Towards a Data-Optimized Coronal Magnetic Field Model (DOC-FM): Statistical Method for Diagnosing the Coronal Magnetic Field

K. Dalmasse, D. W. Nychka, S. E. Gibson, N. Flyer & Y. Fan

NCAR, P.O. Box 3000, Boulder, CO 80307-3000, USA; *dalmasse@ucar.edu*

1. Abstract

- Solar coronal mass ejections (CMEs) and solar flares are the main drivers of space weather. Their potential impact on Earth is determined by the morphology and orientation of the magnetic structure of these events and its evolution as it propagates into the interplanetary magnetic field.
- The Data-Optimized Coronal magnetic Field Model (DOC-FM) is a collaborative project aiming at performing precise space weather forecasting by combining data-constrained modeling of the 3D solar coronal magnetic field that includes coronal polarimetric measurements such as the ones provided by the Coronal Multi-channel Polarimeter (CoMP), and magnetohydrodynamic (MHD) models of solar flares and CMEs.
- In this paper, we present a new method for **fast and efficient model-data fitting** to be used for *near-real-time* reconstructions of the 3D solar coronal magnetic field. The proposed method combines the **computation of a loss function on a sparse sample of the parameter space with function approximation** (Dalmasse *et al.*, submitted).

2. Method

1. Sparse sampling of the parameter space is performed using **Latin Hypercube Sampling** (see Fig. 1; McKay *et al.* 1979): create a sample of n-points, $(\boldsymbol{x}_i)$, out of n^d points (d=# of parameters);

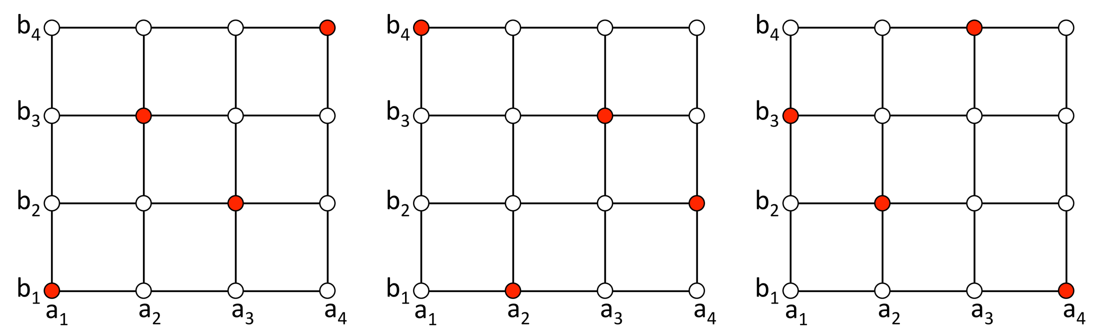

Figure 1: Examples of 2D designs generated by LHS (red points). Note that there is always only one point per row and column, which is a specific property of LHS.

2. The magnetic model is computed for all $\boldsymbol{x}_i$ of the sample and used to generate the set of predicted observations, $\mathrm{Y}(\boldsymbol{x}_i)$, which is then compared to the ground truth, Y_{GT} (either observation or synthetic test bed simulation), by means of the following loss function

$$\mathcal{L}(\boldsymbol{x}_i) = \sum_{\text{k-th pixel}} \left(\mathrm{Y}_k(\boldsymbol{x}_i) - \mathrm{Y}_k^{\mathrm{GT}}\right)^2 \qquad (1)$$

3. We use Polyharmonic Splines (Duchon 1976) to interpolate the loss function using Radial Basis Functions (RBFs; see e.g., Nychka *et al.* 2013) decomposition

$$\mathcal{L}(\boldsymbol{x}) \approx \hat{\mathcal{L}}(\boldsymbol{x}) = \sum_{j=1}^{n} a_j\ \varphi(||\boldsymbol{x} - \boldsymbol{x}_j||) + \sum_{j=1}^{\binom{p+d}{p}} b_j \psi_j(\boldsymbol{x}) \qquad (2)$$

$$\begin{aligned} \varphi(r) &= r^{2m}\ \log(r)\,,\ \text{if } d \text{ is even}\,, && (3a)\\ &= r^{2m-1}\,,\ \text{if } d \text{ is odd} && (3b)\end{aligned}$$

$\{\psi_j\}$ is a set of polynomials up to degree p in dimension d, and $m \geq 1$ is a user-specified integer.

4. Finally, the set of best-fit parameters is obtained by minimizing the RBF-interpolated loss function using the **DFPMIN IDL routine**.

