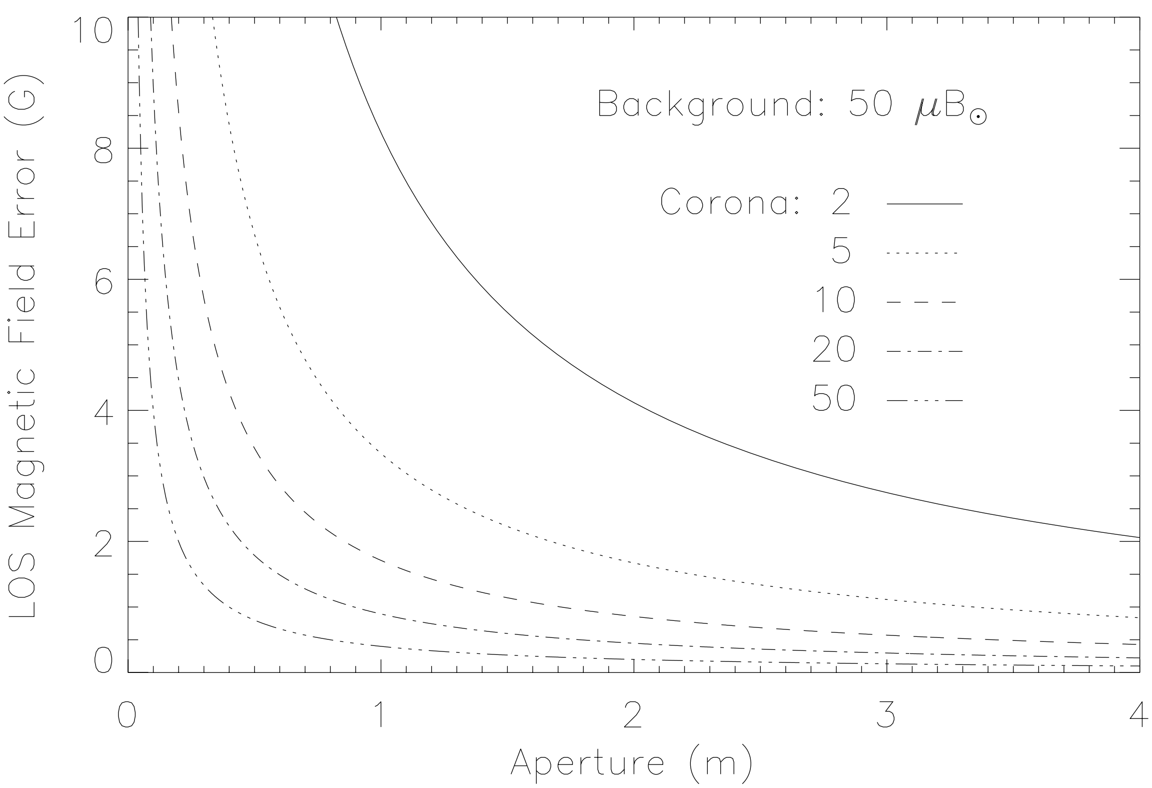
LOS Magnetic Field Error (G)
Aperture (m)
Background: 0 $\mu B_\odot$
Corona: 2
5
10
20
50



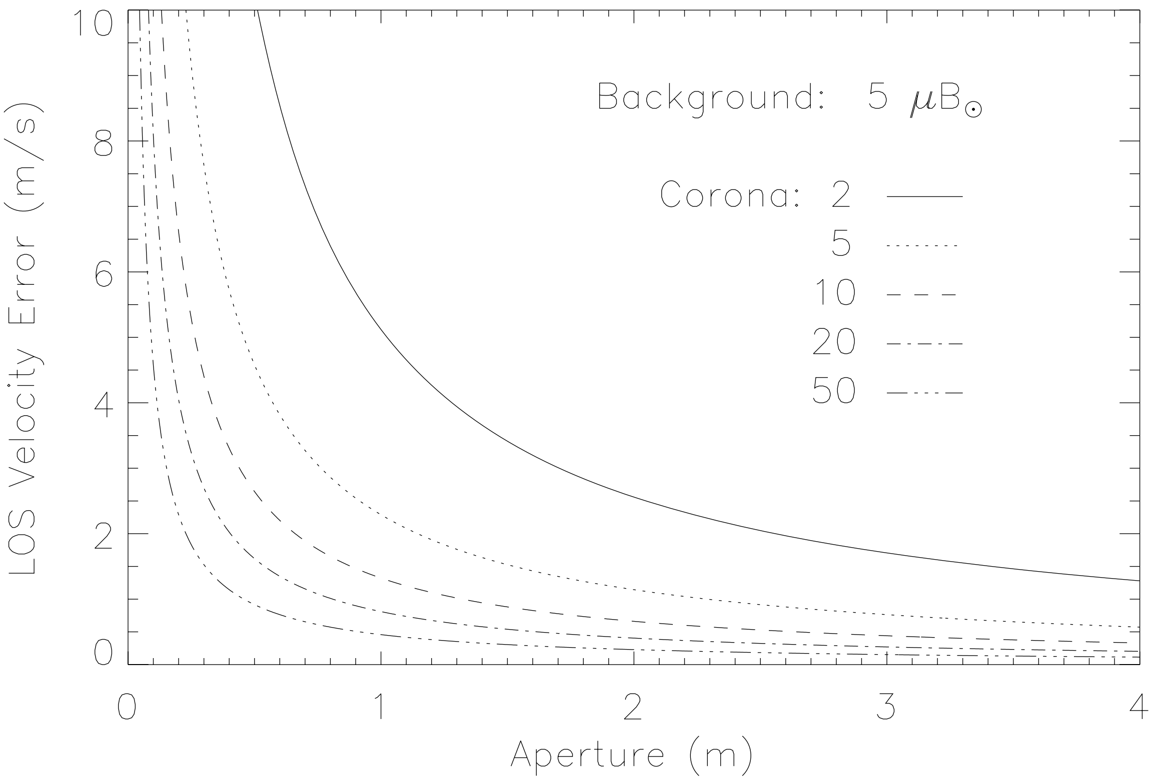
LOS Magnetic Field Error (G)
Aperture (m)