3. Synthetic test bed with a 3D parameter space

- Fig. 2a presents the magnetic field of the synthetic test bed considered to validate the proposed method.
- Fig. 2b displays the three parameters used for testing the model-data fitting method. The parameters relate to the position of the flux rope in the solar corona: (co-latitude ; longitude ; rotation angle) = (θ; ϕ; Ω).
- **$(\theta_{GT}; \phi_{GT}; \Omega_{GT})$ = (45°; 90°; 30°) is chosen as the ground truth.**
- We have constructed the synthetic data set to be informative of how one can use the polarized spectral measurements associated with the Fe XIII-I (10747 Å) line observed by the Coronal Multichannel Polarimeter (CoMP; Tomczyk *et al.* 2008). All line-of-sight (LOS) integrated, synthetic images of the polarization signal are generated using the **FORWARD** package of **SolarSoft IDL** (Gibson *et al.* 2016).

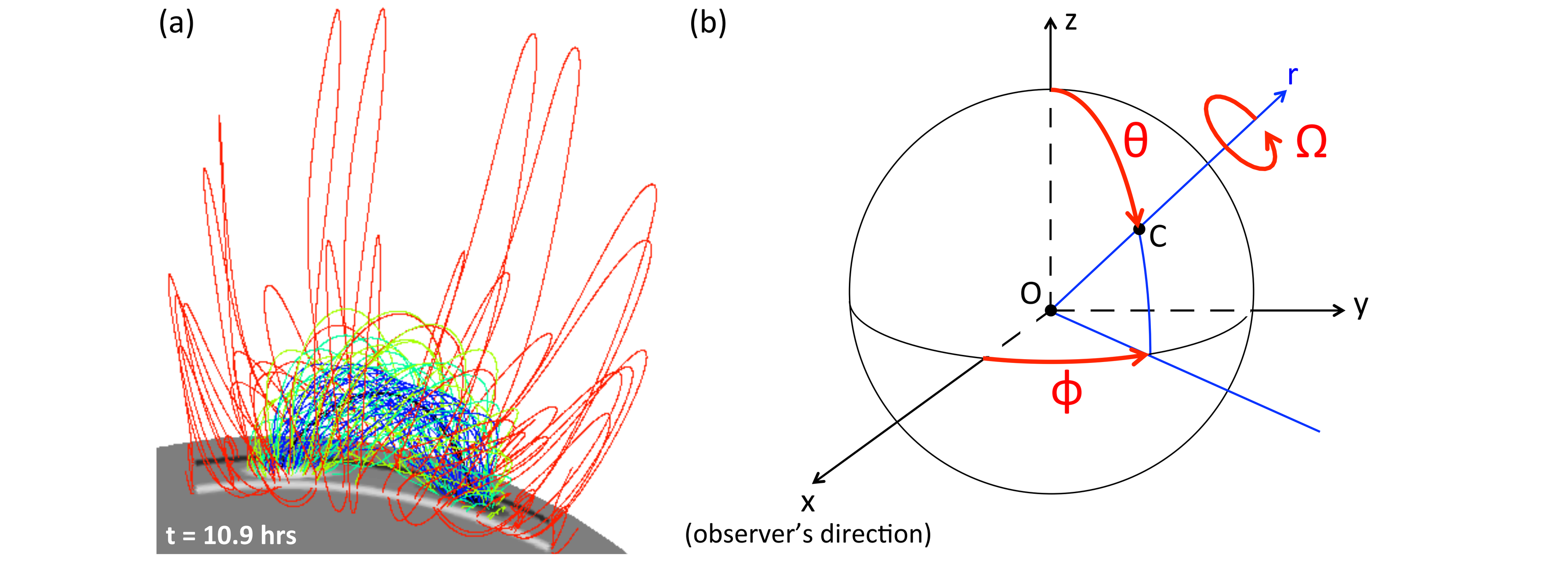

Figure 2: (a) Magnetic field of the synthetic test bed generated from an MHD simulation (Fan 2012). *Blue and green solid lines display the 3D magnetic field lines of the twisted flux tube, while the red solid lines correspond to the surrounding magnetic field. (b) Schematic describing the three parameters considered for the tests performed to validate the model-data fitting method described Section 2. Point C corresponds to the photospheric center of the numerical box containing the magnetic field. Point O is the center of the Sun. The solid black sphere highlights the solar photosphere.*