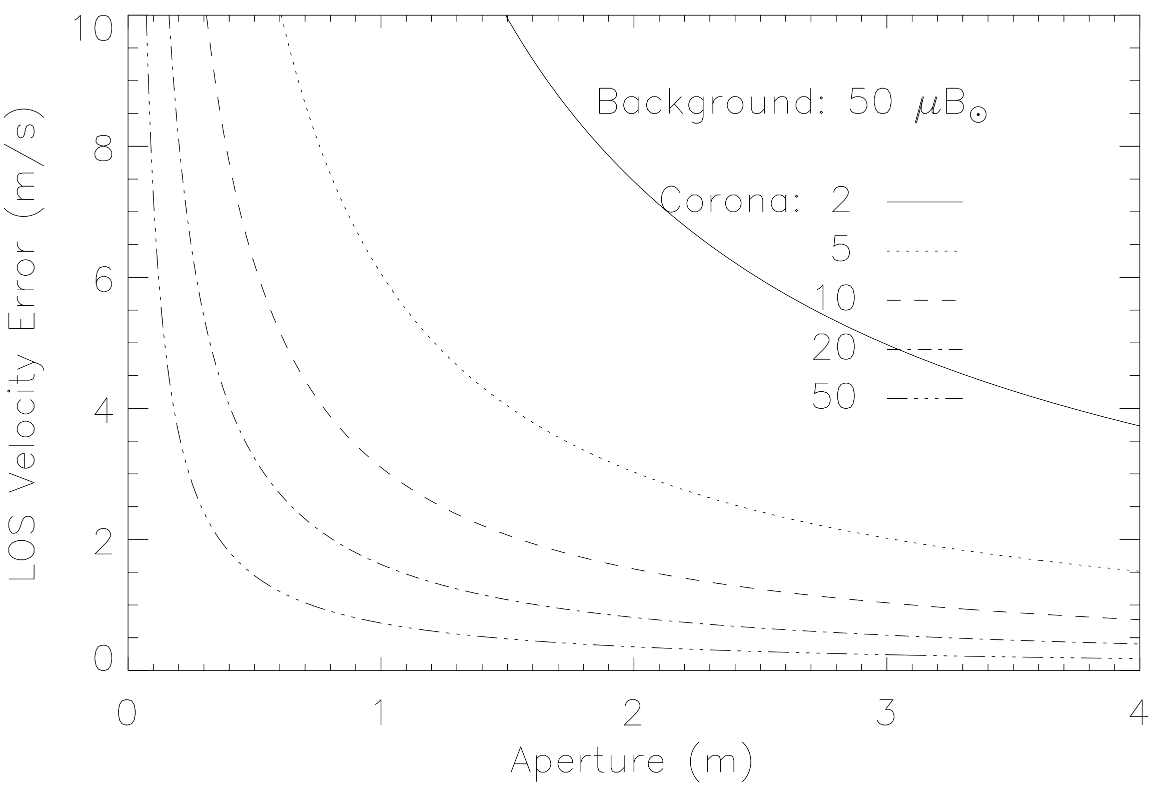


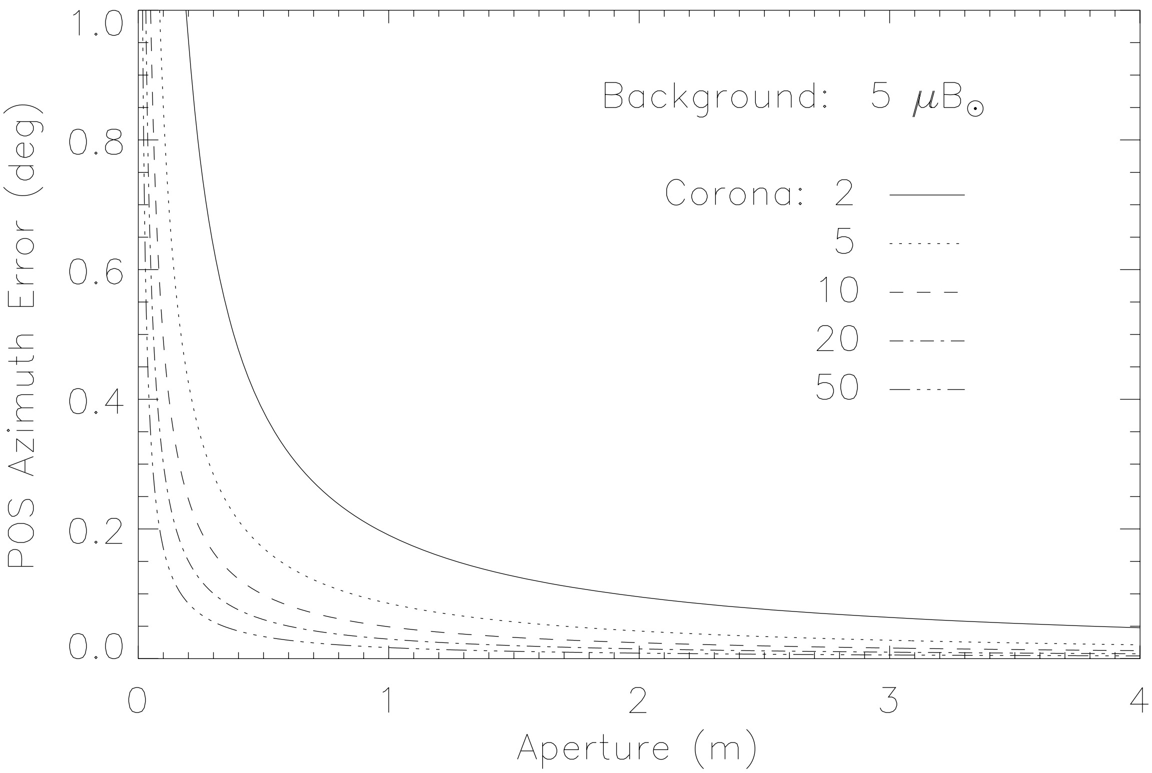
Background: 5 $\mu B_\odot$
Corona: 2
5
10
20
50
LOS Magnetic Field Error (G)
Aperture (m)
0
2
4
6
8
10
0
1
2
3
4

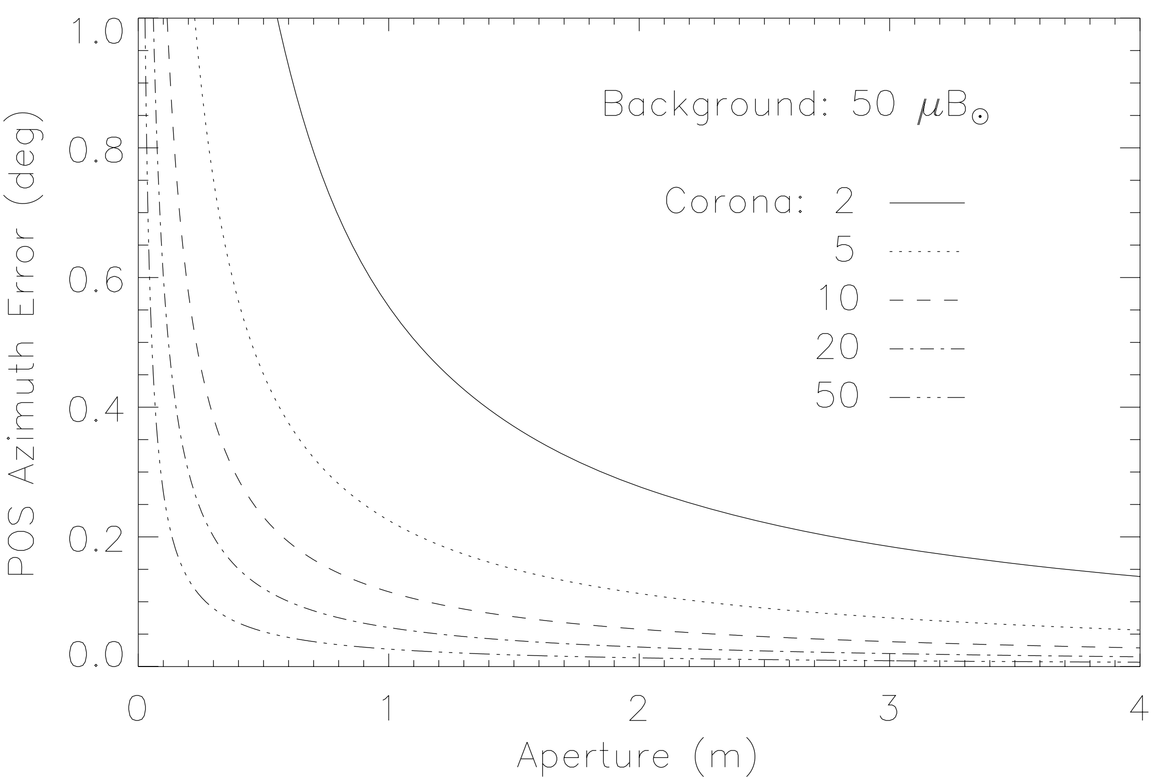


Background: 50 $\mu B_\odot$
Corona: 2
5
10
20
50
LOS Magnetic Field Error (G)
Aperture (m)
0
2
4
6
8
10
0
1
2
3
4


LOS Velocity Error (m/s)
Aperture (m)
Background: 5 $\mu B_\odot$
Corona: 2
5
10
20
50
0
1
2
3
4
0
2
4
6
8
10


LOS Velocity Error (m/s)
Aperture (m)
Background: 50 $\mu B_\odot$
Corona: 2
5
10
20
50
0
1
2
3
4
0
2
4
6
8
10


Background: 5 $\mu B_\odot$
Corona: 2
5
10
20
50
POS Azimuth Error (deg)
Aperture (m)
0.0
0.2
0.4
0.6
0.8
1.0
0
1
2
3
4


Background: 50 $\mu B_\odot$
Corona: 2
5
10
20
50
POS Azimuth Error (deg)
Aperture (m)
0.0
0.2
0.4
0.6
0.8
1.0
0
1
2
3
4