4. Results

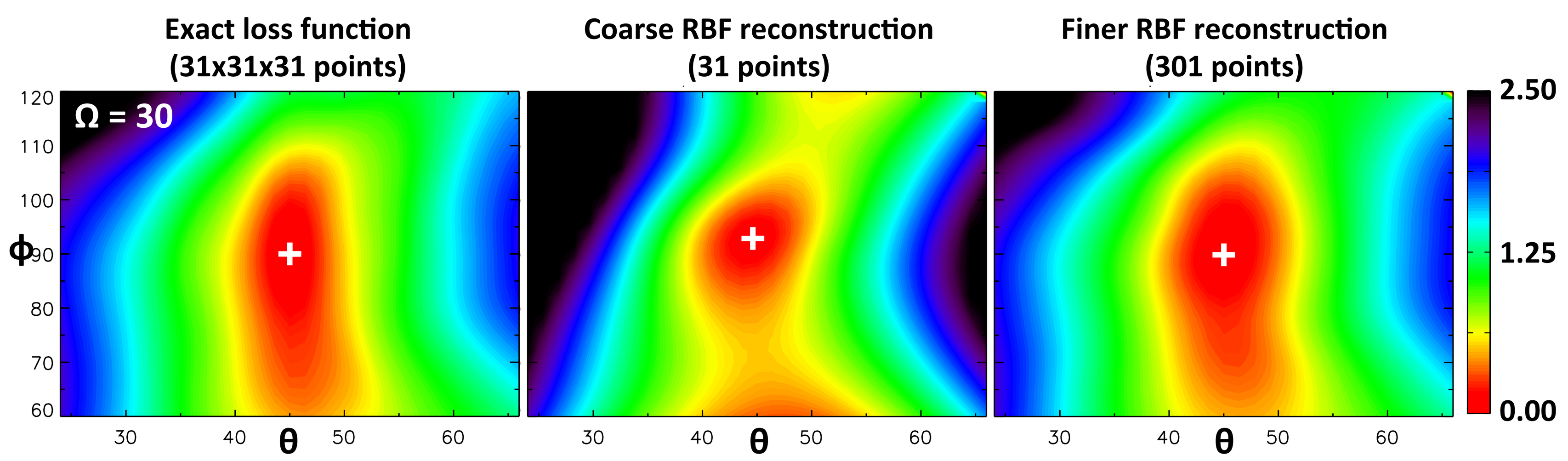

Figure 3: Examples of 2D cuts of the exact and RBF-interpolated, 3D loss function. The white "+" sign indicates the position of the minimum, and hence, best-fit parameters.

- The finer RBF reconstruction gives a very good approximation of the exact loss function (rms error ≈0.05) while using 301 points distributed in the 3D parameter space instead of 31^3, and a much better approximation than the coarse RBF reconstruction (rms error ≈0.24; Fig. 3).
- Both the coarse and finer RBF reconstructions are able to retrieve the ground-truth parameters with a good accuracy. The best-fit parameters are **(θ; ϕ; Ω)≈(44.6°; 92.9°; 30.1°)** for the coarse and **(45.0°; 89.9°; 30.3°)** for the finer reconstruction while the ground truth is **$(\theta_{GT}; \phi_{GT}; \Omega_{GT})$ = (45°; 90°; 30°)**.
- The finer RBF reconstruction (sample of 301 points in 3D parameter space) is:
  - **100 times faster** than computing the exact loss function with 31^3 points;
  - **10^5 times faster** than computing the exact loss function with 301^3 points.

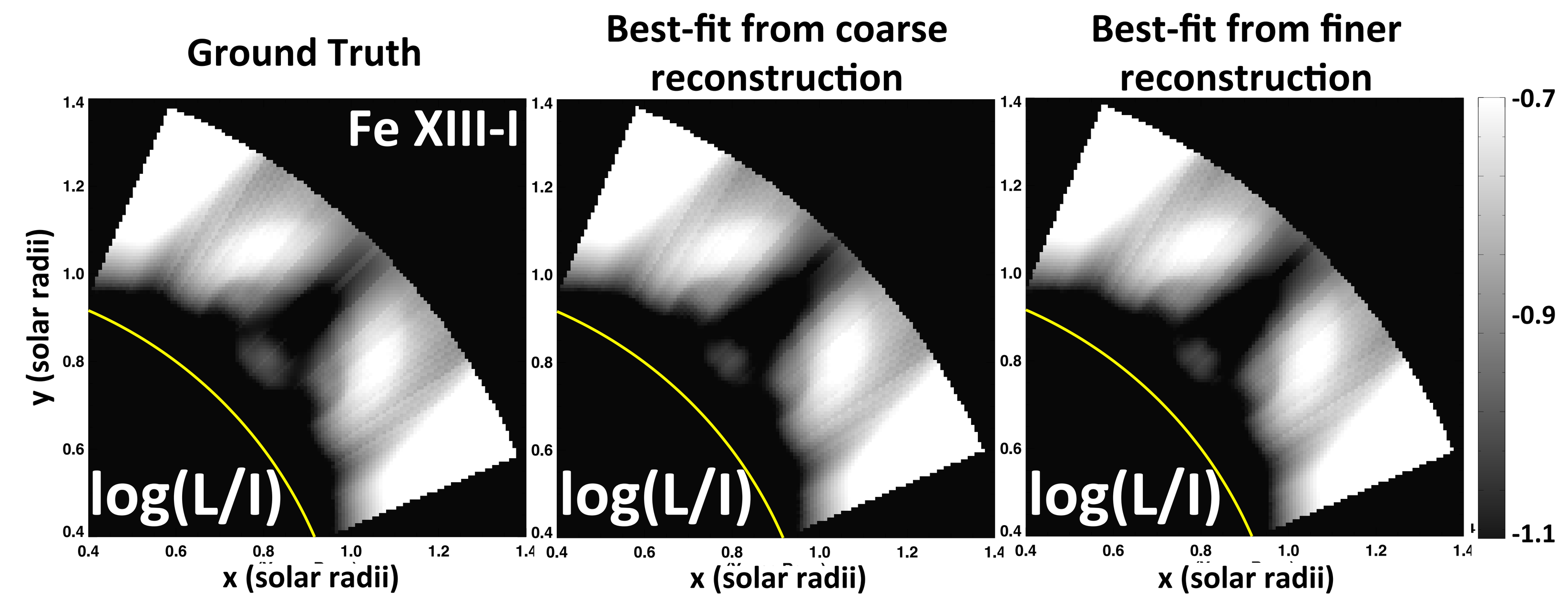

Figure 4: Fraction of linear polarization for each minimum of Fig. 3. The yellow solid line indicates the solar limb.

- Fig. 4 displays the linear polarization signal associated with the parameters of the minimum of the exact loss function (ground truth) and the RBF reconstructions of the loss function.
- The predicted polarization signal for the best-fit parameters obtained with the coarse and finer RBF reconstructions both give a good approximation of the ground truth. The error on the polarization signal is ≈0.025 for the polarization signal associated with the best-fit parameters obtained from the coarse RBF reconstruction, while it is smaller than 0.005 for the polarization signal of the best-fit parameters obtained from the finer RBF reconstruction.

5. Conclusions and work-in-progress

- We developed a new method for fast and efficient model-data fitting to be used for future *near-real-time* reconstructions of the 3D solar coronal magnetic field. Using a synthetic test bed, we showed that the method can retrieve the best-fit parameters with a good accuracy while using a reduced number of model evaluations.
- Reliability and robustness of the optimization method depend on # of RBFs (i.e., # of points per sample), size of the parameter space, and complexity of the exact loss function. An iterative implementation is being tested with preliminary results showing removal of such dependencies and fast convergence towards the ground-truth parameters (typically 4-5 iterations).
- Next step involves a synthetic test bed using the present optimization method to fit a twisted flux tube with the magnetic field model (van Ballegooijen 2004) that will be used for the DOC-FM project. Our goal is to assess the performances of our data-constrained reconstruction method of the solar coronal magnetic field in retrieving key quantities and properties for solar activity (e.g., electric currents, magnetic helicity, volumes of strong magnetic field gradients).

Acknowledgments

K.D. acknowledges funding from NCAR/CISL and NCAR/HAO, as well as support from the Air Force Office of Scientific Research under award FA9550-15-1-0030. NCAR is sponsored by the National Science Foundation.

References

Duchon, J. 1976, Lecture Notes in Mathematics, Vol. 571, pp. 85-100
Fan, Y. 2012, ApJ, 758, 60
Gibson, S. E., Kucera, T., White, S. M., et al. 2016, Frontier in Astronomy and Space Sciences, 3, 8
McKay, M. D., Beckman, R. J., and Conover, W. J. 1979, Technometrics, Vol. 21, No. 2, pp. 239-245
Nychka, D. W., Bandyopadhyay, S., Hammerling, D., et al. 2015, JCGS, 24, 2, pp. 579-599
Tomczyk, S., Card, G. L., Darnell, T., et al. 2008, Solar Physics, 247, 2, pp. 411-428
van Ballegooijen, A. A. 2004, ApJ, 612, 1, pp. 519-529